Proceedings of ICFDP7:
Seventh International Congress on Fluid Dynamics and Propulsion
December 18-20, 2001, Cairo, Egypt

ICFDP7-2001026

NUMERICAL INVESTIGATION OF THE 2D AND 3D INCOMPRESSIBLE FLOW IN A SUDDEN EXPANSION DUCT

D. TOUZOPOULOS AND G. BERGELES

Department of Mechanical Engineering
Technical University of Athens
15773 Athens, Greece
E mail: bergeles@fluid.mech.ntua.gr

ABSTRACT
A numerical experimentation of 2-D time dependent incompressible flow in a plane symmetric sudden expansion duct of area ratio 1:2 was conducted. Computations were performed for Reynolds numbers in the range of 1500 to 4200. The results reveal that the flow is symmetric up to a point in time, depending on the Reynolds number; then becomes asymmetric with two main recirculation zones, at the duct corners, acting as continuous sources of vortices which roll along the channel walls.
The 3-D numerical solution of the same channel flow reveals that the flow first becomes three dimensional at low Reynolds number and then time dependent.

INTRODUCTION
Cherdron et al [1] were among the first to find out experimentally the flow asymmetry developed in symmetric, 2-D plane duct with sudden expansion; the asymmetry of the flow was attributed to the disturbances generated at the edge of the expansion leading to one long and one short separation zones at the duct corners. Sobey and Drazin [2] studied analytically, numerically and experimentally the bifurcation of 2-D channel flows. They found time-periodic flow fields at sufficiently large Reynolds numbers, whilst at lower Reynolds numbers up to eight asymmetric, and stable solutions were found. Fearn et al [3] investigated experimentally and numerically the origin of steady asymmetric flows in a symmetric sudden expansion. They observed that the unsteady flows were a consequence of three-dimensional effects in the channel.
Velocity measurements in the flow through 2-D duct with symmetric sudden expansion 1:2 at Re up to 610 were carried out by Durst et al [4] supported by numerical predictions. The results indicate that flow asymmetry develops at Re around 125 (based on the upstream channel height and the maximum flow velocity upstream). Battaglia et al [5] studied numerically the flow in 2-D channels with various expansions. Their results indicate that the critical Reynolds number for which flow asymmetry occurs decreases with increasing channel expansio ratio.
Drikakis [6] studied the laminar incompressible flow in symmetric 2-D sudden expansions. He found that at Re higher than 600, more than 3 separation bubbles are formed. In our previous work [7] the flow through a plane symmetric sudden expansion for Reynolds numbers less than 1700 was investigated. The calculations were performed with the assumption that the flow is steady. The results indicate that bifurcation phenomena appear at a Reynolds number of 160. At higher Reynolds number up to 2000 the flow is asymmetric with many stable asymmetric solutions appearing with increasing Reynolds number; however after a Reynolds number of 2000 the numerical solution failed to converge under the assumption of steady flow.
In the present work we repeated the previous numerical experiments and present numerical results for 2-D calculations assuming that the flow is time dependent as also results for 3-D calculations assuming that the flow is steady but three dimensional, trying thus to establish the regions where the flow becomes time dependent or develops three dimensional structures in a nominally two dimensional duct.

NOMENCLATURE
A: distance between two moving fluid elements
A_0: initial value of A at time t_0
A_T: final value of A at the end of the channel

C: Chaos indicator = $\frac{1}{\Delta t}\ln\frac{A}{A_o}$

C_T: C value at the end of the channel
d: inlet height
Dt: time step=0.003s
H : step height =d/2
L: the length of the large channel
Re: Reynolds number = Ud/ν
U: mean inlet velocity
u : velocity in the x direction
I,J: grid numbers
Y: distance from bottom wall
Y_{A6} : Y's of the instantaneous positions of the elements which at $t=t_0$ were at A_1, A_6
X1: distance from inlet
X: distance from step
v: velocity in the y direction

Greek letters

$\lambda_1(t)$: the mean velocity $\bar{v}$ along the channel axis

$\lambda(t)$: the mean absolute velocity $\overline{|v|}$ along the channel axis

λ_0: the value of $\lambda(t)$ as $t\rightarrow\infty$

μ: viscosity

ν : kinematic viscosity

ρ: fluid density

SOLUTION PROCEDURE AND BOUNDARY CONDITIONS

The flow through the sudden expansion is assumed to be incompressible, laminar and two-dimensional.

