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NUMERICAL STUDY OF TURBULENT FREE JETS USING *K*-ε MODEL

Ali A. Seif and Fahad M. Almohaisen
Department of Mechanical Engineering, King Saud University
P.O. Box 800, Riyadh 11421, Saudi Arabia

ABSTRACT

This paper examines computational modeling of turbulent self-preserving jets. A standard k-ε model has been considered to predict the flow field of the plane and axisymmetric jets. The equations of motion for the mean and turbulent quantities are transformed into a similarity form. The resulting closed set of ordinary differential equations has been solved numerically. The calculations show that the standard k-ε closure yields reasonable solutions for both the plane and round jets, however, with different sets of coefficients. It has been observed that the model is sensitive to any adjustment in production/destruction parameters, namely $c_{\varepsilon 1}$ and $c_{\varepsilon 2}$. Such behavior might be due to the inadequacy of the modeling of the ε-equation. Examining the k- and ε-equations at the state of equilibrium, a single set of parameters for the k-ε closure is established. The present investigation shows that this modification significantly improved the agreement of the calculations with experiment for both the plane and round jets. This leads us to believe that the present modification is an important step toward the universality of the model. Hence, we may expect that the modified turbulence model employed in the present study would be suitable for predicting more complex flows.

INTRODUCTION

A form of the k-ε model was first proposed by Harlow and Nakayma (1968). Since then, several attempts have been made to establish a set of universal model constants that lead to a good agreement between the predicted profiles and published experimental data for various shear flows. For example, Launder et. al (1973) have done extensive studies on turbulent free shear flows and have re-evaluated the k-ε model constants. Hofman (1975) examined the diffusion constants in the k- and ε-equations for channel flow. Popes (1978) analyzed the turbulent flow field for the plane and axisymmetric jets. He added an extra term in the dissipation rate equation for the round jet case to account for vortex stretching effect. Hassid (1980) predicted momentumless wake using the k-ε closure and suggested another set of constants. Hanjalic and Launder (1980) proposed a modified dissipation rate equation and added second order terms in the production of the k- and ε-equations. They used this modified model to predict plane and round turbulent jets. Their conclusion was that further improvements of the model would widen its application to a larger range of turbulent flows. Seif and Taulbee (1983, 1984) showed that the k-ε model yields solutions that smoothly approach zero at the outer edge of the shear layer if $\sigma_\varepsilon = 2\ \sigma_k$. Based on this, other model parameters have to be changed to obtain reasonable solutions. Malin (1988) used the standard k-ε model to primarily predict the flow field of turbulent radial jet. He also paid attention to the plane and round jets in view of their importance as benchmarks for turbulence-model development. He concluded that a modified production term in the k-equation improved the performance of the model in predicting the spreading rate of the radial jet.

Moreover, recently Chukkapall and Turan (1995) used an improved k-ε model to predict complex, adverse pressure gradient turbulent diffuser flows. Their results indicated that there is a need for a better modeling of the ε-equation. Zhang et al. (1996) proposed a new Low-Reynolds number k-ε model to simulate turbulent flow over smooth and rough surfaces. They reported that they used different values for the model constants (c_μ, $c_{\varepsilon 1}$, $c_{\varepsilon 2}$, σ_k and σ_ε) and model functions for different flows in order to obtain reasonable solutions. Lai and Yang (1997) made comparisons of the performance of four k-ε models, namely, the standard k-ε model and three Low-Reynolds number k-ε models. They investigated the flow field of developing and fully developed turbulent pipe flows. They found that for developing pipe flows the region of turbulence suppression predicted by the three Low-Reynolds number model is much more extensive than for fully developed flow; whereas the standard k-ε model has only predicted turbulence enhancement.

In the present study, an attempt is made to examine the k- and ε-equations based on experimental data for self-preserving turbulent jets. The analysis indicates that $\sigma_\varepsilon = 2\sigma_k$ and that $c_{\varepsilon 1}$ is a function of $c_{\varepsilon 2}$, production and dissipation. It was found that the present modified k-ε model with a variable $c_{\varepsilon 1}$ predicted the plane and axisymmetric jets with the same set of constants equally well.

