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A GEOMETRIC REPRESENTATION OF AIRFOILS USING NURBS

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ABSTRACT

Simulation-based shape optimization is getting a lot of attention at present. The geometric representation of the shape plays an important role in the optimization process; it affects the number of design variables as well as the smoothness of the final profile. This work is on finding an optimum representation of generic airfoil geometry using non-uniform rational B-splines (NURBS), which are currently used to parameterize wings as well as gas turbine blades. This parameterization requires an optimization algorithm; the Simulated Annealing method is currently used. The optimum representation would have the minimum number of control points to approximate the profile within a given tolerance, yet without loosing its smoothness. Generic wing section geometry, compressor, and turbine blade geometry are represented in two-dimensions using an optimal number of NURBS control points and weights. The goodness of this optimized geometric representation is assessed by evaluating its effect on the flow field.

INTRODUCTION

The geometric representation of curves in two-dimensions, or surfaces in three-dimensions, is an important part of any shape optimization procedure. One would like to represent the shape with the least possible number of parameters for a given accuracy. Typical wing sections have a round leading edge (LE), a sharp trailing edge (TE) and very low camber. Compressor blades of gas turbine engines are usually thin, low cambered and have a sharp trailing edge, whereas turbine blades are highly cambered and have round leading and trailing edges. Therefore, the geometric representation of such blades with one function is quite a challenge. Bézier curves (Farin, 1993) were first used in representing airfoil geometry, probably because of their ease of implementation. However, they have two limitations, first they are global in nature, i.e. when a control point is moved the entire blade shape is modified, which results in less control over the local blade profile; second they cannot represent conics (e.g. leading and trailing edge circles) exactly.

To overcome the global nature of the Bézier polynomials, B-splines use the concept of control points introduced by Bézier, but with more complex interpolation functions that can capture local characteristics such that the displacement of a control point introduces a local modification of the curve near that point. However, B-splines cannot represent conics exactly so that e.g. leading and trailing edge circles cannot be captured exactly.

To alleviate the shortcomings of Bézier curves and B-splines, i.e. to allow for local control of the curve and represent conics exactly, NURBS can be used to represent the blade shape. Using a single NURBS function with at most thirteen control points and weights, Trépanier *et al.*, 2000, were recently successful in representing, rather accurately (up to manufacturing tolerance) and efficiently, the geometry of two-dimensional airfoils used in wing sections.

In this work, NURBS functions are used to find the optimum parameterization of generic airfoil shapes used in wing sections and in blades of gas turbine engines. This parameterization involves the solution of an inverse problem for the control points and weights, where the error in the representation is minimized using Simulated Annealing (SA), the latter gives the control points and weights that would approximate a given blade shape up to a given tolerance. The optimum parameterization, involving the minimum number of control points and weights, is identified for a NACA 2412 profile, and a generic compressor and turbine blade shapes. The effect of this parameterization on the airfoils/blades aerodynamic performance is then discussed.

NOMENCLATURE

P	x- and y- coordinates of the control points
u	parameters or knot vector over which the B-spline coefficients are defined
$N_{i,p}$	P^{th} degree B-spline basis function
E(X)	objective function, which is a function of the design variables
X	vector of design variables, it includes the control points P and the weights W
w	weights, see Eq. 1

Subscripts:

ave	average or mean value
i,j	running index
max	maximum value
p	p^{th} degree of the NURBS function, p=2 at present

Acronyms:

IGV	Inlet guide vane
LE/TE	Leading/Trailing Edge
NURBS	Non-Uniform Rational B-Spline
SA	Simulated Annealing

METHODOLOGY

The NURBS curve is given by a sum over all control points, *n*, of a rational B-spline $N_{i,p}(u)$, times the control point coordinates, $\vec{P}_i$, times a weight, w_i, so that the coordinates of the blade profile are determined once the control points and the corresponding weights are specified. The NURBS curve is defined as (Piegl and Tiller, 1995, p. l95):

$$\vec{C}(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i \vec{P}_i}{\sum_{j=0}^{n} N_{j,p}(u) w_j} \quad (1)$$