Figure 1a shows the solution domain and the employed orthogonal Cartesian grid. The calculations start 1.85-inlet duct channel heights upstream from the expansion with a uniform inlet profile; the expansion ratio is 2 and the calculation domain downstream the expansion covers 25 inlet duct heights. Calculations have been performed with numerical grid 100x70 and the convergence criterion was set at 0.1%. The time-step has been taken equal to 0.003s. As discretization scheme for the convection terms was used the BSOU scheme of Papadakis and Bergeles (1995), a second order accurate bounded upwind scheme. The computer code is based on the finite volume methodology, employs the Rhie and Chow (1985) interpolation method for avoiding pressure velocity decoupling and uses the Patankar-Spalding (1972) SIMPLE algorithm for the numerical solution of the equations in their time dependent form. The number of iterations required at each time step ranged from several thousands, at the first time steps, to 10 at the last steps.

PRESENTATION AND DISCUSSION OF THE RESULTS

For continuation purposes with our previous study [7], the present 2D flow was examined as time dependent starting from Reynolds number as low as 500. The results obtained for Reynolds numbers from 500 up to 1700 coincided with those obtained in [7] which were obtained with the assumption that the flow is steady. Therefore the results which are presented here refer to higher than 1700 Reynolds numbers. A general instantaneous view of the flow pattern for reference purposes is shown in figure 1b.

The time development of the flow, for constant Reynolds number, is shown in Figure 2, for Reynolds numbers 1800 and 3200. At time $t=0^+$, just after the start (the duct filled with fluid at rest), the flow is symmetric, the fluid behaving like an ideal fluid. So at point e.g. P1 (see for reference figure 1b) the velocity is u=U/2, because of the doube cross-section of the large duct. As time develops the flow retains its symmetry but two separation regions attached to the expansion corners of the duct appear. Later in time, these vortices roll along the walls of the channel, whilst new vortices are created at the duct corners; at a critical time step of the flow development small asymmetric flow perturbations develop, which eventually due to Coanda effect lead to the establishment of large scale flow asymmetry. Up to this critical time where flow asymmetry develops the velocity v at point P1 is constant. Figure 3 clearly shows the way, the velocities u and v at point P1 are varying with time for Re=1800 and Re=3200.

At critical time t_1 , which is a function of Reynolds number and of the distance X from the step, velocities u and v start to oscillate in time. In order to have a measure of the velocity by which the disturbance of the flow propagates, we consider as disturbance-front the point of the first wall reattachment of the flow. The relation between the distance from the corner of the disturbance-front, as it moves along the wall, and the time t_1 for different Reynolds numbers is shown in Figure 4.

A non dimensional equation can be fitted to the previous results, $\frac{X}{H} = 0,316 \quad \mathrm{Re} \left(\frac{\nu t_1}{H^2}\right)^{1,25}$.

Figure 3 indicates also a periodicity in the flow velocities u and v. An FFT analysis of the velocity signature at point P1(for time beyond t_1) reveals a predominant frequency of 1Hz for Re=3200.

In order to have a measure of the flow symmetry we calculate at every moment the mean value of the velocity v along the channel axis (integral $\frac{1}{L}\int_0^L v dx$), denoting it as $\lambda_1(t)$, as also the mean value of the absolute velocity v along the channel axis (integral $\frac{1}{L}\int_0^L |v| dx$), denoting it as $\lambda(t)$. The way $\lambda_1(t)$ and $\lambda(t)$ vary with time for constant Re is shown in Figure 5.

We can distinguish three time moments:

t_1: up to this moment $\lambda_1(t) = \lambda(t) = 0$, the flow is symmetric and the central line is a streamline,

t_2 : between t_1 and t_2 the central streamline exhibits periodicity in space and time, with growing tendency for the flow disturbances,

t_3 : after this moment $\lambda(t)$ maintains constant value, meaning that the flow disturbances have reached a saturated state.

Considering the variation of λ_0, t_1, t_2 and t_3, with Re, it follows that the flow bifurcates faster with Re, as also that the level of asymmetry increases linearly with Re.