Equations of Motion

For fully developed turbulent free shear flows, it is assumed that: the motion is steady, the fluid viscosity is negligible, derivatives with respect to x are negligible as compared to those with respect to y except for the convective terms, the flow field is self-preserving ($k^2/\varepsilon \sim$ const) and the motion is either two dimensional ($\frac{\partial}{\partial z} = 0$) or axisymmetric ($\frac{\partial}{\partial \theta} = 0$). Based on these assumptions, retaining only first order terms and adopting the turbulence eddy viscosity hypotheses, the final form of the k-ε closure is given by (Seif and Taulbee 1984);

Continuity:

$$\frac{\partial U}{\partial x} + \frac{1}{y^n}\frac{\partial}{\partial y}\left(y^n V\right) = 0 \qquad (1)$$

Streamwise momentum:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{1}{y^n}\frac{\partial}{\partial y}\left(y^n \nu_t \frac{\partial U}{\partial y}\right) \qquad (2)$$

k-equations

$$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial y} = \frac{1}{y^n}\frac{\partial}{\partial y}\left(\frac{y^n \nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + \nu_t\left(\frac{\partial U}{\partial y}\right)^2 - \varepsilon \qquad (3)$$

ε-equation

$$U\frac{\partial \varepsilon}{\partial x} + V\frac{\partial \varepsilon}{\partial y} = \frac{1}{y^n}\frac{\partial}{\partial y}\left(y^n \frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + c_{\varepsilon 1}\frac{\varepsilon}{k}\nu_t\left(\frac{\partial U}{\partial y}\right)^2 - c_{\varepsilon 2}\frac{\varepsilon^2}{k} \qquad (4)$$

where n = 0 for the plane jet case and n = 1 for the axisymmetric jet, ν_t is the turbulence eddy viscosity which is assumed to be constant and is given by;

$$\nu_t = c_\mu \frac{k^2}{\varepsilon} \qquad (5)$$

and c_μ, $c_{\varepsilon 1}$, $c_{\varepsilon 2}$, σ_k and σ_ε are model constants.

PROPOSED MODEL COEFFICIENTS

Near the outer edge of free shear layers, such as turbulent jets, the turbulent flow field reaches equilibrium. That is; production = dissipation, consequently the convection is balanced by the diffusion.

Based on this fact we have from equations (3) and (4) the following equations;

$$\frac{\partial}{\partial x}\left(y^n U k\right) + \frac{\partial}{\partial y}\left(y^n V k\right) = \frac{\partial}{\partial y}\left(y^n \frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) \qquad (6)$$

$$\frac{\partial}{\partial x}\left(y^n U \varepsilon\right) + \frac{\partial}{\partial y}\left(y^n V \varepsilon\right) = \frac{\partial}{\partial y}\left(y^n \frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) \qquad (7)$$

$$P_k = \varepsilon \qquad (8)$$

$$c_{\varepsilon 1} P_k = c_{\varepsilon 2}\varepsilon \qquad (9)$$

where P_k is the production of the turbulent kinetic energy which is given by;

$$P_k = \nu_t\left(\frac{\partial U}{\partial y}\right)^2 \qquad (10)$$

According to the thin shear layer assumption $\frac{\partial}{\partial x} << \frac{\partial}{\partial y}$ in the entire flow field. Furthermore, near the outer edge of the turbulent jet the axial component of the mean velocity approaches zero, while the lateral component approaches constant value for the plane jet and zero for the round jet case. Based on these arguments, the first terms in equations (6) and (7) are negligibly small as compared to the other convective terms, hence they can be neglected. Then equations (6) and (7) can be integrated to give:

$$\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y} - kV = 0 \qquad (11)$$

$$\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y} - \varepsilon V = 0 \qquad (12)$$

where it can be shown that the integration constants are zero in this case.

Dividing equation (11) by k and equation (12) by ε and combining the two equations yield;

$$\frac{\sigma_\varepsilon}{\sigma_k}\frac{1}{k}\frac{\partial k}{\partial y} - \frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial y} = 0 \qquad (13)$$

Direct integration of equation (13) gives:

$$\frac{k^{\sigma_\varepsilon/\sigma_k}}{\varepsilon} = \text{constant} \qquad (14)$$

Based on dimensional ground, and the turbulence eddy viscosity hypothesis (equation 5) we must have;

$$\frac{\sigma_\varepsilon}{\sigma_k} = 2 \qquad (15)$$

According to Seif and Taulbee (1984), with $\sigma_k = 1$, $\sigma_\varepsilon = 2$ and $c_\mu = 0.09$, the k-ε closure yields a reasonable solution for turbulent self-preserving jets with $c_{\varepsilon 1} = 1.45$ for the plane jet calculation and with $c_{\varepsilon 1} = 1.55$ for the axisymmetric case, while $c_{\varepsilon 2} = 2$ for both cases. Hassid (1980), also stated that unless $\sigma_\varepsilon = 2$, a self-similar solution for momentum less wake can not be obtained. So, the choice of $\sigma_k = 1$, as proposed in the standard k-ε closure, fixes $\sigma_\varepsilon = 2$. It is believed that the inadequacy of the modeling of the ε-equation stems principally from the form of the production/ destruction term in which $c_{\varepsilon 1}$ and $c_{\varepsilon 2}$ are assumed to be constants.

