

INFORMATION TECHNOLOGY OF A TURROMACHINERY ROW FORMATION

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ABSTRACT

The statement and method of the multicriteria accepting decisions problem of formation of the 3D rational blade shape of turbomachinery rows are developed. The system of preferences structured as system of rules of control variables formation is based. The generalized algorithm of a turbomachinery row reconstruction (modification) problem solving is developed. The information technology of the aerodynamic analysis and perfection of turbomachinery rows is developed. As a test example, the axial flow compressor stator aerodynamic perfection is executed.

INTRODUCTION

The basic scientific problem of the given research is the development systemtechnical bases of synthesis aerodynamically perfected high-loaded rows by their three-dimensional rational reconstruction on the basis of system structuring, modeling of three-dimensional viscous flow in blade channels and complex use of ways of a flow separation control in them — for use in practice of designing and operational development turbomachines.

Solution of the given problem is based on the following data: the description of object of research; presence of the prototype, purpose of reconstruction (modification) of complicated technik system (CTS), class of allowable ways flow separation control devices, realizing them; criteria of the design decisions quality.

The system purpose of turbomachine reconstruction (modification) — its aerodynamic efficiency increasing, i.e. is necessary to achieve minimization of losses in rows in a wide range of operation conditions at preservation given turbomachine gasdynamical parameters on the steady state modes.

STATEMENT OF PROBLEM

Let's formulate a problem of reconstruction (modification) of a turbomachinery row. Let's characterize subject to reconstruction by different groups of parameters: regime and design Π° , which are set by the designer; phase parameters or parameters of a condition Φ° , determined during accounts on the given closing parities; control or regulating variables U° , which choice is determined by a type of the task. Vector Π° to be in some area D_Π of space Π , the vector U° is limited and to be in some area $D_U = \{ U^\circ = (u_1, \dots, u_l, \dots, u_L) : (\forall l \in [1 \dots L]) a_l \leq u_l \leq b_l \}$ spaces U , that mathematically enters as

$$\Pi^\circ \in D_\Pi \subset \Pi, U^\circ \in D_U \subset U.$$

Area $D_\Pi \subset \Pi$ is area of modes, having physical sense, and area $D_U \subset U$ area of allowable controls. The vector $\Phi^\circ = \Phi^\circ(\Pi^\circ, U^\circ)$ is limited so, that is usual $\Phi^\circ \in [\Phi^\circ_{\min}, \Phi^\circ_{\max}]$.

Let $Q = \{ q_j^\circ \}$, $q_j^\circ = (\Pi_j^\circ, \Phi_j^\circ, U_j^\circ)$, $j = 1 \dots J$ — discrete set of the allowable design decisions (subset of a correctness). From the point of view of the accepting decisions person (ADP) quality of any decision $q_j^\circ \in Q$ is determined by criteria $W = \{ w_i \}$, $i = 1 \dots M$.

Let for each decision there is a representation $A: q_j^\circ \rightarrow W_j$, then value $A_i(q_j^\circ) = w_{ij}$ — estimation of the decision $q_j^\circ \in Q$ by the i -s criterion $w_{ij} \in W$.

Let for criteria $W = \{ w_i \}$ — there is a representation $L: W \rightarrow W^\circ$, $W^\circ = \{ w_i^\circ \}$, then value $L_i(w_{ij}) = w_{ij}^\circ$ — estimation of criterion $w_{ij} \in W$ by the normalized criterion $w_{ij}^\circ \in W^\circ$.

Let $W_j^\circ = (w_{1j}^\circ, w_{2j}^\circ, \dots, w_{Mj}^\circ)$ — set of estimations j -s design decision by criteria $W_j = (w_{1j}, w_{2j}, \dots, w_{Mj})$ on system of preferences G , designed as system of rules of formation U°

vector. Then pair (q_j^0, W_j^0) , $j=1...J$, is alternative v_j^0 . The set of pairs $v_j^0=(q_j^0, W_j^0)$ makes set of alternatives $V=\{v_j^0\}$.

In a considered case the set of alternatives V is contained the prototype $v_0^0 \in V$, and its other elements — v_j^0 can be obtained by the assignment of small deviations from parameters of the prototype.