The calculations have shown that the length of the separation zones are equal to (7-9)H, for the short one, and (24-30)H, for the large one, for the Reynolds numbers investigated (up to 3200). We must mention here that the corresponding lengths in steady state flow (for Re=1000 to 1500) were found equal to 5H and 18H respectively.

In order to study the way, the fluid elements move, we followed the paths of two adjacent fluid elements which in moment $\mathbf{t_o}$ were at points A1(X=0,Y=0.07) and A6(X=-0.00286,Y=0.07). Along their paths we calculated the distance A between the elements $[A^2=(X_1-X_6)^2+(Y_1-Y_6)^2]$ with time. The fluid paths stop at the end of the channel, in different time intervals $(\Delta t)_{Re}$ for different Reynolds numbers; there A takes the value A_T. We define as a chaotic indicator the quantity $C = \frac{1}{\Delta t} \ln \frac{A}{A_o}$ at the end of the channel.

Figure 6 shows for various Re and t_o the paths of the two fluid elements.

The following can be concluded:

- For t_o=0, i.e. with the start of the flow, the paths coincide, independently of Re.
- For increasing t_o the fluid element trajectories differentiate (more with increasing Re) and earlier with increasing t_o .
- Also for t_o =0, C_T is negative, for every Re. This means that the flow initially has a deterministic character. This deterministic character of the flow continues up to time t_1(Re), function of Re.
- for $t_o > t_1$ (t_1 depending on Re) :
  - if Re >2400 , C_T is positive, which means that the paths diverge and the flow becomes chaotic,
  - in the region Re=1800 to 2400, C_T may be negative or positive, indicating a transitional character.

In Figure 6 we have also drawn correlation curve $Y_{A6} = f(Y_{A1})$ for various Re for different initial moments t_0. For Re=2000 the curve is a straight line, no matter what is time t_0 , indicating the deterministic character of the flow. For greater Re the curve is also a straight line for $t_0 < t_1$, but for $t_0 > t_1$ the curve becomes complex indicating a chaotic behavior of the flow. The critical time t_1 decreases to zero for sufficiently large Reynolds number.

3D CALCULATIONS

We conducted also 3-D calculations in the duct with the same dimensions and inlet conditions as the previous one (plane X, Z), but with a large depth of Y=32, i.e lateral aspect ratio of 8. The numerical grid was 66x60x66.The results indicated that bifurcation appears at Reynolds numbers around 200 in agreement with the 2D calculations, with the flow remaining the same (as two dimensional) in the lateral planes (Y direction). However for higher Reynolds numbers (Re>250) three dimensional structures appear in the flow, as it can be seen in Figure 7. Converged solution of the 3D calculations assuming the flow as steady was possible to obtain up to Reynolds number of 450.

CONCLUSIONS

Transient phenomena of incompressible flow in a plane sudden expansion with ratio 1:2 were numerically studied at Reynolds numbers between 1500 and 4200 with uniform inlet velocity distribution. The results indicate that:

- at the beginning (in time) the fluid behaves as ideal one and the flow is symmetric,
- soon, disturbances appear which propagate with a velocity proportional to Re,
- the disturbances reach saturated state (in time) as sooner as higher is Re and their value increases with Re,
- for Re>2400 the flow becomes chaotic.
- However the 3D calculations of the planar duct indicated the following:
  For Re<200 the flow is symmetric 2D,
  For 200<Re<250 the flow is asymmetric but 2D
  For 250<RE<450 the flow is 3D
  For Re>450 the flow is 3D, time dependent

REFERENCES

[1] Cherdron W., Durst F. and Whitelaw J.H., 1978, “Asymmetric flows and instabilities in symmetric ducts with sudden expansions”. J. Fluid Mech.,vol.84, part 1, pp. 13-31.

[2] Sobey I.J. and Drazin P.G., 1986, “Bifurcations of two-dimensional channel flows” J.Fluid Mech., vol.171, pp. 263-287.

