Proceedings ICFDP7:
Seventh International Congress on Fluid Dynamics and Propulsion
18–20 December 2001, Cairo, Egypt

ICFDP7-2001040

SPATIO–TEMPORAL STABILITY OF FLOW THROUGH VISCOELASTIC TUBES

M. Hamadiche
LMFA, Ecole Centrale de Lyon
69131 Ecully CEDEX, France
<hamadich@mecaflu.ec-lyon.fr>

M. Gad-el-Hak
Department of Aerospace & Mechanical Engineering
University of Notre Dame
Notre Dame, IN 46556, U.S.A.
<gadelhak@nd.edu>

ABSTRACT

The stability of the Hagen–Poiseuille flow of a Newtonian fluid in an incompressible, viscoelastic tube contained within a rigid, hollow cylinder is determined using linear stability analysis. The stability of the system subjected to infinitesimal axisymmetric or non-axisymmetric disturbances is considered. A novel numerical strategy is introduced to study the stability of the coupled fluid–structure system. It is found that the system is unstable to both axisymmetric and non-axisymmetric disturbances. For a given azimuthal wavenumber n, it is found that there are no more than two unstable modes. For both the axisymmetric (n=0) and non-axisymmetric instabilities (n=1) one mode is a solid-based, flow-induced surface instability, while the other one is a fluid-based instability that asymptotes to the least-damped rigid-wall mode as the thickness of the compliant wall tends to zero. One of the non-axisymmetric unstable mode represents an absolute instability, while the three remaining modes represent convective instabilities. All four modes are stabilized, to different degrees, by the solid viscosity.

INTRODUCTION

For close to half a century the science and technology of compliant coatings has fascinated, frustrated and occasionally gratified scientists and engineers searching for methods to delay laminar-to-turbulence transition, to reduce skin-friction drag in turbulent wall-bounded flows, to quell vibrations, and to suppress flow-induced noise. Compliant coatings offer the potential for favorable interference with a wall-bounded flow. In its simplest form, the technique is passive, relatively easy to apply to an existing tube or vehicle, and perhaps not too expensive. Unlike other drag reducing techniques such as suction, injection, polymer or particle additives, passive compliant coatings do not require slots, ducts or internal equipment of any kind. Aside from reducing drag, other reasons for the perennial interest in studying compliant coatings are their many other useful applications, for example as sound absorbent materials in noisy flow-carrying ducts in aero-engines, and as flexible surfaces to coat naval vessels for the purposes of shielding their sonar arrays from the sound generated by the boundary-layer pressure fluctuations and of reducing the efficiency of their vibrating metal hulls as sound radiators. An even more important reason to study compliant coatings is the fact that all tubes carrying the bodily fluids of humans and other animals are flexible and the fluid–structure interaction there constitutes a fascinating albeit formidable problem in physics.

THE PRESENT PROBLEM

In this paper, we consider a system where a viscoelastic tube is coupled with a Hagen–Poiseuille flow. The homogeneous, viscoelastic tube is a hollow cylinder surrounded on the outside by a rigid wall, and the basic flow inside the tube is the Hagen-Poiseuille flow in a circular pipe as schematically depicted in Figure 1.

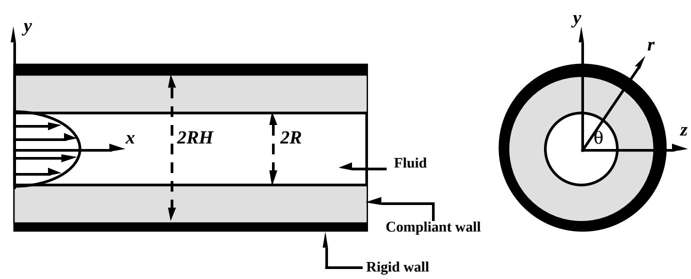

Figure 1. System configuration.