Problems of reconstruction, as a special case of problems of accepting decisions [1, 2] can be represent by a tuple of a kind

$$\{t, V, G, F\}, \quad (1)$$

where t — problem statement (for example, eliminating of the best alternative, eliminating of the ordered or disordered subset of the best alternatives etc.), F -procedure of the rational design decisions choice from set of alternatives V in system of preferences G .

Procedure of a choosing F of representation realizations $F: (V, G) \rightarrow V$, and result of its application - subset $\hat{V} \subseteq V$ of rational alternatives, which make elements, nondominated relatively criteria W , included in used preferences system $G=(W, R)$:

$$\begin{aligned} \hat{V} &= \{v_i^0, i \in [1...J]: v_i^0 \in \hat{V}, (\forall j \in [1...J]) \\ v_i^0 \notin \hat{V} \rightarrow v_i^0 R_l^1 v_j^0: v_i^0 R_\lambda^1 v_0^0, l=2...M, \quad (2) \\ v_i^0 &\in \hat{V}; \\ (\forall i_1, i_2 \in [1...J]) v_{i_1}^0, v_{i_2}^0 \in \hat{V} \rightarrow v_{i_1}^0 R_k^2 v_{i_2}^0, k=1...K \}. \end{aligned}$$

Here $R_\lambda^1 \in R$ — attitude of domination relatively G ; $R_k^2 \in R$ — attitude of an equivalence relatively G .

The record $v_i^0 R_l^1 v_j^0$ means "the decision v_i^0 more preferably, than v_j^0 by criterion w_l^0 , if the binary attitude of a priority is given $R_l^1: w_l^0(q_i^0) < w_l^0(q_j^0)$ ", record of a kind $v_i^0 R_\lambda^1 v_0^0$ — "the decision v_i^0 satisfies to conditions $R_\lambda^1: |w_\lambda^0(q_i^0) - w_\lambda^*| < \varepsilon_l$ (or $w_\lambda^0(q_i^0) \leq w_\lambda^*$)", record of a kind $v_{i_1}^0 R_k^2 v_{i_2}^0$ — "the decision $v_{i_1}^0$ is equivalent to the decision $v_{i_2}^0$, if the condition is satisfied $R_k^2: |w_k^0(q_{i_1}^0) - w_k^*| < \varepsilon_k$ (or $w_k^0(q_{i_1}^0) \leq w_k^*$)". Here w_l^0 — decisive criterion, w_λ^0 — additional criteria (including pseudo-criterion).

As decisive criterion we shall choose

$$w_1^0 = \langle \delta_\sigma \rangle / \langle \delta_\sigma \rangle_0, \quad (3)$$

where $\langle \delta_\sigma \rangle$, $\langle \delta_\sigma \rangle_0$ — averaged losses of total pressure in relative movement of researched variant of the design decision and prototype, accordingly.

As additional criteria can be chosen:

a) for ROTOR_j+STATOR_j:

$$w_2^0 = \max |\Delta \alpha_j^0(r)|, \quad (4)$$

where $\Delta \alpha_j^0(r) = (\alpha_1(r) - \alpha_{1t}(r)) / \Delta \alpha_{1t}(r)$ — relative mismatch degree of flow corners in absolute movement on an input to the following row in comparison with design (theoretical); $\Delta \alpha_{1t}(r)$ — range of working modes on a incidence angle to the following row.

For STATOR_j+ROTOR_{j+1}: $w_2^0 = \max |\Delta \beta_{j+1}^0(r)|$;

b) for ROTOR_j+STATOR_j:

$$w_3^0 = \max |\Delta P_{0j}^0(r)|, \quad (5)$$

where $\Delta P_{0j}^0(r) = (\langle P_{0j} \rangle - P_{0j}(r)) / \langle P_{0j} - P_j \rangle$ — relative degree of total pressure non-uniformity on blade height averaged by pitch in absolute movement behind trailing edge; $\langle P_{0j} \rangle$ — value of total pressure obtained as a result of averaging on a pitch and blade height in absolute movement in the chosen crosssection;

c) for ROTOR_j+STATOR_j:

$$w_4^0 = \max |\Delta \pi_{0k}^0|, \quad (6)$$

where $\Delta \pi_{0k}^0 = (\langle \pi_{0k} \rangle - \langle \pi_{0k} \rangle_0) / \langle \pi_{0k} \rangle_0$ — relative degree of a mismatch by on compression ratio.

