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### NUMERICAL SIMULATION OF ACOUSTIC EFFECT ON THE STABILITY OF A LEAN PREMIXED COMBUSTOR

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#### ABSTRACT

The present research is devoted to study the effect of the acoustic field on the stability of a lean premixed combustor. A theoretical model has been developed to achieve this objective. The development of this model is based upon the postulation that for low Mach numbers the flow field can be treated as a mean time dependent incompressible flow, while taking into account the effect of heat release, and an acoustic perturbations. The derived mathematical model has been solved numerically. The numerical solution uses the random vortex method, RVM, for incompressible viscous 2D flow, the simple line interface calculation, SLIC, for the propagation of the flame interface, and the solution of acoustic model. All of these methods interact to give the solution of the overall mathematical model. The results of the numerical experiments indicate that a coupling among the unsteady heat release, the fluid dynamics and acoustic field is responsible for the flashback instability. This coupling leads to increase the unsteady heat release and the acoustic perturbations.

#### INTRODUCTION

Extremely restrictive emission limits, particularly with respect to nitric oxides have led to the concept of a lean premixed combustion. The operation of a lean premixed combustor is handicapped by potentially harmful combustion instabilities [6]. Mechanisms taking place in the combustion chamber can lead to flashback instability (to be differentiated from the conventional flashback instability occurring at rich limit). A study of the previous investigations indicated the importance of the role of the acoustic field in the combustion instability of a lean premixed combustion.

A lean premixed combustor involves many sources of sound [1], acoustic waves, such as, monopole sources resulting from the unsteady volumetric

expansion associated with unsteady heat release, dipole sources resulting from the interactions between the vortex structures and the edge of the cavity or the walls of the combustor, and quadrupole sources resulting from the motion of vortex structures and the interactions between these structures. The interactions among these acoustic waves, large scale vortex structures, chemical heat release and the confined space of the combustor can result in destructive combustion instability [8]. The understanding of the mechanisms leading to combustion instability problem is therefore of great importance to the future development of lean premixed dump combustors.

Najm [10] carried out a numerical simulation to study the flashback instability. Low Mach number was considered. The reacting flow was studied with regard to the effect of the flame heat release on the overall combustor stability under different conditions of exit pressure modulation. When the exit pressure is forced at low frequency, the recirculation zone dynamics are amplified by an increase in the rate of heat release, ultimately leading to flow reversal and flame flashback. On the other hand, when the pressure is fixed, no acoustic effect, the effect of increasing the rate of heat release is to cause the damping of the circulation zone instability. These results reveal the importance of the acoustic effect on the stability of the premixed combustion.

The aim of the present research is to enhance the understanding of the effect of the acoustic field on the stability of a lean premixed combustor. To achieve this target, a mathematical model is formulated to describe the interactions among the acoustic waves, unsteady heat release and the dynamics of the vortex structures. A numerical solution for this model is presented.

## NOMENCLATURE

$p_o$	Mean pressure
$p_a$	Acoustic pressure
$p$	Total pressure
$Re_b$	Reynolds number for burnt reactants.
$Re_u$	Reynolds number for unburnt reactants
ST	Strouhal number =fD/U
$S_u$	Burning speed for unburnt reactant.
$T_u$	Temperature for unburnt reactant.
t	Time
$u_o$	Streamwise mean velocity.
$u_a$	Streamwise acoustic velocity.
u	Streamwise velocity.
$v_o$	Transverse mean velocity.
$v_a$	Transverse acoustic velocity.
v	Transverse velocity.
$x_f$	Flame location
$\rho_o$	Density of mean flow
$\rho_a$	Acoustic density
$\rho$	Density
$\phi$	Velocity potential
$\phi_a$	Acoustic velocity potential
$\Psi$	Stream function
$\omega$	Vorticity
$\delta$	Delta function

## MATHEMATICAL MODEL

A premixed model combustor can be divided into two zones namely, a combustion zone and an upstream reservoir zone as shown in Fig. (1). The mathematical model for the combustion zone can be divided into two interdependent sub-models namely, mean flow model and acoustic model. The two zones can be coupled at the inlet plane of the combustor at  $x_{min}$ . The instantaneous inlet flow rate for the combustion zone model can be calculated from the upstream reservoir model, while the pressure boundary condition for the upstream reservoir model can be provided from the combustion zone model. The following is a detailed description of the mathematical model.