We study numerically the stability of the coupled fluid–structure system to infinitesimal axisymmetric or non-axisymmetric disturbances. Both the fluid flow and the compliant wall are considered incompressible. It is well known that the Hagen–Poiseuille flow in a rigid circular cylinder is stable to infinitesimal axisymmetric disturbances as well as to infinitesimal azimuthally varying disturbances. The stability of Hagen–Poiseuille flow in a compliant circular pipe has been studied by Evrensel et al. (1993), who considered only low-Reynolds-number flows and therefore neglected inertia. They have shown that the coupled system is unstable to axisymmetric perturbations.

Hamadiche (1997; 1998) showed that there are two unstable axisymmetric modes. Hamadiche and Gad-el-Hak (2001a) have studied extensively the stability of the coupled system to both axisymmetric and non-axisymmetric infinitesimal disturbances. Computing the group velocity of the unstable mode, they concluded that one of the non-axisymmetric unstable mode may represent an absolute instability. In order to study the spatio–temporal stability of the system, a numerical method is developed to compute—without a need for an initial guess—all the normal modes of the fluid–structure system within a given closed region in the complex s-plane, where s stands for the complex frequency, for any complex wavenumber k. This method is an extension of the strategy used by Hamadiche (1999a; 1999b) and Hamadiche and Gad-el-Hak (2001a). Finally, by removing the rigid shell, the compliant tube becomes collapsible. This problem is studied in a companion paper in these proceedings and reported in more details in the article by Hamadiche and Gad-el-Hak (2001b).

SYSTEM INSTABILITIES

From a fundamental viewpoint, a rich variety of fluid–structure interactions exists when a fluid flows over a surface that is able to interact with the flow. Not surprisingly, instability modes proliferate when two wave-bearing media are coupled. Some waves are flow-based, some are wall-based, and some are a result of the coalescence of both kind of waves. What is most appealing about compliant coatings is their potential to inhibit, or to foster, the dynamic instabilities that characterize both transitional and turbulent boundary-layer flows, and in turn to modify the mass, heat and momentum fluxes and change the drag and the acoustic properties. While it is relatively easy to suppress a particular instability mode, the challenge is of course to prevent other modes from growing if the aim is, say, to delay laminar-to-turbulence transition. From a practical point of view, it is obvious that an in-depth understanding of the coupled system instabilities is a prerequisite to rationally designing a coating that meets a given objective.

There are at least three classification schemes for the fluid–structure waves, each with its own merits. The original scheme is attributable to Landahl (1962) and Benjamin (1963). It divides the waves into three classes according to their response to irreversible energy transfer to and from the compliant wall. Both class **A** and class **B** disturbances are essentially oscillations involving conservative energy exchanges between the fluid and solid, but their stability is determined by the net effect of irreversible processes such as dissipation in the coating or energy transfer to the solid by non-conservative hydrodynamic forces. Class **A** oscillations are Tollmien–Schlichting waves in the boundary layer modified by the wall compliance, in other words by the motion of the solid in response to the pressure and shear-stress fluctuations in the flow. The disturbance eigenfunction for class **A** waves has its maximum amplitude within the fluid region. Such waves are stabilized by the irreversible energy transfer from the fluid to the coating, but destabilized by dissipation in the wall.

Class **B** waves is to be found in both the fluid and the wall. However, the disturbance eignenfunction has its maximum amplitude at the fluid–solid interface and thus those waves are principally wall-based modes of instability. Such instability would not exist had the wall been rigid. The instability is due to the downstream-running freewave in the solid being modified by the fluid loading. The destabilization of class **B** waves is effected by the phase-difference between the pressure perturbation and the wall deformation, which allows a flow of energy from the fluid into the wall. The behavior of Class **B** waves is the reverse of that for class **A** waves, stabilized by wall damping but destabilized by the non-conservative hydrodynamic forces. Essentially class **B** waves are amplified when the flow supplies sufficient energy to counterbalance the coating's internal dissipation.