Values of w_2^* , w_3^* and w_4^* are usually nominated by the expert - designer.

The problem of reconstruction (modification) of a turbomachinery row (1) mathematically formulated as follows: the prototype v_0^0 , set of alternatives $V = \{v_i^0\}$ and system of preferences G is given. It is required to allocate such allowable control $U^0(\Phi^0)$ ($U^0 \in D_U$), which would translate system from the given condition v_0^0 in another allowed $\hat{v}^0 \in \hat{V}$ (2) in system of preferences G .

The formulated problem concerns to problems of optimum control synthesis $U^0(\Phi^0)$ with the moving ends at presence of restrictions on regime, phase and control parameters.

Phase variable are determined by computer modeling CTS, based on application of computer interactive system of the engineering analysis of three-dimensional viscous flow in turbomachinery blade channels.

COMPUTATIONAL SEARCH METHOD

As the through turbomachinery designing on the basis of use of three-dimensional viscous models today is inexpedient because of the large expenses of time and computing resources, the development of preliminary turbomachinery shape usually is carried out with two-dimensional models using. The further study of setting geometry and turbomachinery blade rows performed as search of some rational geometry within the framework of the preliminary concept. Thus, the system technology of three-dimensional aerodynamic perfection of turbomachinery rows, based on the analysis of computer modeling results of viscous gas flow, should represent a number consecutive parametrical approximation to some

rational geometry. Achievement of the rational aerodynamic blades form and turbomachine as a whole possible only through competent application of a necessary set of influences on a flow. Measure of expediency of using different influence on a flow is the qualified analysis of flow structure according to entered priority and generalized quality criteria, which can be made both experimental, and computation methods.

The generalized algorithm of the turbomachinery blade reconstruction (modification) problem solving (1) can be submitted, according to [1,3], in the following kind:

- a) computer modeling and analysis of condition parameters $\Phi^0(\Pi^0, U^0)$ of prototype;
- b) on the basis of the system analysis formation of:
 - system functions of CTS elements;
 - is functional-significant relations between them, morphological set of essential classification attributes and parameters, appropriate to them;
- c) composition of a morphological table - set of parameters values appropriate to essential classification attributes. At this stage it is expedient to use the data on analogues and regression models;
- d) formation of knowledge base - system of preferences structured as system rules of formation control variable on the basis of the system technical requirements and restrictions;
- e) formation of discrete set of the allowable design decisions (subset of a correctness) Q by a filtration of the initial morphological table on the basis of use product rules of knowledge base and appropriate to it set of alternatives V;
- f) formation of the ordered discrete set of the allowable design decisions Q' by ordering of subsets and parameters, appropriate to them, in subsets in decreasing order of influences by value priority parameters, based on system of preferences of the expert - designer, quality criteria of the technical decision sets, (3-6) and appropriate to it set of alternatives V';
- g) formation of set of rational alternatives \hat{V} (2) and definition of the best alternative φ^0 in the chosen system of preferences by parametrical synthesis of the system rational decisions.

The generalized algorithm can be presented as the following sequence of sets maps: $Q \rightarrow V \rightarrow Q' \rightarrow V' \rightarrow \hat{V} \rightarrow \varphi^0$.

The convenient form of discrete set representation of the allowable design decisions Q (subset of a correctness) — alternative variants of a subject to updating technical object design shape, is the morphological table, to which lines correspond regime and design Π^0 , and also control U^0 parameters (object as a whole and its components), and elements — possible values (realization of the technical decisions) each of them e_{ij} , $i=1 \dots n$, $j=1 \dots m$, n — total amount of assigned parameters Π^0 and U^0 .