[3] Fearn R.M., Mullin T. and Cliffe K.A., 1990, “Nonlinear flow phenomena in a symmetric sudden expansion”. J.Fluid Mech.., vol.211, pp. 595-608

[4] Durst F., Pereira J.C.F. and Tropea C., 1993, “The plane symmetric sudden-expansion flow at low Reynolds numbers”. J.Fluid Mech., vol.248, pp.567-581

[5] Battaglia F., Tavener S.J., Kulkarni A.K. and Merkle C.L., 1997, "Bifurcation of low Reynolds number flows in symmetric channels". AIAA Journal, Vol.35, N^0 1, pp.99-105.
[6] Drikakis D., 1997, "Bifurcation phenomena in incompressible sudden expansion flows". Physics of Fluids, Vol. 9, N^0 1, pp.76-86.
[7] D. Touzopoulos and G.Bergeles., 1998, " Numerical Investigation of the Laminar Steady Incompressible Plane Sudden-Expansion Flow"., ECCOMAS 98, Proceedings, John Wiley and Sons, pp 906-912.

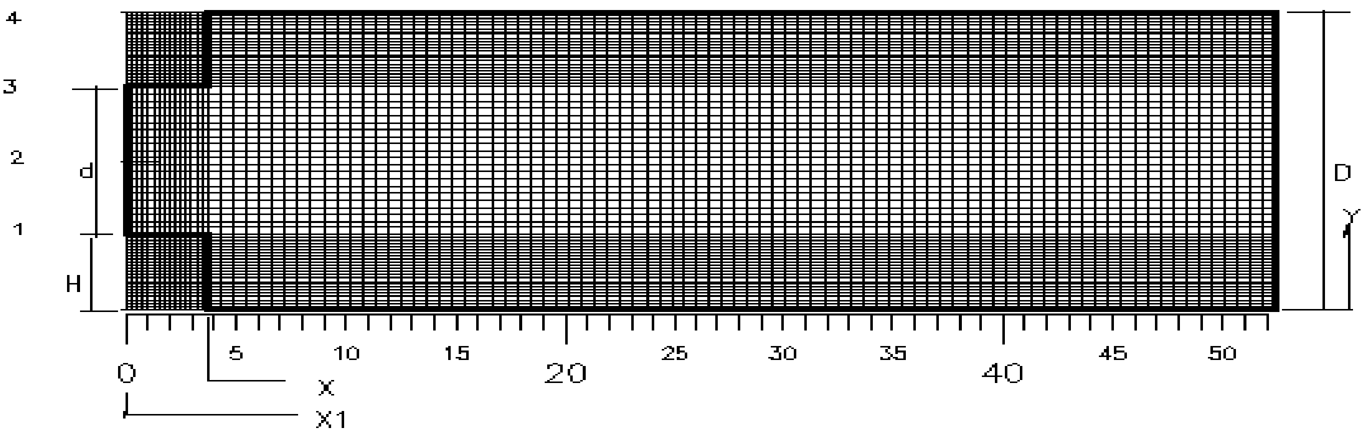


FIGURE 1a. Mesh for the sudden expansion flow