Finally, class **C** waves are akin to the inviscid Kelvin-Helmholtz instability and occur when conservative hydrodynamic forces cause a unidirectional transfer of energy to the solid. The pressure distribution in an inviscid flow over a wavy wall is in exact antiphase with the elevation. In that case, class **C** waves can grow on the solid surface only if the pressure amplitude is so large as to outweigh the coating stiffness. Class **C** waves are the result of a modal-coalescence instability where the flow speed is sufficiently high that the originally upstream-running wall freewaves are turned to travel downstream and merge with the modified downstream-running wall waves. Irreversible processes in both the fluid and solid have negligible effect on class **C** instabilities.

If one considers the total disturbance energy of the coupled fluid–solid system, a decrease in that energy leads to an increase in the amplitude of class **A** instabilities, class **B** is associated with an energy increase, and virtually no change in total energy accompanies class **C** waves. In other words, any non-conservative flow of activation energy from/to the system must be accompanied by disturbance growth of class **A**/**B** waves, while the irreversible energy transfer for class **C** instability is nearly zero.

The second classification scheme is due to Carpenter and Garrad (1985; 1986). It simply divides the waves into fluid-based and solid-based. Tollmien–Schlichting instability (TSI) is an example of fluid based waves. The solid-based, flow-induced surface instabilities (FISI) are closely analogous to the instabilities studied in hydro- an aeroelasticity, and include both the traveling-wave flutter that moves at speeds close to the solid free-wave-speed (class **B**) and the essentially static—and more dangerous—divergence waves (class **C**). The main drawback of this classification scheme is that under certain circumstances the fluid-based T–S waves and the solid-based flutter can coalesce to form a powerful new instability termed the transitional mode by Sen and Arora (1988). According to the energy criterion advanced by Landahl (1962), this latest instability is a second kind of class **C** wave. In a physical experiment, however, it is rather difficult to distinguish between static divergence and the transitional mode.

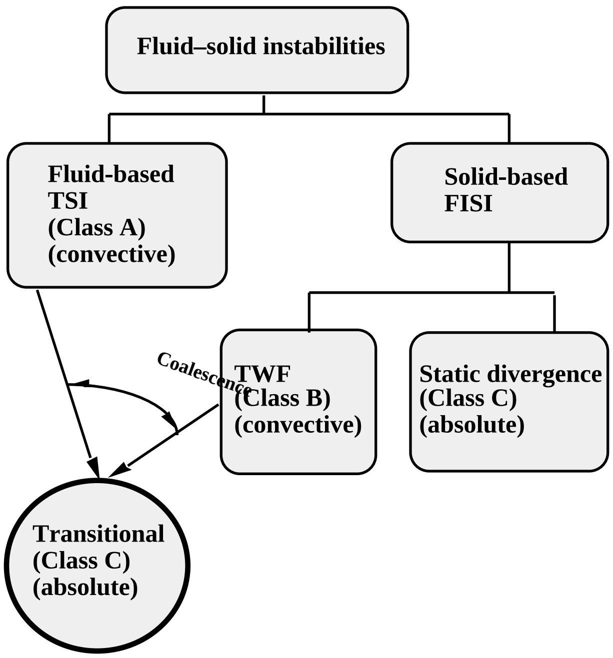

Figure 2. Summary of all three classification schemes.

The third scheme to classify the instability waves considers whether they are convective or absolute (Huerre and Monkewitz, 1990). An instability mode is considered to be absolute if there is a pinch point in the Fourier contours which prevents the temporal amplification rate from being reduced down to zero. This is equivalent to the existence of a cusp point in the Laplace contours (Kupfer et al., 1987). In this case, the unstable mode propagates upstream as well as downstream and often has a very small (or even zero) group velocity in comparison with the velocity of the mean flow. On the other hand, the unstable development of a disturbance is said to be convective when none of its constituent modes possesses zero group velocity. Both classes **A** and **B** are convective, while class **C** divergence and the transitional modes are absolute. As Carpenter (1990) points out, the occurrence of absolute instabilities would lead to profound changes in the laminar-to-turbulence transition process. 'It is therefore pointless to consider reducing their growth rate or postponing their appearance to higher Reynolds number; nothing short of complete suppression would work.' Figure 2 combines and summarizes all three classification schemes.