### Combustion zone model

The development of the combustion zone model [1] is based upon the postulation that for low Mach numbers the majority of the flow is calculated

incompressibly taking into account the heat release from the combustion process and one can only consider acoustic properties as perturbations about the incompressible flow. Hence, the unknown variables are decomposed into time dependent mean flow and acoustic perturbations components:

$$\begin{aligned} \mathbf{u} &= \mathbf{u}_o + \mathbf{u}_a, \quad \mathbf{p} = \mathbf{p}_o + \mathbf{p}_a, \\ \rho &= \rho_o + \rho_a, \quad \mathbf{v} = \mathbf{v}_o + \mathbf{v}_a \end{aligned} \quad (1)$$

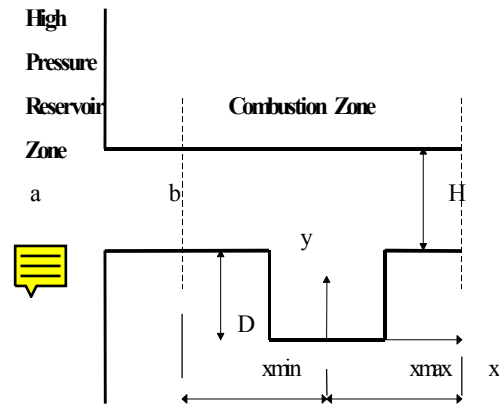


Figure (1) Diagram of the Model Combustor

### Mean flow model

The mean flow model is divided into combustion dynamics model and flow dynamics model. The derivation of the mean flow model is obtained based on the low Mach number approximation, and the thin flame thickness assumption [13].

The instantaneous relative motion of the fluid near any point is considered a combination of  $u_e$ ,  $u_p$ , and  $u_\omega$ . The two flow fields  $u_e$ , and  $u_\omega$ , volumetric expansion and rotational velocity fields respectively, contain no information about the domain boundaries while  $u_p$ , potential velocity field, provides the required correction to have  $u_o$  conform to a specified normal velocity at boundaries.

Hence, the flow dynamics model can be expressed as follows:

$$\frac{D\omega}{Dt} = \frac{\partial \omega}{\partial t} + u_o \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega \quad (2)$$

$$\omega = \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y} \quad (3)$$

$$\mathbf{u}_0 = \mathbf{v} + \mathbf{u}_\omega, \quad \mathbf{v} = \nabla\phi, \quad \mathbf{u}_\omega = (\partial_2\psi - \partial_1\psi) \quad (4)$$

$$\nabla^2\psi = -\omega \quad (5)$$

$$\nabla^2\phi = \frac{Q S_u}{T_u} \delta(x - x_f) \quad (6)$$

The equation governing the evolution of the flame surface by advection and burning is given by:

$$\frac{\partial F}{\partial t} + \mathbf{u}_u \cdot \nabla F = S_u |\nabla F| \quad (7)$$

### **Acoustic model**

Neglecting viscous and heat conduction effects, the linearized acoustic model can be expressed in the following form:

$$\mathbf{u}_a = \nabla\phi_a \quad (8)$$

$$p_a = -\rho_0 \frac{\partial\phi_a}{\partial t} \quad (9)$$

$$\frac{\partial^2\phi_a}{\partial t^2} = c^2 \nabla^2\phi_a \quad (10)$$

$$\nabla^2\phi_a = \Delta' \quad (11)$$

where  $\Delta'$  is the unsteady volumetric expansion.

### **The Upstream Reservoir Model**

The essential requirement from the upstream reservoir is to provide a flow rate into the channel. This depends on the flow picture inside the computational domain at every time step. The model of the reservoir flow involves either one of two cases, depending on the direction of the inlet flow rate. For more detail see [1].

### **NUMERICAL SCHEME**

The above mathematical model has been solved numerically to study the effect of the acoustic field on the stability of a lean premixed dump combustor. The numerical solution methods include the random vortex method, RVM [2,3 & 4], for incompressible

viscous 2D flow, the simple line interface calculation, SLIC[5], for the propagation of the flame interface, and the solution of acoustic model [9]. All of these methods interact to give the solution of the overall mathematical model.

