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ON NUMERICAL SIMULATION OF SUPERSONIC FLOW USING BOW-SHOCK-FITTING TECHNIQUE

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ABSTRACT

The calculation algorithm for supersonic flow of the hollow cylinder with bow shock wave fitting was applied in this work in a way of a mobile grid use. The model of inviscid non-heat-conducting gas in accordance with the modified finite-difference scheme of Godunov was used in order to describe the flow.

The calculations on mobile grids with Godunov's scheme showed that the pressure pulsed in the cavity had practically decayed. However, using the modified scheme of Godunov it was possible to pick out the oscillations, and the main features of the process corresponded both to the calculation and the experiment.

Keywords: supersonic flow, shock wave, pulsation, cavity.

INTRODUCTION

Great attention is traditionally paid to the methodical experiment in the area of computation hydrodynamics. The problems of the methodical experiment include evaluation the influence of a difference grid step, disposition of nodes and their quantity in a calculation domain, an order of approximation scheme of discretization the initial equation, boundary conditions and also determination of the calculation domain dimension, especially for flow the bodies detached with stream. Now a day the role of methodical experiment becomes more important and its functions are extended, since numerical simulation becomes an instrument to predict flow characteristics during constructing or determining the exploitation regimes of various apparatus and devices.

Numerical simulation of many gas-dynamics problems of quite complex flows uses the model of an inviscid non-heat-conducting gas and difference schemes for start-to finish calculation. Both steady and unsteady flows with more or less numerous strong and weak (including interacting) discontinuities are normally considered in this formulation. Numerical methods, using simplified motion equations, do not fit in a number of cases, since they don't provide registration of the most important physical laws of the being researched flows and don't have the necessary accuracy. At the same time a wide class of a detached flows with free interaction exists and their investigation don't require use of the Navier-Stocks equations [1,2]. Results comparison of both numerical and physical simulation points to the fact that while numerical simulating of a flow, diffusion transfer may be reproduced due to both physical viscosity (molecular, turbulent) and scheme viscosity, input during the transfer discretization of the equations [3].

The majority of engineering problems, which require solutions, are unsteady. Auto-oscillation regimes of pulse flows, including interaction of free displacement layer with body surface, appear very often in engineering applications. Mechanism of inverted connection has to be present in order to maintain oscillations. It was noticed that selective amplification of oscillations of certain frequencies occurs in unstable free shear layer. During the inleakage of a shear layer on the body surface pressure vibrations arise, which are transmitted up along the flow through the subsonic flow area that represent a mechanism of inverted connection. Pressure pulsation in open cavities and also pressure pulsation arising while flow the needle-shaped bow of the body is related to this class of hydro-dynamical phenomena [1].

NOMENCLATURE

$\overline{A}$ - the flow vector of conservative variables;
a_∞ - the velocity of the incident flow;
d - the external diameter of the cylinder;
E - the specific total energy;
$\bar{\mathrm{i}}_x$, $\bar{\mathrm{i}}_y$ - the unit vectors of a Cartesian system coordinates;
M - the Mach number;

Re - the Reynolds number;

STATEMENT OF THE PROBLEM

Finite-difference model of a continuum based on non-linear equations of steady motion of an ideal gas (Euler equations) very often provides good compliance to a real shock-wave flow [5]. In contrast to stationary models unsteady difference model in through count doesn't form ideal tangential breaks but numerical zones of displacement and establishing a mutual influence of gas-dynamics parameters in the whole flow area through the zones in the unsteady transient process while solving the boundary value problem for the initial conditions.

The Navier-Stocks equations were used in the previous researches for simulation of supersonic flow cone with concave bow ($\mathrm{M}_\infty = 10$, $\mathrm{Re}_\infty = 7.1\times10^6$ [6]) and a hollow cylinder ($\mathrm{M}_\infty = 3.7$, $\mathrm{Re}_\infty = 5\times10^4$ [7]). Good coincidence of numerical and experimental results was obtained in [6] to distribute static pressure in the bottom of the hollow, however some divergence in pulse amplitude of the bow shock concentration appears. The average deviation of a shock wave and the pulse amplitude of shock concentration in front of the hollow cylinder, which were obtained in the calculation [7] for $\mathrm{Re}_\infty = 5\times10^4$, satisfy the experimental data [8] for $\mathrm{Re}_\infty \geq 10^5$.