In the present study, the stability of the Hagen–Poiseuille flow of a Newtonian fluid in an incompressible, viscoelastic tube contained within a rigid, hollow cylinder is determined using linear stability analysis. The stability of the system subjected to infinitesimal axisymmetric or non-axisymmetric disturbances is considered. A novel numerical method is introduced to study—without a need for an initial guess—the spatio–temporal stability of the coupled fluid-structure system. We will focus on the classification of the unstable modes in order to distinguish the modes representing convective instabilities from those representing absolute instabilities. Hamadiche and Gad-el-Hak (2001a) provide more details of the temporal stability of the same system studied herein.

STABILITY OF AXISYMMETRIC DISTURBANCES

For all the wall and fluid parameters explored here in the closed region in the complex s-plane, we do not find more than two unstable axisymmetric modes, which we term Mode I and Mode II. The axisymmetric unstable Mode I is strongly stable for $H<1.7$, and diverges, i.e. becomes extremely stabilized, as the coating thickness goes to zero. For $H>1.7$, Mode I is weakly unstable (see Hamadiche and Gad-el-Hak, 2001a). At small values of H, Mode I wavespeed is much larger than 1. Recalling that the rigid-walled tube result of Salwen and Grosch (1972) gives a wavespeed that tends to either unity or zero as the Reynolds number goes to infinity. It follows then that Mode I instability does not exist in rigid tubes and is due to the viscoelastic wall. According to Carpenter and Garrad (1985), this is a solid-based, flow-induced surface instability.

The second axisymmetric unstable mode, Mode II, is identified with the least-damped mode in the rigid-walled tube when the depth of the viscoelastic coating goes to zero (see Hamadiche and Gad-el-Hak, 2001a). This is then a fluid-based instability according to the classification scheme of Carpenter and Garrad (1985).

STABILITY OF NON-AXISYMMETRIC DISTURBANCES

For all the wall and fluid parameters explored here in the closed region in the complex s-plane, we do not find more than two unstable non-axisymmetric modes for each value of the azimuthal wavenumber n. For $n=1$, we term the corresponding modes Mode III and Mode IV. The non-axisymmetric Mode III becomes extremely stabilized when the depth of the viscoelastic material goes to zero. This is a solid-based, flow-induced surface instability. The second non-axisymmetric unstable mode, Mode IV, is identified with the least-damped mode in the rigid-walled tube when the depth of the viscoelastic coating goes to zero. This is a fluid-based instability. For more information on the dependence of those unstable modes on different system parameters, see Hamadiche and Gad-el-Hak (2001a). A résumé of the proprieties of those unstable modes is given in Table 1 and in the next section.

DISCUSSION

We examined in more detail the unstable modes obtained with the two azimuthal wavenumbers $n=0$ and $n=1$. For $n=0$, we found two unstable axisymmetric modes which we termed Mode I and Mode II. For $n=1$, there are two unstable non-axisymmetric modes: Mode III and Mode IV. We found that for high Reynolds numbers, only Mode III persists. When the depth of the viscoelastic layer goes to zero, Mode II and IV asymptote to the least-damped modes of Hagen–Poiseuille flow in a rigid-walled tube, while Mode I and III become extremely stabilized. It is thus concluded that Mode II and IV are fluid-based instabilities, while Mode I and III are solid-based, flow-induced surface instabilities.

The effect of the compliant-wall viscosity was examined by Hamadiche and Gad-el-Hak (2001a). It was found that wall dissipation damps, to different degrees, all four unstable modes.

	Mode I	Mode II	Mode III	Mode IV
	A.S.	A.S.	Non A.S.	Non A.S.
Base	Solid-based	Fluid-based	Solid-based	Fluid-based
n	0	0	1	1
H-1<<1	Diverges	Least-damped rigid-wall mode	Diverges	Least-damped rigid-wall mode
Re>>1	Stable	Stable	Unstable	Stable
Group velocity	Positive	Negative	Varies strongly with *k*	Nearly zero
Solid viscosity	Stabilizing	Stabilizing	Stabilizing	Stabilizing
Nature	Convective	Convective	Convective	Absolute

Table I. Summary of the properties of Mode I–Mode IV instabilities. A.S. stands for Axisymmetric.