It is expedient to structure classifying elements CTS attributes and parameters, appropriate to them, by splitting set

of parameters $K=\{K_1, \dots, K_n\}$ on T of independent or poorly cooperating subsets L_t ,

$$t = 1..T : \bigcup_{t=1}^T L_t = K; \quad (7)$$

$$\forall t, r : L_t \cap L_r = \emptyset, t \neq r, r=1..T.$$

Further carry out ordering of subsets and parameters, appropriate to them, in subsets in decreasing order influences on priority values, proceeding from system of preferences of the expert - designer, quality criteria of the technical decision.

In result of control parameters ordering (at the given form of a flowing part and number of blades), according to the entered principles, the sequence of the morphological table lines was chosen, which elements are the values of parameters determining:

- profiles in k crosssections along blade span — $\alpha'_1, \alpha'_2, B, x^0_c, C^0_{max}, r^0_{LE}, r^0_{TE}, l^0_{TE}$. The decision — to form blades along span by a symmetric profile with wedge-shape near a site of trailing edge with an average line in crosssections in the form of an arch of a circle connected to a straight line on an output here was accepted;
- meridian outlines of the hub and casing within the limits of a row — $(x^0, \Delta r^0)_i, i=1..5$ — in meridian crosssection;
- complex bowled blades at the hub and casing - displacement of the maximal thickness centers line for stator (centers of gravity for rotor) in sections along blade span relatively the axis, directed along radius, of technological (uniform) system of coordinates - height of area h^0 , angles γ_r and γ_ϕ between tangential to the centers line and radial direction at a end surface.

For example, if $k=5$ — the total number of parameters is equal 66.

At creation of the morphological table of a set of control parameters - were used:

- information about analogues;
- regression models (on the basis of generalizations of the experimental and computation data achieved by other authors);
- knowledge base (system of preferences structured as system of rules for control variables formatting on the basis of the system technical requirements and restrictions).

The received thus morphological table represents the ordered subset of a correctness Q' of combinations of the probable technical decisions of technical object components.

In view of the large number of control variables, as a procedure of parametrical synthesis of the system rational technical decisions the heuristic procedure of the consecutive analysis on serial scales (2) and elimination of variants based on ideas a of directing neighborhood [4,5], method was used. The search of rational alternatives is carried out by transition from a point v_j^0 of set V (generally) to a point $v_{j+1}^0 \in \hat{V}$ with smaller value of decisive criterion of quality w_1^0 (3). The unpromising directions were excluded from the further

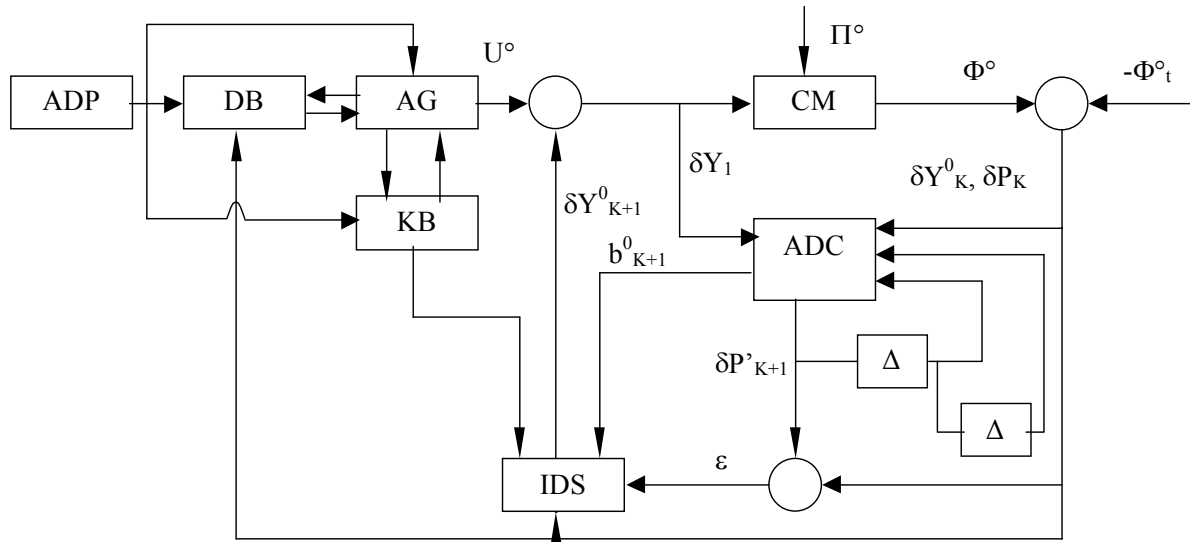


Fig. 1 Structure of adaptive accepting decisions support system

consideration. As initial approximation v_0^0 the data on the prototype were used.