Lumley and Khaja-Nouri (1974) assumed that the production/destruction term in the ε-equation can be expressed as a single term, namely $\psi(\varepsilon^2/2k)$, in which the dimensionless invariant ψ is a scalar function that depends on the local state of turbulence. Hence, it can be expanded in terms of the invariants of the anisotropic stress tensor and the mean velocity gradient to give two parameters that are equivalent to $c_{\varepsilon 1}$ and $c_{\varepsilon 2}$ where both $c_{\varepsilon 1}$ and $c_{\varepsilon 2}$ are assumed to be functions of turbulence. They argued that the production of dissipation must be related to the anisotropy of the Reynolds stress tensors, turbulence Reynolds number, velocity gradient and viscous dissipation. Now since the k-ε model does not provide detailed information about the invariants that can be formed on which $c_{\varepsilon 1}$ and $c_{\varepsilon 2}$ might depend, we may instead use the ratio P_k/ε as functional dependence for these parameters. Note that for high turbulence Reynolds number flows, P_k/ε provides nearly the same effect that can be generated by the invariants sited above. Examining the experimental data of self-preserving jets, we see that the production of the turbulent kinetic energy, k, increases considerably from a minimum at the axis of symmetry to a maximum around half way toward the outer-edge and then decreases to zero at the outer edge of the shear layer. On the other hand, the dissipation decreases from a maximum (near the axis of symmetry) to zero at the outer edge of the jets. Hence the ratio P_k/ε increases from a minimum at $\eta = 0$ approaching unity at the state of equilibrium near the outer edge of the jets. On the other hand, a constant value for $c_{\varepsilon 1}$ that has been used in previous works is in the range $1.32 \leq c_{\varepsilon 1} \leq 1.55$. This suggests that P_k/ε might be the proper mechanism that allows $c_{\varepsilon 1}$ to increase from a minimum at the axis of symmetry of the turbulent jet to a maximum of $c_{\varepsilon 1} = c_{\varepsilon 2}$ at the state of equilibrium. A functional form for $c_{\varepsilon 1}$ that satisfies these conditions can be obtained by combining, equations (8) and (9) leading to the following equation:

$$c_{\varepsilon 1} = 1 + \frac{P_k}{\varepsilon}\left(c_{\varepsilon 2} - 1\right) \qquad (16)$$

Note that when $P_k = \varepsilon$ we have $c_{\varepsilon 1} = c_{\varepsilon 2}$ which satisfies the equilibrium conditions. Evaluating equation (16) for $c_{\varepsilon 1}$ using the data of Panchapakesan and Lumley (1993) and Hussein et al. (1994) across the turbulent jet we found that the values of $c_{\varepsilon 1}$ are in the range $1.24 \leq c_{\varepsilon 1} \leq c_{\varepsilon 2}$. This range of $c_{\varepsilon 1}$ covers almost all the suggested constant values for $c_{\varepsilon 1}$ by numerous authors that are referenced in this paper, (see also Taulbee (1989)).

Chen and Jaw (1998) examined the k- and ε-equations based on homogeneous pure shear flow experiment and proposed a similar form of equation (16). However, when their form for $c_{\varepsilon 1}$ was tested with reference to the axisymmetric jet's data it did not give realistic values for $c_{\varepsilon 1}$. In the present study it is found that equations (15) and (16) yield excellent results in predicting the plane and the round jets equally well with no need to tune any of the model constants. On the other hand, $c_{\varepsilon 2}$ has been obtained from the data of isotropic decaying turbulence (Reynolds (1976), Taulbee (1989)). The data indicates that $c_{\varepsilon 2}$ is in the range $1.4 \leq c_{\varepsilon 2} \leq 2$, where 2 is the asymptotic value of $c_{\varepsilon 2}$ for the case of high turbulence Reynolds number flows. Almohaisen (2001) used different values for $c_{\varepsilon 2}$ in his recent study of turbulent free jets and reported that $c_{\varepsilon 2} = 2$ gives fairly good results for turbulent jet calculations.

SIMILARITY FORMULATION

An important feature of turbulent free shear flows is their tendency to become self-similar (at least in principle) after certain development region downstream. For self-preservation of turbulent free jets, the turbulence motion must be characterized by a single length and velocity scales. Hence all of the turbulent quantities must be so related such that:

$$\frac{\overline{uv}}{U_c^2} = f\left(\frac{y}{\ell}, \frac{U}{U_c}\right) \qquad (17)$$