This suggests that these modes do not belong to class **A**, introduced by Benjamin (1963). For all four unstable modes, we plot in Figure 3 the group velocity and amplification rate as functions of the wavenumber. Mode I has a significant positive group velocity, although consistently lower than the mean flow velocity at the center of the tube. It may thus be inferred that the solid-based, axisymmetric Mode I is a flutter mode and belongs to class **B**. Mode II has a significant negative group velocity which is higher than the velocity of the mean flow at the center of the tube. The group velocity of Mode III is positive for low wavenumbers (long wavelengths) and negative for high wavenumbers. The non-axisymmetric Mode III is very dispersive. The group velocity of Mode IV is close to zero for most of the range of unstable wavenumbers. This suggests that Mode IV, though fluid-based, is a standing-wave divergence instability belonging to class **C**. Hamadiche and Gad-el-Hak (2001a) suggested an examination of the spatio-temporal instability, along the lines described by Yeo et al. (1999) for boundary layers, in order to reveal the presence of a pinch point in the Fourier contours and thus to confirm the absolute nature of Mode IV instability.

Figure 4 shows the Laplace contours related to Mode IV. The cusp point in one of those contours indicates that Mode IV represnts, in fact, an absolute instability, confirming the prediction of Hamadiche and Gad-el-Hak (2001a). Figure 5 shows the Laplace contours of the unstable mode III representing an example of a convective instability. No cusp point exists in this case.

An inspection of the energy balance equation related to the viscoelastic wall shows that the system is unstable when the energy flux is from the fluid to the wall. The energy flux from the flowing fluid to the viscoelastic wall across the interface was numerically computed and compared to the amplification rate.