Computer modeling, analysis of state parameters $\Phi^0(\Pi^0, U^0)$ of prototype, and also alternative variants from a subset of a correctness are carried out on the basis of information technology of the three-dimensional viscous flow analysis in blade channels of turbomachinery rows with complex use of flow separation control. The preparation of the input data and analysis them on consistency is carried out in an interactive mode by tool means developed CAE-system ensuring also graphic representation of results.

The effective decision of the multicriteria accepting decisions problems is possible by hierarchical organization of algorithm: the initial problem of the large dimension is reduced to the solving of smaller dimension subproblems. Thus the set of the allowable decisions is broken into a number of subsets. The exception of the further consideration of unpromising subsets allows to reduce number of problem solving variants.

According to the basic principles of the system approach the decomposition of a general multicriteria accepting decisions problem of formation of blade row shape is carried out on a number of problems: synthesis of the system rational design decisions on the basic design data of a row and blade cascades in sections along blade span (with use of some typical forms family profiles) and synthesis of the profiles forms in crosssections. The basis for the chosen hierarchy of subproblems is accepted control parameters ordering in decreasing order by influence on value of priority parameters, proceeding from system of preferences, criteria of the technical decision quality.

Statement and method of a variation of aerodynamics problem for blade cascades are shown in [6]. The offered method was distributed on a three-dimensional case. For computer modeling of three-dimensional viscous flow the adaptive spatial computation grid obtained by connection of grids such as «H» at surfaces of rotation in a radial direction was used. The coordinates of originally taken profiles in sections along blade span were defined as a points of crossing of suction and pressure profile sides with surfaces of rotation,

on which a computation grid nodes are located. The distributions of pressure $P_0(s)$ as functions arc abscise of an originally taken contour γ_0 for each of the profile sides separately in sections along blade span obtained by computer modeling of three-dimensional viscous flow in blade channels of rows. The new profile form in crosssections was obtained as quasisolution of a variation aerodynamics problem on surfaces of rotation close or conterminous with axisymmetrically surfaces of a current in complete conformity with a method [6].

INFORMATION TECHNOLOGY

The information technology of the aerodynamic analysis and perfection of turbomachinery rows is developed. The information technology of perfection process assumes presence of the initial row - prototype, which further is reconstructed with the purpose of its aerodynamic efficiency increasing. The structure of adaptive accepting decisions support system of the turbomachinery row rational form formation at a stage of conceptual designing is submitted in a fig. 1. In general structure of designing it is possible to allocate two contours - external and internal.

In an external contour the parametrical synthesis of the system rational design decisions on the basic design data of a row and blade cascades in crosssections along blade span is made. According to the generalized algorithm the following order of operations is established. The expert - designer on the basis of own experience and database (DB) chooses from a set of design parameters a subset of control parameters, based on which there will be a updating of initial variant of row shape. Besides the variant is possible, at which the subset of allowable control parameters is formed by the alternatives generator (AG) and knowledge base (KB) with use of the information from DB. Further to be made computer modeling (CM) of row design alternative variant with given regime and design (Π^0) and chosen control parameters (U^0). Phase variable of alternative variant (Φ^0), obtained on the basis of CM, are compared with given Φ^0_t , then to be carried out visualization and analysis of computation results by the accepting decisions person (ADP). Further or the new variant of row design shape

is generated, or the parametrical synthesis of the solutions proceeds on an internal contour.