Where $\vec{P}_i$ are the x- and y-coordinates of control point *i*, w_i is the corresponding weight, $N_{i,p}$ is the p^{th} degree B-spline basis function, and $\vec{C}_i(u)$ are the x- and y-coordinates of control point *i* on the curve, which corresponds to u_i, the i^{th} element of the knot vector $\vec{u}$. The latter is determined using the chord length method (Piegl and Tiller, 1995, p. 364). The basis functions $N_{i,p}$ vanish everywhere except in the vicinity of point *i*, where the size of this vicinity depends on the order p. The weight w_i provides control on the curve attraction towards control point *i*. The NURBS are defined on the non-uniform parameters called knots, so that some of the control points affect a larger region of the curve while others affect a smaller region depending on the knot vector distribution.

The key feature of a NURBS curve is that its shape is determined/controlled by the set of control points and the corresponding weights. Moreover, placing and moving either one or more of the control points, the knots or the weights can accomplish either a local or a global change of the target shape. However, using the NURBS to model a desired shape is a very challenging task. A NURBS curve also represents exactly conics, e.g. circles, ellipses, hyperbolas, cylinders, cones. This implies that NURBS functions can represent a much wider family of curves compared with what B-spines or Bézier curves can represent, while simultaneously ensuring the profiles smoothness.

NURBS are becoming an industry standard tool for the representation and design of geometry; here are some of the reasons:

- They can represent exactly conics, e.g. circles, and provide the flexibility to design a large variety of shapes
- They can be evaluated reasonably fast by numerically stable and accurate algorithms
- They are invariant under affine as well as perspective transformations
- They are generalizations of B-splines and Bézier curves and surfaces

Note that a NURBS curve with e.g. ten points will have 24 design variables, namely *x*, *y*, and *w* at each control point excluding the two end-points, which are fixed in all results presented in this paper. This number of design variables would correspond to 14 control points, had we used B-splines or a Bézier polynomial. The reduction in the number of design variables in this case seems in favor of using B-splines, however, it has been shown that with the same number of control points, a NURBS function represents a given curve more accurately than a B-spline representation of the same curve, (Trépanier *et al.*, 2000).

Given the blade shape as a list of *x*- and *y*-coordinates, the optimum NURBS function that would represent it is obtained as follows:

- Some carefully selected points are chosen from the given data.
- An initial guess is obtained for the NURBS control points assuming all the weights are set equal to one, this is simply a B-spline.
- The position of the selected control points and the corresponding weights are optimized using Simulated Annealing so as to minimize the error between the representation and the original data.

The initial guess for the control points is of great importance since this problem is a strongly non-linear one. It was found (Trépanier *et al.*, 2000) that the best results were obtained when the selected data points are concentrated in areas of high curvature, typically in the leading and trailing edge regions, where a high accuracy is also required. This initial guess is then obtained by fitting these selected data points with a non-uniform B-spline function, which is a NURBS with weights equal to one. The control points can be found directly by solving a linear system of equations (Piegl and Tiller, 1995)

For a given number of control points, the Simulated Annealing method is used to determine the set of values for these parameters such that the error in the geometric representation falls within a predetermined limit. The set with the minimum number of control points and weights is the optimum one.

In this optimization problem, the error in the geometric representation is the objective function; it can be defined in different forms. One way of defining this error is (Trépanier *et al.*, 2000):

$$E(\vec{X}) = k\varepsilon_{ave} + \varepsilon_{\max} \tag{2}$$

where $\varepsilon_{ave} = \frac{1}{m}\sum_{j=1}^{m} e_j$ *and* $\varepsilon_{\max} = \underset{1 \le j \le m}{Max}\, e_j$

$E(\vec{X})$ is the objective function, $\vec{X}$ is the vector of design variables, which contains the control points and the corresponding weights, e_j is the distance between the original curve and its approximation computed at m locations, and k is a weight factor. In the present calculations, m=300 and k=2.