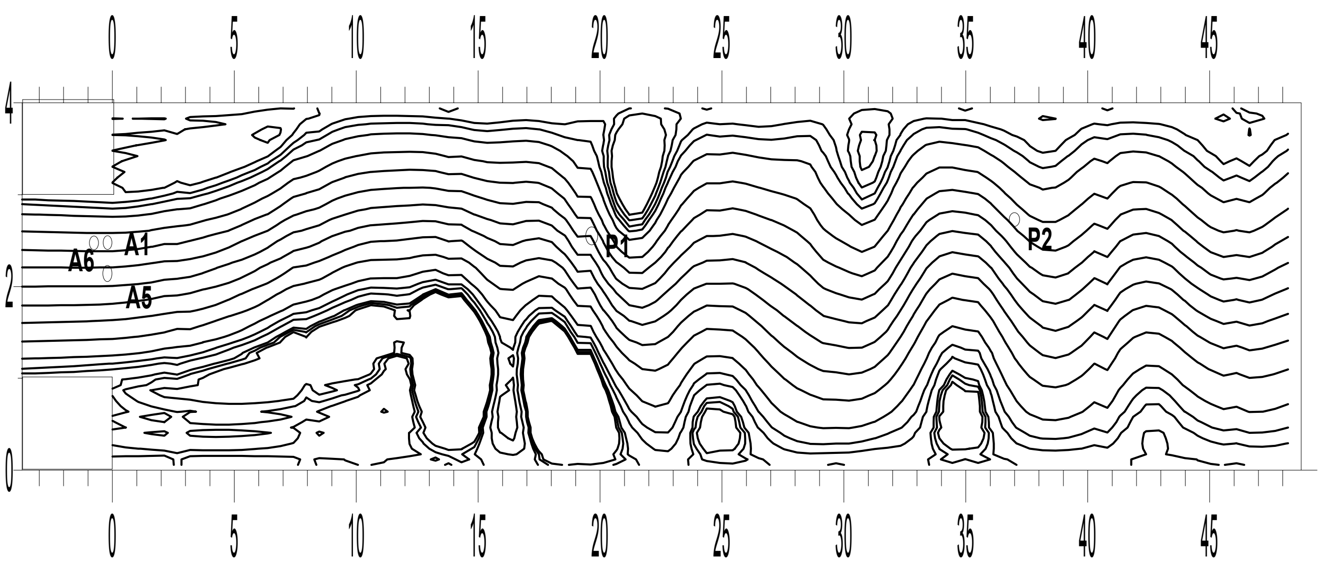


FIGURE 1b. General view of the flow pattern

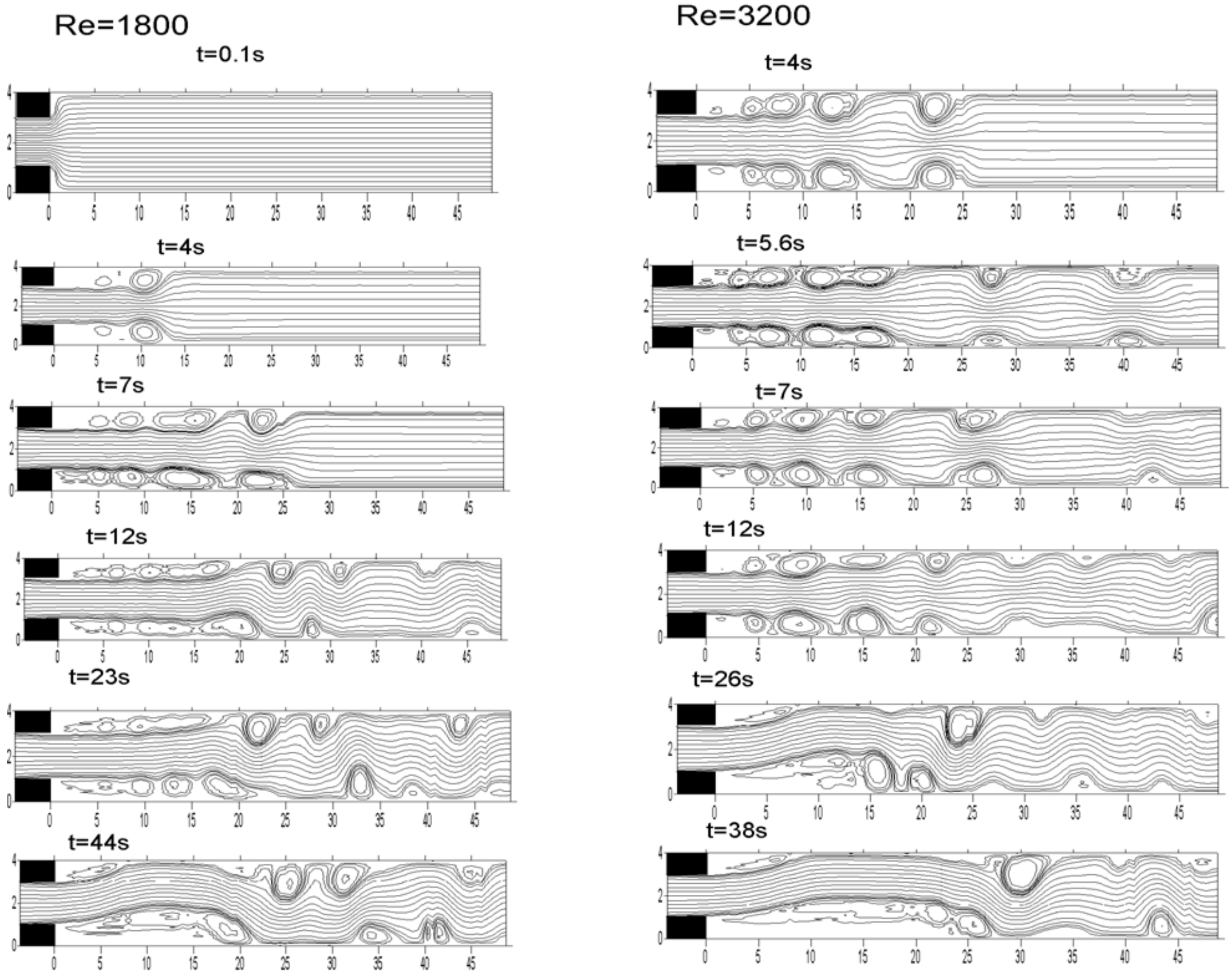


FIGURE 2. Calculated streamlines at various moments

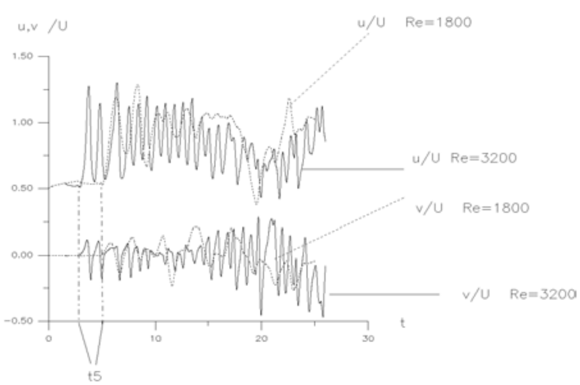


FIGURE 3. Non-dimentionalised velocity u & v at point P1(20, 2.4) for Re=1800 and 3200

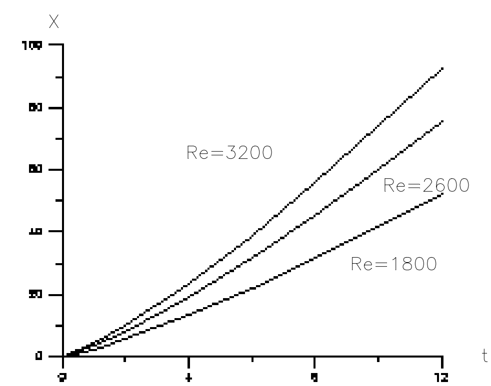


FIGURE 4. Disturbance-Front distance X from the Step (as a function of Re and time t)