The Random vortex method employs viscous splitting [10] of vorticity transport equation (2) in two fractional steps. In the first step, the discretization of a continuous field of vorticity into a set of vortex elements, each carrying a prescribed value of circulation that is calculated by integrating the local vorticity over a finite area element. These vortex elements are updated every time step according to the flow equations. In the second fractional step, the other part of the equation of motion, diffusion, is implemented (Chorin [3] & Ghoniem & Sherman [7]) by adding an extra displacement to the location of the vortex elements simulating their dispersion due to molecular diffusion.

Equation (7), which describes the motion of the flame front in terms of advection with velocity field and normal propagation due to burning, is integrated numerically in two fractional steps. In the first step, the flame front is transported with advection of the velocity field that is determined from the mean model and from the acoustic field model. In the second step the flame front is moved normal to itself in the direction of the unburned medium with the velocity equal to the laminar burning speed  $S_u$ . For more detail of SLIC see [1 & 11].

The solution of the acoustic mode is dependent on the postulation that the dipole and quadruple sources could be neglected with respect to monopole sources. So it is considered that the acoustic field is produced only by the unsteady volumetric expansion field associated with the unsteady heat release. If the volumetric expansion field is discretized into a number of volumetric expansion field sources coincident with the centers of the combustion cells. The strength of these sources is considered as the summation of the strength of all volumetric expansion sources within the cells. Thus the solution of the two dimensions wave equation (10) is given by lighthill [9].

The solid boundary condition, i.e. the normal acoustic velocity vanishes at the wall, is accomplished by a set of image sources, symmetrically placed with respect to the boundary plane. Both source and its image are radiating into unbounded space. The normal acoustic velocity vanishes at the upstream boundary. On the other hand, the acoustic pressure vanishes at the downstream boundary.

## RESULTS AND DISCUSSIONS

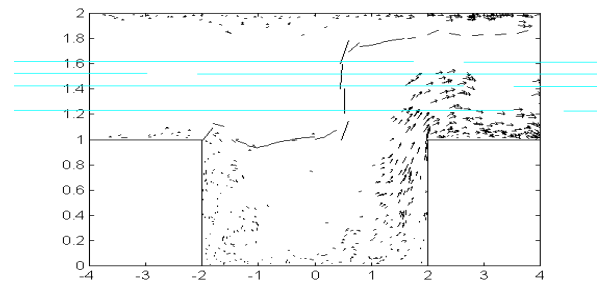
To study the role of the combustor acoustics in the mechanisms of combustion instability, two numerical experiments at the same equivalence ratio have been carried out. The first one has been devoted to study the flow features inside a lean premixed combustor taking the effect of the acoustic field on the flow field into consideration. While the second experiment has been devoted to study the flow features through a lean premixed combustor neglecting the effect of the acoustic field on the flow field.

The results have been obtained for a fixed cavity geometry,  $H/D=1.00$ ,  $x_{min}/D=-4.0$ ,  $x_{max}/D=4.00$  and aspect ratio of the cavity  $L/D=4.00$ . The following dimensionless parameters are considered to be the same for the two experiments described above: Reynolds number in reactants,  $Re_u = 10,000$ , Reynolds number in products,  $Re_b = 1250$ , normal burning speed,  $S_u / U_r = 0.05$ , upstream stagnation pressure,  $p_o / p_r = 0.4$ , and density ratio across the flame,  $\rho_u / \rho_b = 4.0$ .

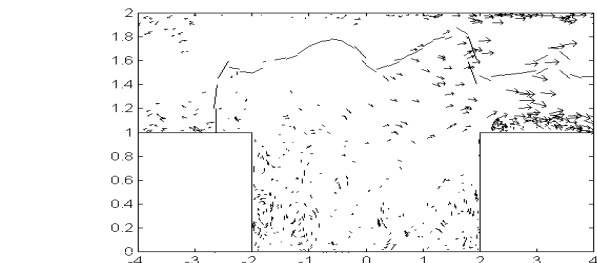
The results of these experiments indicate that a coupling between the unsteady heat release and acoustic field of the combustor is responsible for the combustion instability as shown from figure (2). Figure (2-a) shows the flow field of reacting flow in the case of neglecting the coupling between the acoustic field and the unsteady heat release while figure (2-b) shows the flow field at the same condition and the same time step in the case of considering this coupling. It is clear that the flame moves back into the inlet section in figure (2-b) which means that flashback occurs under this condition. Moreover figure (2) shows that the flame surface becomes less corrugated when the acoustic effect is neglected. This means that the length of the flame surface becomes shorter. Consequently, the heat release rate is decreased since the amount of the volume generated at the flame front per time step is proportional to the length of the flame,  $(\rho_u / \rho_b - 1)$  and  $S_u$ . And the burning speed  $S_u$  and the density ratio  $\rho_u / \rho_b$  are the same for the two experiments. Also it is clear that the coherence of the eddies becomes more stronger when the acoustic effect is considered. These results agree with what was found in [6 & 8].