In an internal contour to be made synthesis of the profile form in crosssections along blade span. Its structure includes three blocks: CM of a row, adaptive differential connection (ADC) and identification of disturbance sources (IDS). CM of a flow in blade channels of turbomachinery row characterized by rather high computing complexity, therefore for increase of generalized algorithm efficiency in system where entered ADC and IDS. Use ADC and IDS is caused by that of small indignation of the profile form generated by AG, can be enough precisely predicted as quasisolutions of a variation aerodynamics problem. According to the generalized algorithm the following order of operations in an internal contour is established: adaptation of differential connection, definition of optimum initial pressure distributions (IPD) $\hat{P}_1(s)$, finding of the amendments to contours coordinates for each of the profile contours sides separately and their updating on the obtained sizes $\hat{\delta}_1^*(s)$ in crosssections along blade span. On the initial stage of system work on the chosen contour the process of training ADC is realized. The correction of factors b_{ik} of form disturbance influence δY_{ik} in k-s profile contour points is made by means of algorithm of adaptation, which as a feedback realizes the control of a deviation ADC of predicted pressure variation $\delta P'_k$, caused by the profile form variation $\hat{\delta}_1^*(s)$, from calculated — δP_k by CM. When the given accuracy of a prediction is achieved, is carried out IDS on the basis of designed in the block ADC factors of influence b_{ik} . Further are determined optimum IPD $\hat{P}_1(s)$ in sections along blade span. IDS consists in quasisolution construction of the inverse boundary problem of aerodynamics (IBPA) — hydrodynamics of expedient pressure distribution (HEPD), satisfying to a initial data compatibility condition (IDCC) and conditions of a constructive realizability and solving (CR) of IBPA, and profile form variation, appropriate to it $\hat{\delta}^*(s)$. The input data for IDS except for optimum IPD are the design and phase parameters of blade cascades in crosssections along blade span. During synthesis of the decisions on an internal contour IDS generates disturbance of the profile form and again, already for the new profile form the pressure distribution by means of CM provided. On each step of iterations prediction accuracy ADC of pressure variation $\delta P'_k$ is estimated on the data of CM and in case of necessity the repeated training ADC is made. At achievement with the given accuracy ε value of quality criterion $w(\Delta\delta^*) = \max(\text{abs}(\hat{\delta}_L^*(s) - \hat{\delta}_{L-1}^*(s))) < \varepsilon$ at construction of quasisolution of IBPA on the chosen correctness set the process of calculations on an internal contour stops. The resulting data in the form of a file of points specifying the profile form in sections along blade span, get back in DB. Phase variable of alternative variant Φ^0 , obtained on the basis of CM, are compared with given Φ^0_t , then the visualization and analysis of calculation results ADP is made. Further, or the parametrical synthesis of the decisions proceeds on an external contour (AG generates new variant of design shape), or the process of global iterations stops.

In completion, ADP according to the generalized algorithm will carry out the analysis of rational alternatives set \hat{V} the information about which is stored in DB, and chooses the best alternative φ^0 — system rational design decision.

RESULTS

For the illustration, the complex calculation researches of aerodynamic row perfection of the block of stages for the perspective air engine are executed. As a result of flow numerical research in rows the zones of a flow separation are revealed: on the hub near suction side; on a suction side near hub of rotors and stators; on suction side near casing - to an output from stators. The occurrence of flow separation zones is caused by the diagonal form of a multistage compressor flowing part and radial non-uniformity of flow parameters caused by character of flow in near-hub area of rotor blade channel of fan.

As a result of flow numerical research and aerodynamic characteristics of initial variants of rows it is accepted to the expedient decision reconstruct the stator blade of a fan in internal contour.

The main design data of initial stator and data of a blade cascade at midspan:

- Casing diameter (D_{1c})	0.6407m;
- Hub-Tip Ratio ($d_{1h}^0 = r_{1h}/r_{1c}$)	0.7688;
- Chord Length (B)	0.0465m;
- Pitch Chord Ratio ($T^0 = T/B$)	0.6587;
- Aspect Ratio ($H^0_1 = H_1/B$)	1.5976;
- Stagger Angle (ν)	26.14°;
- Camber Angle (Θ)	35.72°.

The calculations were carried out for mode of operations appropriate to the air flow $G_a = 29.49$ kg/s. The boundary conditions on an input of stator were set on the basis of the given calculations of three-dimensional viscous flow in near-hub area of the fan rotor.

The updating was carried out according to described CAE — technology under obtained in initial flow angles (in absolute movement) on an input of stator and on required (in relative movement) design angles - on an input in next rotor, following for stator. Some variants of reconstruction are executed.