The calculation algorithm for supersonic flow of the hollow cylinder with bow shock wave fitting was applied in this work in a way of a mobile grid use. The model of inviscid non-heat-conducting gas in accordance with the modified finite-difference scheme of Godunov [9] was used in order to describe the flow. Substantiation of the scheme choice and advisability of its modification were represented [9-11].

In the frames of the finite volumes [10] the differential equations, which describe inviscid non-heat-conducting gas flow, are noted in a form of integral conservation laws. The equations may be presented in a form analogous to [9] for the two-dimensional gas flow in Cartesian system coordinates:

$$\frac{\partial}{\partial t}\int_{\Omega} F d\Omega + \int_{\sigma} \overline{A}\,\overline{n}\, d\sigma = 0 , \qquad (1)$$

where

$$F = \begin{bmatrix} \rho \\ \rho u \\ \rho \upsilon \\ \rho E \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} \rho(\overline{q} - \overline{\lambda}) \\ \rho u(\overline{q} - \overline{\lambda}) + p\bar{\mathrm{i}}_x \\ \rho \upsilon(\overline{q} - \overline{\lambda}) + p\bar{\mathrm{i}}_y \\ \rho E(\overline{q} - \overline{\lambda}) + p\overline{q} \end{bmatrix}. \qquad (2)$$

Here $\bar{\mathrm{i}}_x$, $\bar{\mathrm{i}}_y$ are the unit vectors of a Cartesian system coordinates, F is the vector of conservative variables, $\overline{A}$ is the flow vector of conservative variables, σ is the boundary surface of the given volume element Ω, having external norm $\bar{\mathrm{n}}$ and moving at velocity $\overline{\lambda}$, $\overline{q} = u\bar{\mathrm{i}}_x + \upsilon\bar{\mathrm{i}}_y$ is the velocity vector of gas flow, u and υ are the components in the x and y directions, p and ρ are the pressure and density of the gas and E is the specific total energy, t is the time. The system is completed with the equation state of an ideal gas.

Unlike the usual representation, the defining equations include the cell velocity $\overline{\lambda}$ and the volume of each element depends on time. For the numerical calculations the equations have advantages such that they give the opportunity to work with cells of an arbitrary form with mobile planes.

Making the values dimensionless was realized as follows:

$$y = \overline{y}d/2 ,\ x = \overline{x}d/2 ,\ t = \overline{t}d/2a_\infty ,\ a = \overline{a}a_\infty ,\ u = \overline{u}a_\infty ,$$
$$\upsilon = \overline{\upsilon}a_\infty ,\ \rho = \overline{\rho}\rho_\infty ,\ p = \overline{p}\rho_\infty a_\infty^2 \qquad (3)$$

(a_∞ is the velocity of the incident flow , d is the external diameter of the cylinder).

Dimensionless parameters of unperturbed entering flow were accepted in calculations as the initial data:

$$p = p_\infty = 1/\gamma ,\ \rho = \rho_\infty = 1 ,\ u = u_\infty = \mathrm{M}_\infty ,\ \upsilon = 0 \qquad (4)$$

(γ is the ratio of specific heat capacities of the gas, $\gamma = 1.4$ in calculations). A line above the dimensionless parameters $y, x, t, a, u, \upsilon, \rho, p$ are absent in the text hereinafter.

Conditions of non-leakage on the body surface and conditions characterizing the entering flow [10] are represented as the boundary conditions.

The calculation were carried out on rectangular (Figure 1) immobile (60×40, 100×40) $ABCD$ and mobile (38×40, 65×40) grids $A^{`}B^{`}CD$ with $M_\infty = 3.7$. Relative depth of the cylinder cavity l/d was changing from 0 to 1.6 (width of the cylinder wall is $\delta/d = 0.04$). The node distribution inside the cylinder $DEFG$ was uniform $\Delta x = 0.1$, $\Delta y = 0.04$ for grids 60×40, 38×40 and $\Delta x = 0.06$, $\Delta y = 0.06$ for the 100×40, 65×40 grids. The grid steps for x and y were increased with distance from the cylinder to allow a sensible choice of the boundaries of the calculation domain. The mobile grid was based in the same way as in [9]. Motion calculation of the grid boundary, which tracks the disposition of the bow shock wave, is done analogous to [10].