Where $\overline{uv}$ is the turbulent shear stress, U is the downstream mean velocity, U_c is the centerline mean velocity and ℓ is a turbulence characteristic length scale. Hence the mean and turbulent quantities may be normalized as follows;

$$U = U_c(x) f(\eta), V = U_c(x) h(\eta), \overline{uv} = U_c^2(x)\, g(\eta),$$
$$k = U_c^2(x) k(\eta), \varepsilon = U_c^3(x)\, E(\eta) / \ell(x)$$

and $\eta = y/\ell(x)$ for plane jet and $\eta = r/\ell(x)$ for the round jet. Where f (η), h (η), g (η), k (η) and E(η) are the normalized mean axial velocity, lateral component of the mean velocity, turbulent shear stress, turbulent kinetic energy and turbulent dissipation respectively and $\ell(x)$ is a characteristic length scale, which is usually taken as the distance from the line of symmetry of the jet to a point where $U/U_c = {}^1/_2$. Based on these assumptions and using the continuity equation to eliminate the lateral component mean velocity, V, equations (2 - 4) are transformed into a self-similar form yielding;

$$\left(\eta^n \nu_t f'\right)' + \frac{n+1}{2}\left(G f' + \eta^n f^2\right) = 0 \qquad (18)$$

$$\left(\eta^n \frac{\nu_t}{\sigma_k} k'\right)' + \frac{n+1}{2}\left(Gk' + 2\eta^n kf\right) + \eta^n \nu_t f'^2 - \eta_n E = 0 \qquad (19)$$

$$\left(\eta^n \frac{\nu_t}{\sigma_e} E'\right)' + \frac{n+1}{2}\left(GE' + (5-n)\eta^n Ef\right) + c_{\varepsilon 1}\nu_t f'^2 \frac{E}{k}\eta^n - \eta^n c_{\varepsilon 2}\frac{E^2}{k} = 0 \qquad (20)$$

where G is obtained by integrating the continuity equation and is given by;

$$G(\eta) = \int_o^\eta \eta^n f(\eta)\, d\eta \qquad (21)$$

The prime denotes differentiation with respect to η and n = 0 for the plane jet and n = 1 for the round jet case. From similarity analysis, the spreading and decay rates of the jet are given by (Seif and Taulbee (1984));

$$\ell(x) = ax \qquad (22)$$

$$U_c(x) = c\, x^{-(n+1)/2} \qquad (23)$$

Where a and c are the spreading and decay rate constants of the turbulent jet respectively. The constants a and c are to be determined from the numerical solution of the turbulent jets by imposing the momentum integral constraint (George et al. (1988)). Now the governing equations (18 - 20) along with the integral equation (21) constitute a closed set which can be solved numerically. The boundary conditions that can be imposed on the system of equations is that the flow is symmetric about the centerline, so that the gradients of k and E are zero at $\eta = 0$ and the normalized centerline mean velocity is equal unity at $\eta = 0$. At the outer edge of the shear layer ($\eta \to \infty$) f, k and E approach zero values. Note that the values of k and E at the centerline ($\eta = 0$) are not known. However, when equations (18-20) are evaluated at the centerline we obtain the following system of equations;

$$2\nu_t f'' + f^2 = 0 \qquad (24)$$

$$(n+1)\frac{\nu_t}{\sigma_k} k'' + (n+1)k\, f - E = 0 \qquad (25)$$

$$(n+1)\frac{\nu_t}{\sigma_\varepsilon} E'' + (n+1)\left(\frac{5-n}{2}\right) Ef = 0 \qquad (26)$$

Hence the values of k and E at $\eta = 0$ will be determined from equations (24) - (26) at each cycle of iteration.

NUMERICAL SOLUTION

Numerical solution of the system of equations (18)-(20) for both the plane and round jets is not a trivial matter. Since the outcome of the solution is sensitive to how the various terms are treated in the computations. In the present computations all terms that involve the gradient of k and ε are treated implicitly, while the other terms are treated explicitly as source terms. This numerical treatment seems to work well for this kind of ordinary non-linear and coupled differential equation. Using a finite-difference technique, the equations are transformed into a system of algebraic equations, which are solved simultaneously using Newton's method (Almohaisen (2001)). An iterative procedure was set up until convergence to a given tolerance is achieved. With a tolerance of 0.0001, the solutions reached a convergence after 27 iteration cycles for the plane jet and 65 iteration cycles for the round jet case.