The precision level required to represent a given shape is determined based on the manufacturing tolerances and/or the change in aerodynamic performance. For a gas turbine blade, the manufacturing tolerance would be around $2*10^{-4}$ for a blade of unit chord.

OPTIMIZATION SCHEME

The Simulated Annealing technique is used to ensure that a global minimum is reached when optimizing problems of large scale, especially one where a desired global extremum is hidden among many local extrema (Kirkpatrick *et al.*, 1983). SA tries to find the global optimum of an N-dimensional function. This method is reported to perform well in the presence of a large number of variables (up to tens of thousands) (Kirkpatrick *et al.*, 1983, and Corana, *et al.*, 1987). The method requires many function evaluations, but it is able to find the global minimum despite the presence of a large number of local minima (Corana, *et al.*, 1987). The search moves both uphill and downhill and as the optimization process proceeds, it focuses on the most promising area. The search starts with a user supplied initial guess and a step size. The method then randomly chooses a trial point at which the objective function is evaluated. All downhill moves are accepted and the algorithm continues from that trial point. Uphill moves may also be accepted; the decision is made based on the Metropolis criterion (Metropolis, *et al.*, 1953). The optimization convergence rate depends upon the SA parameters, which are problem dependent and have to be carefully tuned to reduce computing time.

The problem under consideration was found to be strongly nonlinear with many local optima. When using a gradient-based optimization algorithm, the Sequential Quadratic Programming method (Vanderplaats, 1984), it was frequently observed that, depending on the initial guess, a different solution was encountered. It was also found that, when a random search method was used, namely SA, a transition out of local minima into the neighborhood of the global minimum was possible. The SA, however, does not guarantee to find the global optimum, it will locate a good, near optimal value, if not the global minimum value (Corana, *et al.*, 1987). In all cases tested, the SA gives approximately the same value for the objective function for different initial guesses of the design variables. This verifies that the solution is near the global optimal. Tables 1 and 2 show the optimum control points and weights obtained using SA for two different sets of tuning parameters, whereas Table 3 shows the same parameters but for a different initial guess of the design variables. Comparison of Tables 2 and 3 indicate that the optimum parameters are not identical however let us note two facts. First, the maximum errors obtained in these two solutions are $2.9*10^{-4}$ and $2.3*10^{-4}$; second when a gradient search method was used, two completely different solutions were obtained.

The methodology, described in this section and the previous one, was programmed in C++ and the resulting program was used to calculate the results presented in the following section.

RESULTS AND DISCUSSION

Method Validation

The methodology and optimization scheme described above was first validated by representing a quarter circle. This case has an exact solution, where the quarter circle is exactly represented using NURBS function with three control points and weights. In principle, one should be able to drive the objective function to zero and recover the exact control points and weights. Since the first and last points are fixed, the design variables reduce to three, the *x*- and *y*-coordinates and the weight at the remaining point. The present methodology was able to approximate the quarter circle of radius one with a precision of $\varepsilon_{max}=10^{-7}$.

The method is then used to represent different shapes of interest. In all cases tested, it was possible to represent the airfoil or blade shapes with a tolerance of $2*10^{-4}$ with nine to thirteen control points.

Generic Wing Section

A NACA 2412 airfoil was first used in the evaluation of the NURBS parametric representation. Different number of control points, as well as initial guesses was used to obtain an optimum representation of the airfoil shape. The optimization convergence history is given in Fig. 1, for 7, 9, and 11 control points. As we increased the number of control points from 7 to 11, the maximum error ε_{max} (which is about half the value of the objective function) decreased from $5*10^{-3}$ to $1.1*10^{-4}$, at the cost of an increased computation time. Note that the reduction in error decreases as the number of control points increases.

As mentioned earlier, the various SA parameters need to be carefully chosen to minimize computing time. When the number of control points is increased from 9 to 13, the CPU time required to achieve the allowable tolerance increases from

1 and 4.5 seconds per control point per SA cycle. All calculations were carried out on a PC Pentium III-700MHz.