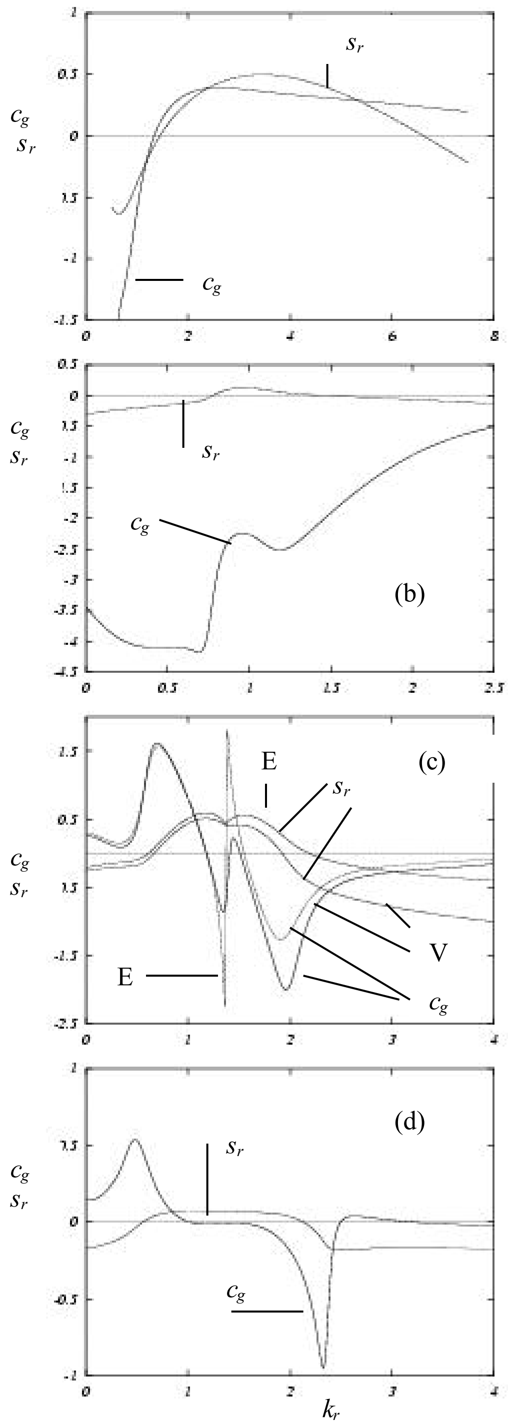

Figure 3. Group velocity c_g and amplification rate versus the wavenumber. E: elastic wall, μ_r=0; V: viscoelastic wall, μ_r=0.3. (a) Mode I; Γ=6, *H*=2, *k*=2.5, *n*=0, *Re*=23. (b) Mode II; Γ=6, *H*=2, *k*=1, *n*=0, *Re*=100. (c) Mode III; Γ=7, *H*=2, *k*=1, *n*=1, *Re*=100. (d) Mode IV; Γ=7, *H*=2, *k*=1, *n*=1, *Re*=100. In figure 3c the group velocity is plotted for a viscoelastic wall as well as an elastic wall in order to verify the high variation of the group velocity of an elastic wall near the wave number *k*=1.4.

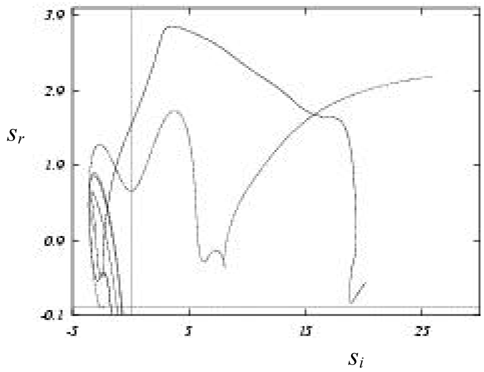

Figure 4. Laplace contours related to the unstable mode IV. The presence of a cusp point in these contours confirmes the prediction of Hamadiche and Gad-el-Hak (2001a) about the absolute instability introduced by this mode.

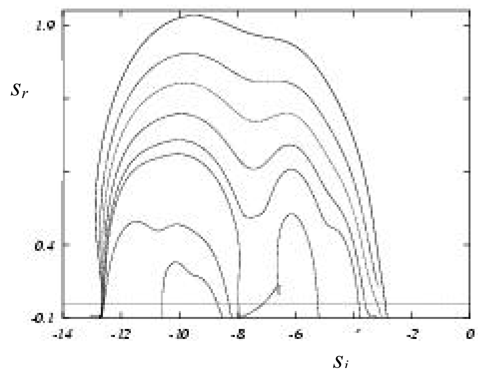

Figure 5. Laplace contour related to the unstable mode III. As one can see, there is no cusp point in this Laplace contours so the instability is convective. A similar Laplace contour for the Modes I and II show that the instabilities representing by those modes are convective.

It was found that the system is unstable (stable) when the energy flux is from (to) the fluid to (from) the viscoelastic medium. The power developed by each force component at the interface was computed by Hamadiche (1999a; 1999b). It was found that at high Reynolds numbers, the energy transfer from the fluid to the solid is mostly done by the work of the component of the stress perpendicular to the wall (pressure force), while the tangential stress plays a more important role at low Reynolds numbers.

Examination of the total energy balance for the coupled system showed that the fluid-based disturbance energy is produced, to different degrees, by the work of the viscous shear stress at the interface between the fluid and the solid and the Reynolds stress in the fluid. Computation of the work done by the shear stress at the interface and the velocity of the disturbance showed that the energy of the unstable axisymmetric Mode I and non-axisymmetric Mode IV is produced by the work of the shear stress at the interface, while the Reynolds stress tends to stabilize these disturbances. On the other hand, the energy of the unstable axisymmetric Mode II and non-axisymmetric Mode III is produced by both the Reynolds stress and the shear stress at the interface. Thus, the Reynolds stress seems to have a stabilizing effect on Modes I and IV but a destabilizing effect on Modes II and III. It is worthy to note that there is no difference in energy supply of Mode I and Mode IV even though the latter mode represents an absolute instability.