DISCUSSION OF THE RESULTS

It was defined that the result of the calculations on the 100×40 fixed grid are in fairly good agreement with the results of [7] ($l/d = 1.6$) and correspond to a regime of flow at the Reynolds number of the incident flow $Re_\infty \geq 10^5$, where the influence of Re on the basic characteristics of the process (average withdrawal of the shock wave, pulsation amplitude of the shock wave, oscillation period, mean-square values of pressure pulsation) is not substantial according to [8]. The average withdrawal of the shock wave $\Delta^0 = \Delta/d$, pulsation amplitude $\Delta^* = \Delta'/d$ and Strouhal number $Sh = s/(a_0 t^0)$ for the different version are given in Table ($Sh \approx 0.25$ in the experiments of [8]) Here t^0 is the period of oscillation, a_0 is the velocity of sound at the braking temperature and $s = (l + \Delta)$ is the characteristic length (cf. Figure 1).

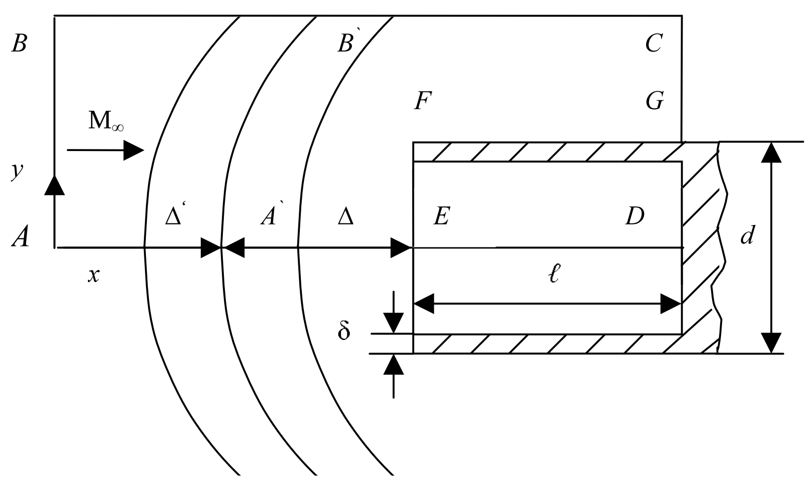

Fig.1 The calculation grid

A change in the ratio of lengths of the cell sides in the cavity in the fixed grid 60×40 $(\Delta x/\Delta y = 0.4)$ led to increase in the mean withdrawal of the shock wave and pulsation amplitude. This must obviously be explained by influence of the approximation viscosity. In the experiment [8] a sharp increase in the mean withdrawal and pulsation amplitude was observed when Re_∞ was reduced in the region of $Re_\infty = 10^5$. The main features of the pulsation process obtained on the fixed 60×40 grid (see Table 1) are similar to the experimental results [8] for $\mathrm{Re}_\infty = 5\times10^4$.

Table 1 The main features of the pulsation process

Δ^*	Δ'	Sh	Grid size	Difference scheme
0.3	0.08	0.246	81×81 (fixed)	[7]
0.33	0.12	0.252	100×40 (fixed)	Godunov
0.3	no	no	38×40 (mobile)	Godunov
0.32	0.09	0.251	38×40 (mobile)	modified
0.4	0.3	0.237	60×40 (fixed)	Godunov
0.3	no	no	65×40 (mobile)	Godunov
0.3	0.07	0.250	65×40 (mobile)	modified

The calculations on mobile 38×40, 65×40 grids with Godunov's scheme showed that the pressure pulsed in the cavity had practically decayed by the time $t\approx40$ (cf. [10]). However, using the modified scheme [10] it was possible to pick out the oscillations, and the main features of the process (see Table) corresponded both to the calculation [7] and the experiment [8] for $\mathrm{Re}_\infty > 10^5$.

With a relative cavity depth $l/d < 1.6$ the use of a mobile grid in the calculations permitted to determine the pulse amplitude of the shock wave (it is difficult to realize this on the immobile grid, since the pulse amplitude was commensurable to the grid dimension of a cell).

It is clear from an analysis of the results that the modified scheme [9] can be used to calculate complicated unsteady flows. It is good qualitative agreement with experiment [8] and with a calculation that allows for the viscosity of the gas [7].

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