Figure 2 provides the optimum airfoil representation: the initial and final airfoil profiles, and the control polygons (joining the control points).

The accuracy of the representation can also be measured by the aerodynamic performance of the approximate shape as compared with the original one. At present, the pressure distribution along the airfoil was compared for the approximate and the original profiles using a second order vortex panel method (Anderson, 2001).

Figure 3 shows the pressure distribution corresponding to the exact shape and the optimum approximation, which uses a NURBS function with 9 control points. One can see from that figure the excellent agreement between the pressure distributions, corresponding to the optimum and the original airfoil shapes.

Generic Blades of Gas Turbine Engines

In this section, generic shapes of gas turbine engine blades are represented by NURBS and are tested with the proposed method. Two blade shapes have been tested, one typical of turbine blades and one typical of compressor blades. The generic turbine blade is an inlet guide vane (IGV) that has a 25% maximum thickness, a turning angle of 50º, and has round leading and trailing edges, whereas the generic compressor blade has 12% maximum thickness, a turning angle of 40º, and has a round LE and a sharp TE. These shapes are represented analytically with the following profile:

$$y^{\pm}(x) = f(x) \pm T(x)/2$$

where $$f(x) = \frac{1}{2}(\tan\beta_2 - \tan\beta_1)x^2 + x\tan\beta_1$$

and $$T(x) = 2T_{\max}\sqrt{x(1-x)}$$

or $$T(x) = \frac{3\sqrt{3}}{2}T_{\max}\sqrt{x}(1-x)$$

The blade shape on the upper and lower blade surfaces is given by $y^{\pm}$, f is the camber line, and T is the thickness distribution given for round leading and trailing edges, or round LE and sharp TE, respectively; β_1 and β_2 are the blade angles at LE and TE respectively.

The optimization method is repeated for several number of control points in order to find the optimum approximation, namely the one that would satisfy the required approximation accuracy with the minimum number of control points. The optimal number for the compressor geometry was found to be nine whereas for the turbine, this number was found to be thirteen. Note that the turbine IGV has a round edge at both ends, which requires relatively more control points to resolve the high curvatures at both blade edges. Figures 4 and 5 show the initial and final NURBS profile and the control polygon for both turbine inlet guide vane and compressor blade.

Figures 6 and 7 show the pressure distributions, for the target and the optimum representation of the blade shape for the compressor and turbine blades, respectively. The optimal shape representation is able to reproduce almost the same pressure distribution obtained for the target shape.

CONCLUSION

A methodology for an optimum geometric representation of generic airfoil/blade shapes used in some aerospace applications has been presented. This geometric representation, which is intended for aerodynamic shape optimization, uses a NURBS function with minimum number of parameters that would represent different shapes, within a specified error. This minimum number of parameters involves an optimization process, where the Simulated Annealing method provided a good performance. This minimum number of parameters is shape-dependent and varies between nine and thirteen for the examined cases. The effect of the approximation on the aerodynamic performance of these airfoils, as measured by the pressure coefficient, reflects the goodness of the geometric approximation.

ACKNOWLEDGMENTS

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P_x	P_y	w
0.633050	0.080515	0.859738
0.219993	0.078206	1.152000

0.039372	0.043446	0.576033
-0.002289	0.007812	0.350043
0.007884	-0.028195	0.508588
0.194758	-0.035232	1.953110
0.492436	-0.025217	1.999940

Table 1. The optimized design variables for NACA 2412 with nine control points (excluding the first and last ones, which are always fixed)

P_x	P_y	w
0.631243	0.080901	0.868889
0.221925	0.078045	1.192710
0.037875	0.043733	0.547348
-0.002148	0.007301	0.350000
0.007886	-0.028675	0.480485
0.198942	-0.035139	1.868120
0.490628	-0.025256	1.907980

Table 2. The optimized design variables for NACA 2412 for a different set of SA optimization parameters