RESULTS AND DISCUSSIONS

In the present calculation no attempt was made to tune any of the k-ε model constants to fit either the plane or the axisymmetric jet data. A similarity solution using the present k-ε closure with $\sigma_k = 1$, $\sigma_\varepsilon = 2$, $c_\mu = 0.09$ and $c_{\varepsilon 2} = 2$ along with the proposed variable form of $c_{\varepsilon 1}$ (equation (16)) were obtained. To display the quality of the agreement, the results are presented together with some of the best available data set for turbulent jets. For the plane jets results, the comparisons are made with reference to the experimental work of Gutmark and Wygnanski (1976) and Bradbury (1965). For the round jet, the most recent comprehensive measurements are those of Panchapakesan and Lumley (1993), and Hussein et al. (1994). Figure 1 shows a plot of the normalized mean axial velocity for the plane jet. The agreement with the experimental data can be considered fairly good except at the outer edge where the computations allow the velocity profile to tail off as $\eta \to \infty$. The Reynolds stress profile for the plane jet is calculated using the eddy viscosity hypothesis ($\overline{uv} = \nu_t \frac{\partial U}{\partial y}$) and plotted in figure 2. The agreement with the experimental data is perfect in the region $0<\eta<0.14$. The discrepancy between calculation and data appears to be near the outer edge of the shear layer. One of the likely reasons is that the calculations do not account for the second order production terms. The other reason is that estimate of $\overline{uv}$ at the outer edge by the data is not quite accurate, because near the outer edge the relative turbulence intensity is very high. As a consequence, the measurements become increasingly unreliable toward the edge. On the other hand, the agreement of the calculated kinetic energy profile with plane jet data is perfect as displayed in Figure 3. Figures 4 and 5 show the calculated kinetic energy and dissipation rate budget respectively across the plane jet. The results show that the k- and ε-equations are well balanced. Figures 6 and 13 shows the calculated variation of the parameter $c_{\varepsilon 1}$ across the plane and round jet respectively. Note that the results of figure 6 and 13 are similar, but not necessarily identical. It can be seen from both figures that $c_{\varepsilon 1}$ varies considerably in the range $0<\eta<0.08$ and approaches an asymptotic value toward $c_{\varepsilon 1} = c_{\varepsilon 2}$ at equilibrium. This range of $c_{\varepsilon 1}$ covers almost all suggested constant values for this parameter in previous studies as stated earlier. Note that $P_k = 0$ at $\eta = 0$, since the present calculation does not account for second order production terms. The calculated eddy viscosity ($\nu_t = c_\mu \frac{k^2}{\varepsilon}$) seems to be nearly constant across the entire shear layer for both plane and round jet as displayed in figures 7 and 14. Note that $\nu_t \approx 0.0042$ for plane jet calculation and $\nu_t \approx 0.0024$ for the axisymmetric jet case.

The overall results for the turbulent axisymmetric jet can be considered excellent. Figure 8 shows a plot of the mean axial velocity profile which is in good agreement with the measured data. The Reynolds stress profile for the round jet seems to be in good agreement with data as shown in Figure 9. Figure 10 shows a comparison of the calculated kinetic energy profile with round jet data. The agreement can be considered excellent within the accuracy of the data themselves. Figures 11 and 12 show the turbulent kinetic energy and dissipation rate budget across the round jet, where it can be seen that both k- and ε-equations are well balanced.

Table 1: A comparison of flow constants for turbulent self-preserving jets.

Flow	Reference	Spreading Rate (a)	Decay Rate (c)
Plane Jet	Bradbury (1965)	0.109	2.34
	Gutmark and Wygnanski (1976)	0.102	2.35
	Present Result	0.11	2.4
Round jet	Hussein et al. (1994)	0.094	5.8
	Panch. And Lumley (1993)	0.096	6.06
	Present Result	0.094	5.95

Table 1 shows a comparison with the measured data of the jet spreading rate and the centerline mean velocity decay rate for both plane and axisymmetric turbulent jets. The spreading and decay rates for both cases display good agreement with the data. The value of the constant c seems to be slightly overestimated as compared with Hussein et al. (1994) experiment and under-estimated by nearly 5% as compared with Panchapakesan and Lumley (1993) experiment.

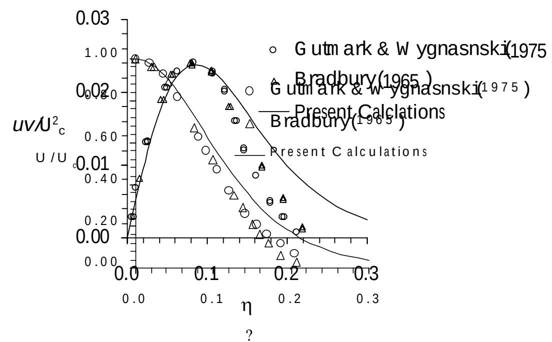
CONCLUSIONS

From the present investigation it was found that the standard k-ε model is not so sensitive to the changes in the diffusion constants as long as $\sigma_\varepsilon = 2\sigma_k$. However, any slight change in either $c_{\varepsilon 1}$ or $c_{\varepsilon 2}$ affects not only the profile shapes but also the spreading and decay rates of turbulent jets. Correct predictions of the spreading and decay rates of turbulent free jets is certainly an indication of the accuracy of the predictions of any turbulence model, since these are important constraints to the conservation of mean momentum. As the most important results of this investigation, a single set of coefficients for the k-ε closure was determined. This modification significantly improved the agreement of the prediction with experimental data for both the plane and round jets.