CONCLUSIONS

In the present study, the stability of the incompressible Hagen–Poiseuille flow of a Newtonian fluid in a compliant tube contained within a rigid, hollow cylinder was determined using linear stability analysis. The wall was modeled as a homogeneous, incompressible, viscoelastic medium, where the stress has an elastic component proportional to the strain and a viscous component proportional to the strain rate. The dynamics of the system is influenced by four dimensionless parameters: the Reynolds number $Re=\rho VR/\mu$; the ratio of radii H; the ratio of the viscosities of the wall material and the fluid $\mu_r = \mu_s / \mu$, and the dimensionless velocity $\Gamma=(\rho V^2/G)^{1/2}$, where G is the coating's shear modulus.

We examined in more detail the most unstable modes obtained with the two azimuthal wavenumbers n=0 and n=1. For n=0, we found two unstable axisymmetric modes which we termed Mode I and Mode II. Those modes represent convective instabilities. For n=1, there are two unstable non-axisymmetric modes: Mode III represents a convective instability while Mode IV represents an absolute instability.

It was shown that the most excited mode may be an axisymmetric or non-axisymmetric one according to the values of several parameters. We found that when the Reynolds number is close to zero, the first excited mode is the axisymmetric Mode I. When the Reynolds number increases, the non-axisymmetric Mode IV with an azimuthal wavenumber n=1 becomes the most unstable mode. By further increasing the Reynolds number, the previous mode is replaced by the non-axisymmetric Mode III with an azimuthal wavenumber n=1. For larger values of the Reynolds number, the first excited mode is the axisymmetric Mode II.

The unstable axisymmetric and non-axisymmetric eigenfunctions were plotted versus the radial coordinate r by Hamadiche (1999a; 1999b). As intuitively expected, it was found that the radial velocity component is small in comparison to the streamwise and azimuthal velocity components. The streamwise and azimuthal velocity components are, however, of the same order of magnitude. The form of the unstable disturbance shows that the Reynolds stress tends to stabilize the unstable non-axisymmetric Mode IV when the instability takes place at low Reynolds numbers and the energy of the disturbance is supplied by the deformation work done at the interface.

The form of the unstable non-axisymmetric Mode III shows that the Reynolds stress tends to destabilize the system and, together with the deformation work, supplies energy to the unstable mode. The low-Reynolds-number Mode I instability whose growth is energized only by the work of the viscous shear stress at the fluid–solid interface is perhaps related in physical origin to the surface-shear-dominated instability termed B_2 mode by Yeo (1988) who identified it while investigating the stability of the canonical boundary layer flow over the same type of compliant coating analyzed in here.

Yeo's mode would occur in the developing region of a pipe flow and might be the natural precursor to Mode I instability in the fully-developed region. Although the B_2 instability mode may not exist right down to zero Reynolds number for a boundary layer due to the strong divergence of the mean flow in that regime, the instability can exist to significantly high *Re* and may even merge or coalesce with other instabilities as the surface becomes highly compliant, producing perhaps the like of the coalescence Mode IV identified in our investigation.

In the present formulation, we neglected the effect of pressure gradient along the undisturbed flow on the compliant wall deformation. Owing to this favorable pressure-gradient, the deformation would increase along the tube, thus slightly reducing the effective tube radius *R*. A variable-radius pipe is a challenging, spatially inhomogeneous problem which we defer to another study, and the radius *R* was taken as constant in the present formulation.

The results described here should be useful when designing a compliant coating to effect certain beneficial flow-control goals in man-made pipe flows. The present stability analysis is linear, but extending the present exercise to nonlinear stability calculations may be a useful next step. Finally, applying the compliant tube studies to the understanding of bodily-fluid flows in various biological systems is certainly a worthwhile endeavor.

REFERENCES

Benjamin, T.B. (1963) "The Threefold Classification of Unstable Disturbances in Flexible Surfaces Bounding Inviscid Flows," *J. Fluid Mech*. **16**, pp. 436–450.

Carpenter, P.W. (1990) "Status of Transition Delay Using Compliant Walls," in *Viscous Drag Reduction in Boundary Layers*, eds. D. M. Bushnell and J. N. Hefner, pp. 79–113, AIAA, Washington, D.C.