- The profiles with controlled diffusion on all blade height were obtained according to a technique [6] (is reduced chord);
- The form of hub surface meridian outlines form is changed;
- The structures with bowed blade ends, providing a obtuse angle between hub, casing surfaces and suction side.

The results of researches in the form of distribution of averaged relative losses of total pressure along blade span $\Delta\delta_\sigma^0 = (\sigma_{in} - \sigma_{ver}) / (1 - \sigma_{in})$ in a fig. 2 shown, that in the modified row the reduction of total losses on 52 % is achieved in comparison with initial variant. The comparative analysis of calculations results of three-dimensional viscous flow in blade

channels of initial and modified variants of stator shows, that a combination of a number of means of indirect flow separation control in rows, to which number concern: using blades with bowed ends, providing an obtuse angle between hub, casing surfaces and suction side; the new form of hub surface meridional outlines, first of all; special type of profiles with controlled diffusion, in the second turn on a degree of influence - results in reduction of a longitudinal pressure gradient in an outlet part of the blade channel of row, and, hence, to reduction of the sizes of flow separation areas.

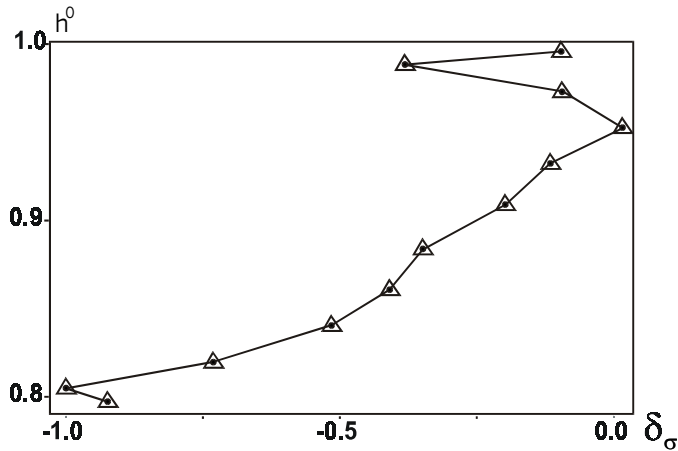


Fig. 2 Distribution of averaged relative losses of total pressure along blade span of trailing edges plane

At fig. 3 shown comparison of relative flow velocity q/q_{LE} for initial and obtained profile of midspan (1-initial variant, 2-quasisolution of variation aerodynamics problem).

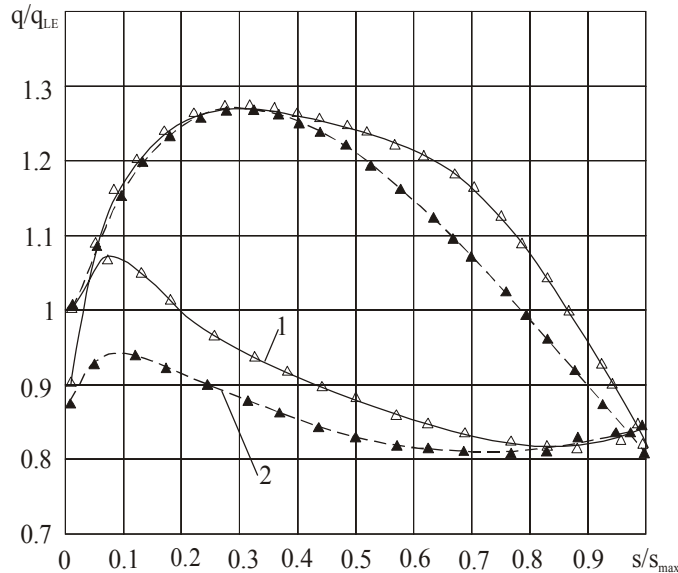


Fig. 3. Distribution of relative flow velocity along profile contour

CONCLUSION

The information technology of an aerodynamic analysis and perfection of turbomachinery rows which includes 3D viscous flow direct and 3D inverse design problems solvers is developed. As a test example, the axial flow compressor stator aerodynamic perfection is executed. It is shown, that a combination of a number of means of indirect flow separation control results in essential reduction of losses in this stator.

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