From the results presented in this paper, we may conclude that the present modified k-ε model adequately predicts the mean and turbulent profiles as well as the spreading and decay rates for both plane and round jets with no need to tune any of the model coefficients. This leads us to believe that this new model form is suitable to predict more complex and critical turbulent flows.

Figure 1 Axial Mean Velocity Profile Across the Plane Jet.

Figure 2 Reynold Stress Variations Across the Plane Jet.



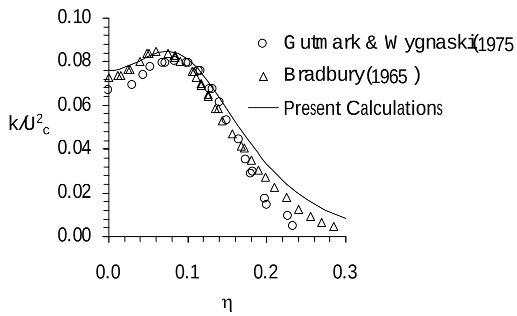


Figure 3 Comparisons of The Normalized Turbulent Kinetic Energy Across the Plane Jet.

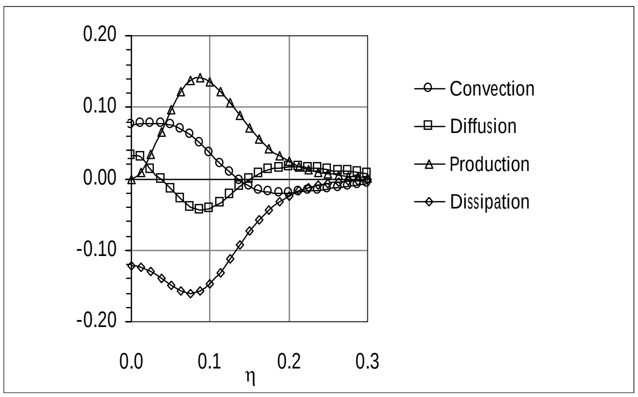


Figure 4 Calculated Turbulent Kinetic Energy Budget Across the Plane Jet.

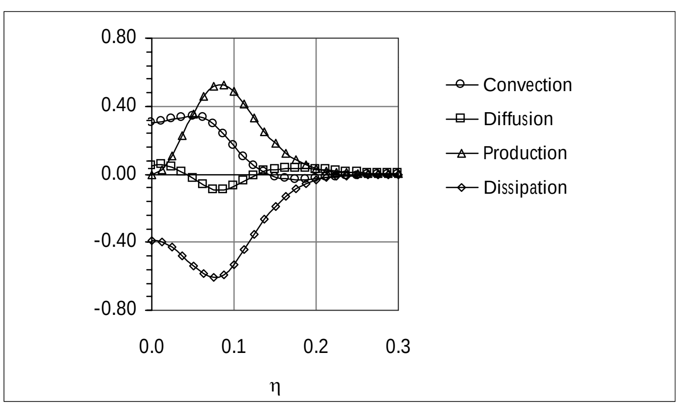


Figure 5 Calculated Turbulent Dissipation Rate Budget Across the Plane Jet.

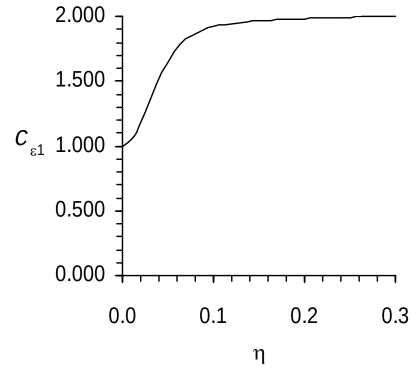


Figure 6 Calculated Distribution of $C_{\varepsilon 1}$ Across the Plane Jet.

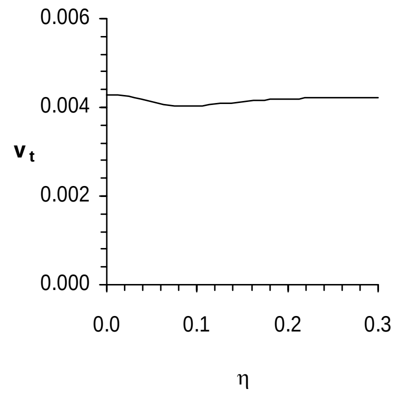


Figure 7 Calculated Turbulent Eddy Viscosity Across the Plane Jet ($v_t = C_\mu k^2/\varepsilon$).