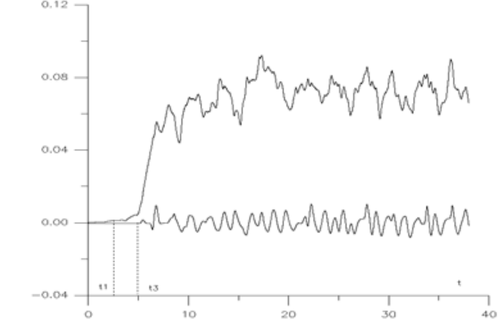


FIGURE 5.
Mean value along the channel center-line of V velocity, Re=3200

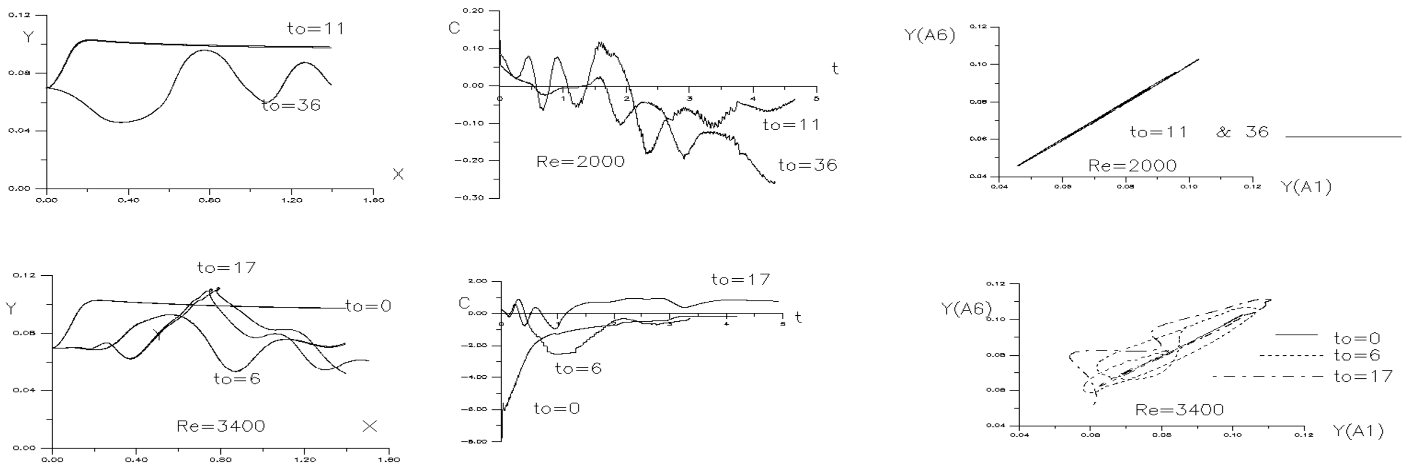
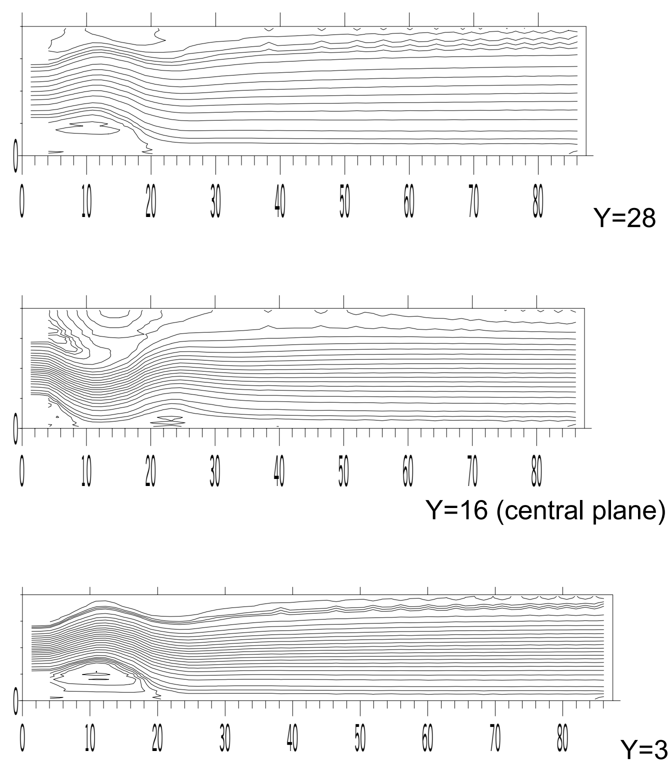


FIGURE 6. The paths of the elements (in moment to) A1(0/0.07) and A6(−0.00286/0.07) for various Re and to, the relevant coefficient C and the phase−diagrams with coordinates [Y(A1), Y(A6)] .



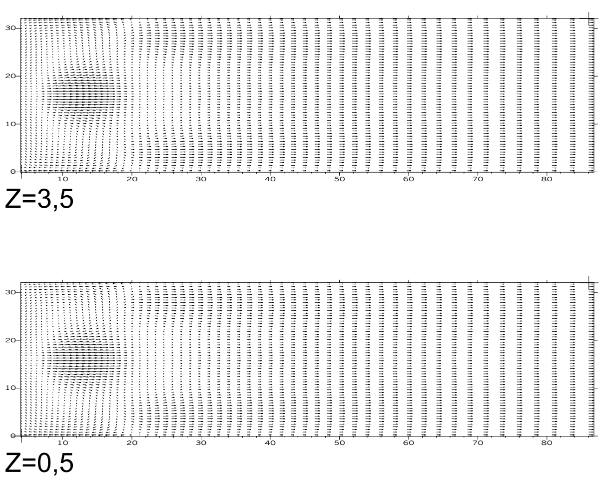


FIGURE 7: Stream lines in the planes Y=28, 16 (central plane) 3 and velocity vectors in Z=3,5 and 0,5 for Re=300