The time trace and the corresponding spectra analysis of number of flamelets, volume release rate and streamwise at  $(x/D, y/D) = (0, 1.2)$ ,  $(2, 1.2)$ , are represented in figures (3, 4, 5, & 6) for the two cases under investigation. The figures show that the unsteady heat release and the amplitude of the acoustic oscillations increase when the effect of the acoustic field on the flow field is considered. Comparing figure (5) with figure (6), one can find that the dominant frequency of streamwise acoustic

velocity is located at Strouhal number,  $ST$ , approximately  $= 0.045$ , which corresponds to the first subharmonic of the natural frequency of the combustor, for the case of considering the acoustic effect with higher content of energy than the case of neglecting the effect of the acoustic field. The flame flapping phenomenon occurred at a frequency significantly lower than that of the dominant duct acoustic mode, as had been observed before [6,10]. In this case, a large vortex is shed from behind the step [1]. Najm & Ghoniem [11] refer to this phenomenon as a 'wake mode' instability, in order to distinguish it from the hydrodynamic shear layer instability. They describe it as an inherent instability of the recirculation zone that occurs at strouhal numbers of 0.05-0.2 based on step height. They also state that this phenomenon is consistent with the Rayleigh criterion, through which heat release and pressure fluctuations coupling will lead to instability.



a) Neglecting the effect of acoustic field.



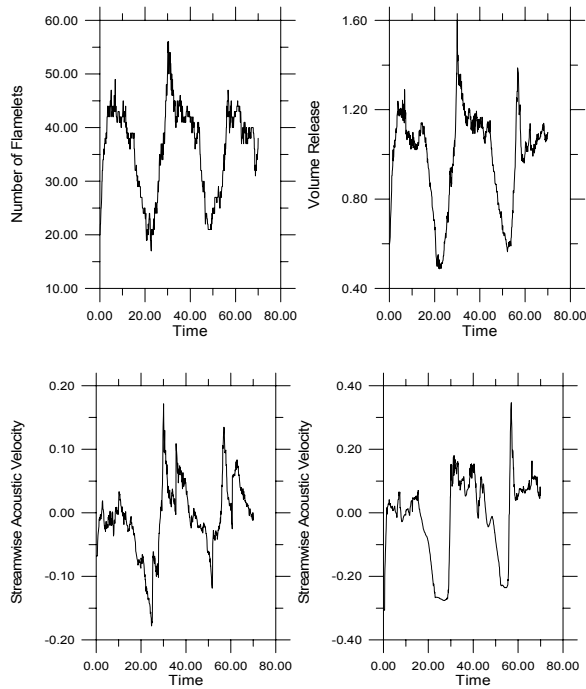
b)

b) Considering the effect of acoustic field.

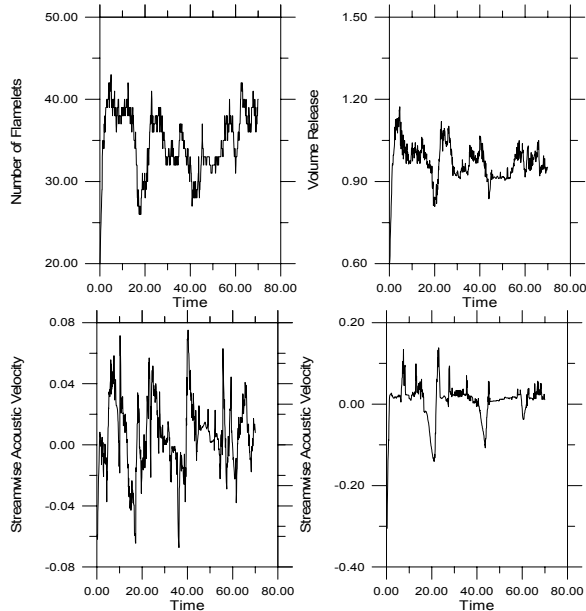
Fig.(2) Flowfield presented in terms of vortex elements and their velocity vectors and flame interface at time  $t=41$ , density ratio=4, and burning speed=0.05.