Carpenter, P.W., and Gajjar, J.S.B. (1990) "A General Theory for Two- and Three-Dimensional Wall-Mode Instabilities in Boundary Layers over Isotropic and Anisotropic Compliant Walls," *Theor. Comput. Fluid Dyn*. **1**, pp. 349–378.

Carpenter, P.W., and Garrad, A.D. (1985) "The Hydrodynamic Stability of Flow over Kramer-Type Compliant Surfaces. Part 1. Tollmien–Schlichting Instabilities," *J. Fluid Mech.* **155**, pp. 465–510.

Carpenter, P.W., and Garrad, A.D. (1986) "The Hydrodynamic Stability of Flow over Kramer-Type Compliant Surfaces. Part 2. Flow-Induced Surface Instabilities," *J. Fluid Mech.* **170**, pp. 199–232.

Evrensel, C.A., Khan, M.R., Elli, S., and Krumpe, P.E. (1993) "Viscous Airflow through a Rigid Tube with a Compliant Lining: A Simple Model for the Air-Mucus Interaction in Pulmonary Airways," *J. Biomech. Eng.* **115**, pp. 262–270.

Hamadiche, M. (1997) "Stabilité de l'Ecoulement de Poiseuille en Conduite Viscoélastique Cylindrique," in *Le troisiéme Congres de Mécanique*, 23–25 Avril, Faculté des Sciences de Tétouan-Maroc, France.

Hamadiche, M. (1998) "Instabilité Causée par l'Interaction Fluide/Structure," in 34[eme] *Colloque d'Airodynamique Appliqué de la A.A.A.F.,* 23–25 Mars, ESM2, Marseille, France.

Hamadiche, M. (1999a) "Flux d'Energie d'un Ecoulement de Poisuille vers la paroi d'un Tube Elastique. a. Modes d'Instabilité Axisymetrique," *C. R. Acad. Sci. Paris*. **327**, Serie II b, pp. 1155–1161.

Hamadiche, M. (1999b) "Flux d'Energie d'un Ecoulement de Poisuille vers la paroi d'un Tube Elastique. b. Modes d'Instabilité non Axisymetrique," *C. R. Acad. Sci. Paris*. **327**, Serie II b, pp. 1163–1170.

Hamadiche, M., and Gad-el-Hak, M. (2001a) "Temporal Stability of Flow through Viscoelastic Tubes," *J. Fluids & Structures*, to appear.

Hamadiche, M., and Gad-el-Hak, M. (2001b) "Spatio–Temporal Stability of Flow through Collapsible, Viscoelastic Tubes," *J. Fluids & Structures*, to appear.

Huerre, P., and Monkewitz, P.A. (1990) "Local and Global Instabilities In Spatially Developing Flows," *Annu. Rev. Fluid Mech.* **22**, pp. 473–537.

Kupfer, K., Bers, A., and Ram, A.K. (1987) "The Cusp Map in the Complex-Frequency Plane for Absolute Instabilities," *Phys. Fluids* **30**, pp. 3075–3082.

Landahl, M.T. (1962) "On the Stability of a Laminar Incompressible Boundary Layer over a Flexible Surface," *J. Fluid Mech.* **13**, pp. 609–632.

Salwen, H., and Grosch, C.E. (1972) "The Stability of Poiseuille Flow in a Pipe of Circular Cross Section," *J. Fluid Mech.* **54**, pp. 93–112.

Sen, P.K., and Arora, D.S. (1988) "On the Stability of Laminar Boundary-Layer Flow over a Flat Plate with a Compliant Surface," *J. Fluid Mech.* **197**, pp. 201–240.

Yeo, K.S. (1988) "The Stability of Boundary-Layer Flow over Single- and Multi-Layer Viscoelestic Walls," *J. Fluid Mech.* **196**, pp. 359–408.

Yeo, K.S., Khoo, B.C., and Zhao, H.Z. (1999) "The Convective and Absolute Instability of Fluid Flow over Viscoelastic Compliant Layers," *J. Sound & Vibration* **223**, pp. 379–398.