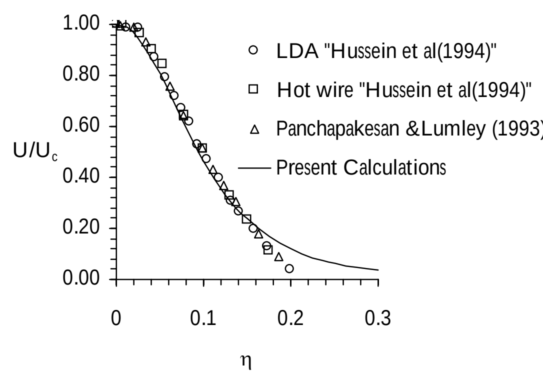


Figure 8 Axial Mean Velocity Profile Across the Axisymmetric Jet.

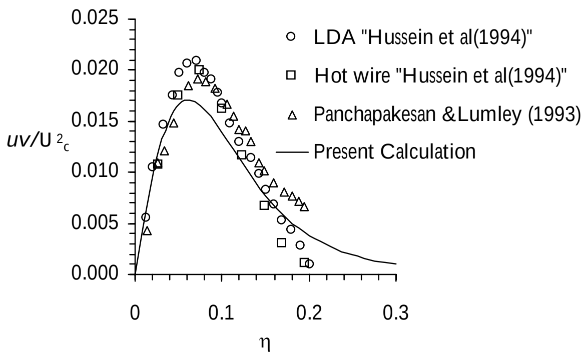


Figure 9 Reynolds Stress Distribution Across the Axisymmetric Jet.

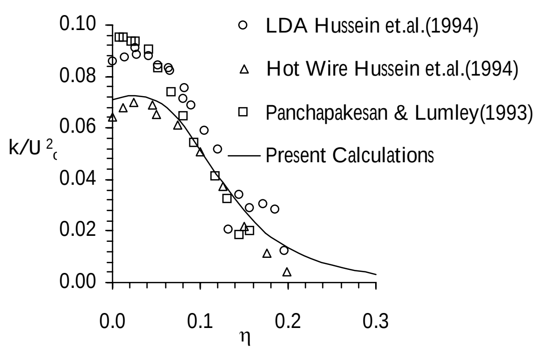


Figure 10 Calculation of the Normalized Turbulent Kinetic Energy Across the Axisymmetric Jet.

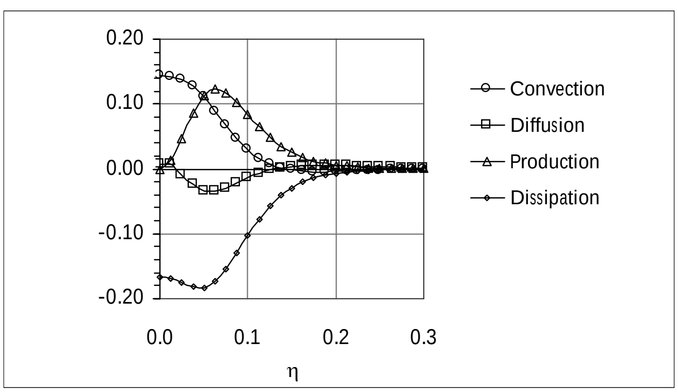


Figure 11 Calculated Turbulent Kinetic Energy Budget Across the Axisymmetric Jet.

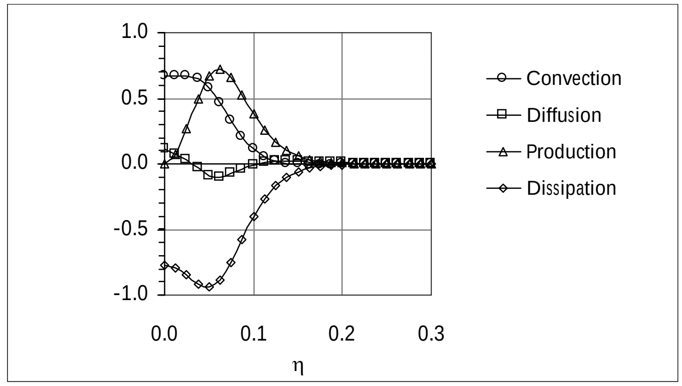


Figure 12 Calculated Dissipation Rate Balance Across the Axisymmetric Jet.

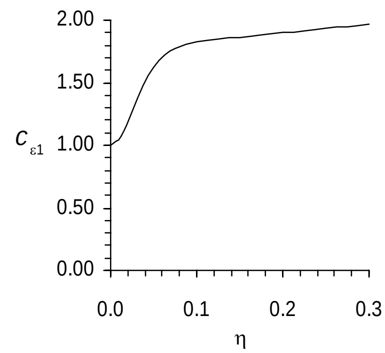


Figure 13 Calculated Distribution of $C_{\varepsilon 1}$ across the Axisymmetric Jet.