P_x	P_y	w
0.695162	0.066180	1.007090
0.287155	0.082593	1.231850
0.031312	0.050216	0.499447
-0.002432	0.003532	0.545103
0.016929	-0.036218	0.590458
0.344193	-0.031617	1.696200
0.879111	-0.005797	2.000000

Table 3. The optimized design variables for NACA 2412 for a different initial guess of the profile

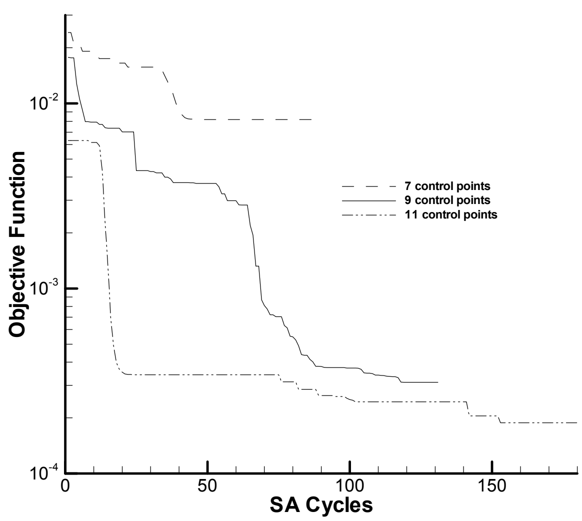

Figure 1. Convergence history for NACA 2412 with different number of control points

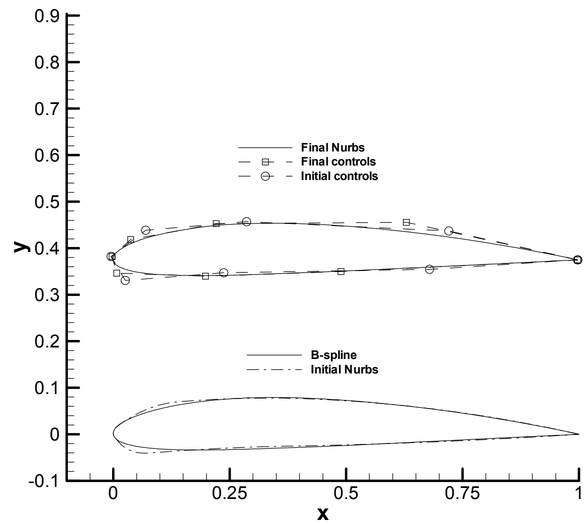

Figure 2. Optimal NACA 2412 profile

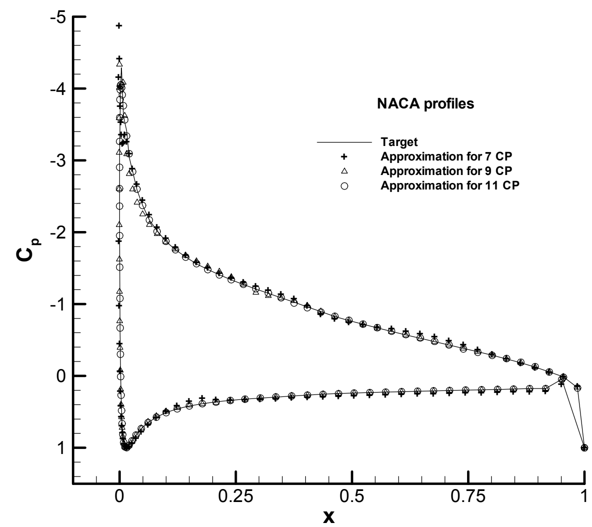

Figure 3. C_p over approximated NACA 2412 profile for different number of control points