## CONCLUSION

A two dimensional mathematical model has been developed to study the effect of the acoustic field



Fig(3) Time Trace of The Number of Flamelets, Volume Release Rate and Streamwise acoustic velocity at  $(x, y) = (0, 1.2)$  and  $(2, 1.2)$  respectively at Density Ratio =4, and Burning Speed=0.05, Considering the Effect of the Acoustic Field.



Fig(4) Time Trace of The Number of Flamelets, Volume Release Rate and Streamwise acoustic velocity at  $(x, y) = (0, 1.2)$  and  $(2, 1.2)$  respectively at Density Ratio =4, and Burning Speed=0.05, Neglecting the Effect of the Acoustic Field.

on both the flow dynamics and flame dynamics assuming a thin flame and neglecting the acoustic

field due to the flow dynamics. Moreover the low Mach number approximation has been considered.

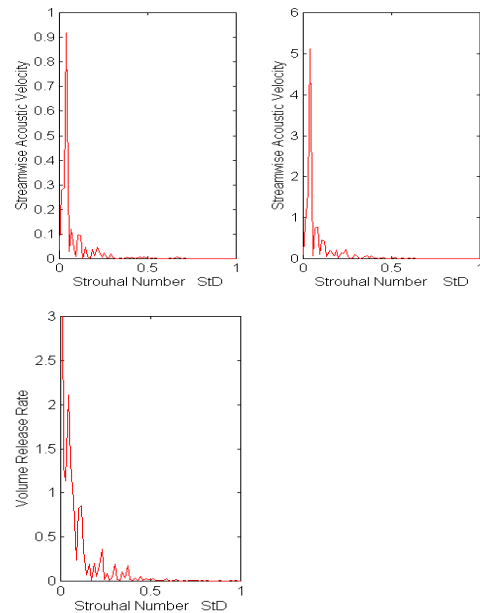


Fig.(5) Spectrum of Streamwise Acoustic Velocity at Density Ratio=4.0 Burning Speed=0.05, and at  $(x, y) = (0, 1.2)$ ,  $(2, 1.2)$ , and volume release rate respectively, Considering the Effect of Acoustic Field.

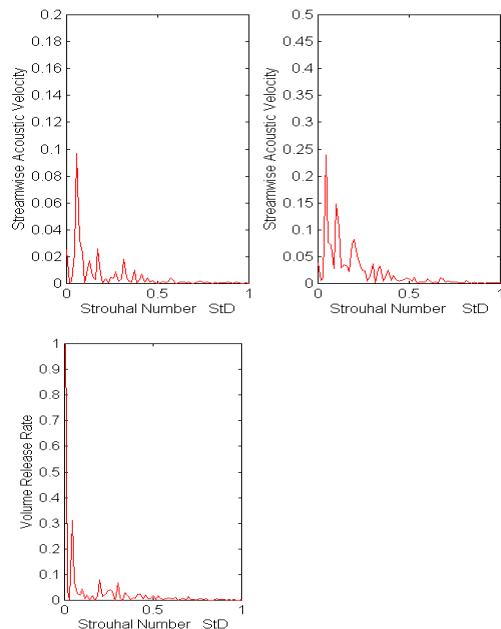


Fig.(6) Spectrum of Streamwise Acoustic Velocity at Density Ratio=4.0 Burning Speed=0.05, and at  $(x, y) = (0, 1.2)$ ,  $(2, 1.2)$ , and volume release rate respectively, Neglecting the Effect of Acoustic Field.

The solution methods include the random vortex method, RVM, for incompressible viscous 2D flow,

the simple line interface calculation, SLIC, for the propagation of the flame interface, and a method for computing the acoustic field. Numerical experiments show that flashback instability occurs when the effect of the acoustic field is considered. The heat release rate increases in the case of considering the effect of the acoustic field. Velocity fluctuations increase due to the interactions among the flow dynamics, combustion dynamics and the acoustic field.

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