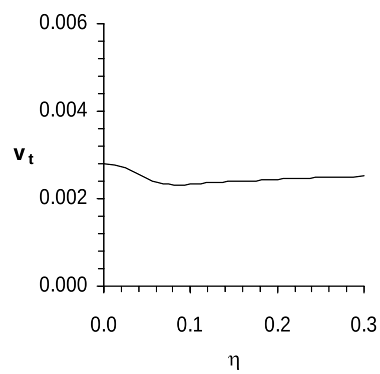


Figure 14 Calculated Eddy Viscosity Across the Axisymmetric Jet.

REFERENCES

1- **Almohaisen F.M.** (2001)," Numerical Prediction of Turbulent Free Jets Using *k*-ε model", Master Thesis, King Saud University.

2- **Bradbury, L. J. S.**, (1965) " The Structure of Self-Preserving Turbulent Plane Jet", *J. Fluid Mech.,* Vol. 23, pp,31-64.

3- **Ching-Jen Chen, Sheng-Yuh Jaw**, (1998) " Fundamentals Of Turbulence Modeling", *Taylor & Francis*, USA.

4- **Chukkapalli G. and Turan O.F.** (1995)," Structural Parameters and Prediction of Aderse Pressure Gradient Turbulent Flows, An Improved *k*-ε Model", Trans. *ASME*, Vol. 117.

5- **George W.K.,Capp S.P., Seif A.A., Baker C.B. and Taulbee D.B.**,(1988)" A Study of the Turbulent Axisymmetric Jet", *J. Engr. Sci.* King Saud University, 14(1), 85-93.

6- **Gutmark, E. and Wygnanski, I.**, (1975) " The Planar Turbulent Jet", *J. Fluid Mech.,* Vol. 73, pp, 465-495.

7- **Hanjalic, K. and Launder B.E.** (1980)," Multiple-time-scale Concepts in Turbulent Transport Modeling", *Turbulent Shear Flows: II*, pp 36-49, Berlin.

8- **Harlow F.H. and Nakayma P.** (1968)," Transport of Turbulence Energy Decay Rate", Los Alamos Science Lab., Report LA-3854.

9- **Hassid, S.** (1980)," Similarity and Decay Laws of Momentumless Wakes", Phys. Fluids, 23.

10- **Hoffman, G.H.** (1975)," Improved Form of The Low Reynolds Number *k-ε* Turbulence Model", *Pysics of Fluid* 18: 309-12.

11- **Hussein, J.H., Capp,S.P. and George,W.K**., (1994) "Velocity Measurements in a High-Reynolds-Number, Momentum-Conserving, Axisymmetric, Turbulent Jet. *J. Fluid Mech.,*Vol.258, p 31-75.

12- **Lai J.C.S. and Yang C.Y.** (1997)," Numerical simulation of Turbulence Suppression, Comparisons of the Performance of Four *k*-ε Turbulence Models", *Int. J. Heat and Fluid Flow*, Vol. 18, No. 6.

13- **Launder B.E., More A.P., Rodi W. and Spalding B.D.** (1973)," The Prediction of Free Shear Flows, A comparison of Six Turbulence Models, Proceedings of *NASA*-Longley Conference on Free Turbulent Shear Flows, *NASA-SP*312,

14- **Lumley, J.L., and B. Khaja-Nouri (1974),** "Computational Modeling of Turbulent Transport", *Adv. In Geophysics,* 18, Eds. H.E. Landsberg and J. Van Miegham, Academic Press, NY, 186-192.

15- **Malin M.R.**(1988)," Prediction of Radially Spreading Turbulent Jets", *AAA J.* Vol. 26, No. 6.

16- **Panchapakesan, N.R. and Lumley, J.L.**, (1993) "Turbulence Measurements in Axisymmetric Air Jet", *J. Fluid Mech.* Vol. 246, pp 197-223

17- **Pope, S. P.**, (1978) " An Explanation of The Turbulent Round-Jet/Plane-Jet Anomaly", *AIAA J.,* Vol. 16, pp 279-281.

18- **Reynolds, W. C**., (1976)" Computation of Turbulent Flows", *Ann. Rev. Fluid Mech.,* Vol. 8, pp. 198-208.

19- **Seif A.A. and Dale B. Taulbee** (1983)" Similarity Solution of Turbulent Free Jets", *GAMM-* Tagung, Hamburg, Germany.

20- **Seif A. A. and Dale B. Taulbee**, (1984) " Prediction of Self-Similar Turbulent Jet Using k-ε", *J. Eng. Sci.,*Vol. 10,*Nos. 1,2*. King Saud University.

21- **Taulbee, D. B.**, (1989) " Engineering Turbulence Models", A State-of-the –art review, *Advances in Turbulence* (ed. W.K. George and R.E. Arnt), Hemisphere.

22- **Zhang H., Faghri M. and White F. M.** (1996)," A New Low-Reynolds-Number *k*-ε Model for Turbulent Flow Over Smooth and Rough Surfaces", *J. Fluid Eng.*, Vol. 118.