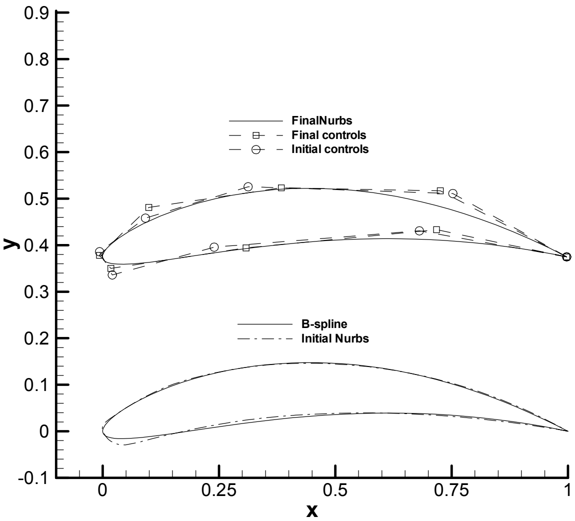

Figure 4. Generic compressor profile: Initial and final NURBS and control polygons

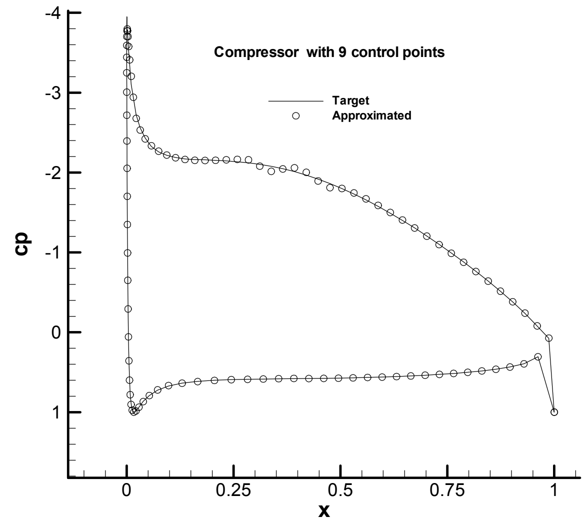

Figure 6. C_p over approximated compressor profile

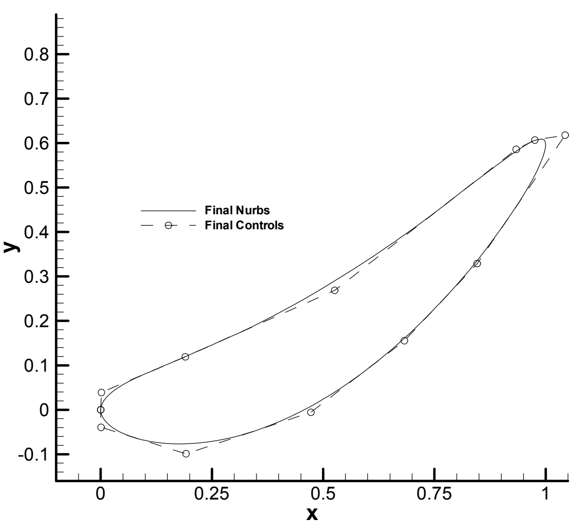

Figure 5. Generic turbine IGV: Optimum NURBS and control polygon

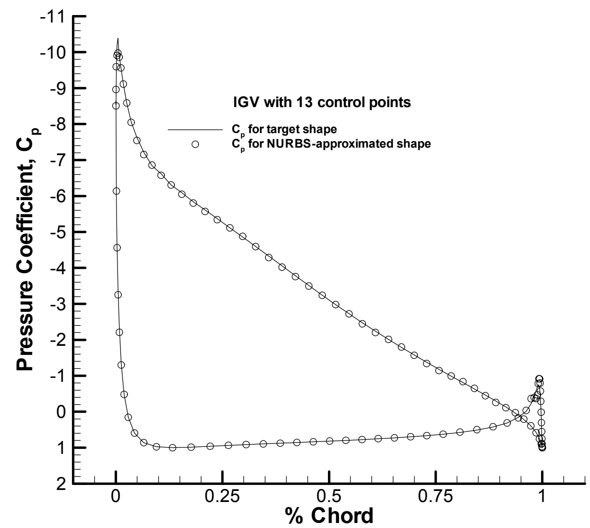

Figure 7. C_p over approximated IGV profile