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INVESTIGATION OF THE NEAR WALL SINGULARITIES THROUGH EXPERIMENTS AND DIRECT NUMERICAL SIMULATIONS

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ABSTRACT

Detailed space-scale study of the near wall local singularities defined as inflectional points of the velocity field are investigated by experiments (turbulent boundary layer, $Re_{\delta} = 12000$ -based on the boundary layer thickness- and direct numerical simulations in a turbulent channel flow. Emphasis is put on the oscillating singularities of the streamwise velocity component related to the meandering of the streaks. New detection techniques are developed and analyzed.

Keywords: Near wall turbulence, streaks, singularities, wavelets, instantaneous phase and amplitude.

INTRODUCTION

Considerable amount of work has been accomplished on the fine structure of the near wall turbulence since the beginning of 1960's with the experimental finding of coherent structures and the process labeled at this time as *bursting*. The direct numerical simulations have subsequently contributed to a better understanding at the end of 1980's through the interrelationships between the coherency and the vortical structures difficult to determine experimentally. More or less sophisticated detection and identification schemes have been proposed so far. However, the question remains open and some new representations of the fluctuating signals may be helpful both for practical and fundamental reasons.

We apply the *instantaneous amplitude-frequency* (phase) formulation to the near wall turbulence here. This representation has been widely used so far, both in particle physics and signal analysis, but never been applied to turbulence to the author's knowledge. We combine it with a particular class of wavelet transforms used for edge detection in image analysis. That allows us to objectively define the rapid variations in time or space of the velocity fluctuations, often associated with the bursting process. These strong local time-space gradients are referred as singularities. One of the main aims of this research is to study the locally oscillating zones of the streamwise singularities in the buffer layer. These regions correspond to the meandering of the streaks, a phenomenon, which is fundamental in the understanding of the regeneration process of the quasi-streamwise vortices. The instantaneous frequency-amplitude representation is quite efficient to achieve this goal. It also has to be recalled that the streak instability was one of the earliest definitions of the bursting process.

DEFINITION OF THE INSTANTANEOUS FREQUENCY

Any signal, moreover the wavelet coefficients $\Omega(k,t)$ may be expressed as:

$$\Omega(k,t) = r(k,t)\cos\left[\int_0^t \omega_i(k,t)\right] \qquad (1)$$

where $r(k,t)$ stands for the instantaneous amplitude and $\omega_i(k,t)$ is the instantaneous angular frequency at scale k. The representation (1) is not unique and different characterizations are possible, depending upon the choice of the dual processes. In the Rice canonical representation that is optimum in the sense of minimizing the average rate of the signal envelope, one has:

$$r^2(k,t) = \Omega^2(k,t) + \breve{\Omega}^2(k,t) \quad (2a)$$

and:

$$\omega_i(k,t) = \frac{\Omega(k,t)\breve{\Omega}'(k,t) - \Omega'(k,t)\breve{\Omega}(k,t)}{r^2(k,t)} \quad (2b)$$

where ' denotes the time derivative. The corresponding optimum carrier frequency, equals:

$$\omega_c(k) = \frac{\overline{r^2(k,t)\omega_i(k,t)}}{\overline{r^2}(k,t)}$$

Moreover, the dual of $\Omega(k,t)$ is obviously its Hilbert transform:

$$\breve{\Omega}(k,t) = r(k,t)\sin\left[\int_0^t \omega_i(k,t)\,dt\right]$$

The instantaneous frequency may also be written as:

$$\omega_i(k,t) = \omega_c(k) + \frac{d\varphi(k,t)}{dt}$$

where $\varphi(k,t)$ is the random phase at scale *k*. It is straightforward that $\omega_i(k,t)$ governs directly the behavior of $\Omega(k,t)$ near the zero-crossings.

EXPERIMENTS: INSTANTANEOUS FREQUENCY OF THE SINGULARITIES DETECTED BY THE HAAR WAVELET NEAR THE WALL.

Interesting results have been obtained thanks to this *time-frequency-scale* representation applied to the near wall turbulence. We showed for instance that, the optimum carrier frequency $\omega_c^+(k^+)$[1] related to the singularities detected by Haar wavelet of both the fluctuating streamwise velocity *and* the shear stress is *constant* in the *whole* buffer layer whatever is the scale parameter k^+. Furthermore, $\omega_c^+(k^+)$ is found to depend only slightly upon k^+ in particular in the small scale range. We also noticed that the Hilbert transform of the multiscale wavelet coefficients of any kind may be obtained by quadrature filtering of either the low pass scaling function or the generating high pass interscale basis coefficients.

The transitions from ejections to sweeps detected by the Haar wavelet at large eddy scales behave as a discontinuous phase frequency shift keying process (Papoulis, 1983; Aulin and Sundberg, 1981), with random, yet somewhat coherent and regular periodicity. Fig.1 shows some traces of the phase $\varphi(k^+,t^+)$ and amplitude $r^+(k^+,t^+)$ of the fluctuating streamwise velocity signal u' at $y^+=10$, for the wavelet scale parameter $k^+=0.057$ (corresponding to the wavelet window duration $T_W^+=26$). The optimum angular carrier frequency is $\omega_c^+=0.08$ in this case. It is seen in Fig.1 that the instantaneous phase $\varphi(k,t)$ consists of line segments that are discontinuous at points B and D where random phase jumps occur. The phase increases first at A-B, remains constant during a large period C-D, jumps again and increases at D-F. The constancy of the phase indicates that the instantaneous frequency is sensibly equal to the carrier frequency. The periods like C-D wherein $\omega_i \approx \omega_c$ coincide generally with large amplitudes $r^+(k^+,t^+)$ as clearly seen in Fig.1. Strong ejection-sweeps transitions marking the arrival of coherent structures are, therefore, merely constant phase events. The time intervals as A-B wherein $\varphi(k^+,t^+)$ increases while $r^+(k^+,t^+)$ decreases are reminiscent of apparition of small scales. The slop of A-B is $\frac{d\varphi^+}{dt^+} = \frac{\omega_c^+}{3}$ indicating that $\omega_i(k,t)$ is jumped by a factor 4/3. The jumps in frequency with the *same* fraction of ω_c^+ are often and repetitively observed. It is asked whether this behavior can be partly explained by the multifractal nature of the cascade process or not (Argoul and al., 1989). The epochs as E-F, wherein both the instantaneous phase and the amplitude increase from small values, are presumably related to the arrival of smaller scale active structures. Note finally that, the duration of the segments is about 100-200 wall units that is close to the ejection (bursting) period. The occurrence of these long periods is particularly interesting. These characteristics may, eventually be used in the decision loop of some drag-reduction control schemes.

The near wall singularities identified by the Haar wavelet (and for wavelets of any kind such as the classical Mexican Hat) at large integration time may consequently be modeled as a discontinuous frequency shift keying process with random phase discontinuities.

DNS DATA. TWO DIMENSIONAL SINGULARITIES

Methodology

We used a DNS data basis ($\mathrm{Re}_\tau = 180$), multiple scale edge detection (Mallat and Zhong, 1992) and directional Hilbert transforms together with 2D Hardy wavelets to detect two dimensional singularities. We consequently extracted information related to the local amplitudes and phases versus the scale, in a manner similar to the section 2. Let 2 wavelets defined by:

$$\psi^1(x,z) = \partial\theta(x,z)/\partial x \qquad \psi^2(x,z) = \partial\theta(x,z)/\partial z \quad (3)$$

[1] + denotes the inner scaling with the shear velocity and the viscosity

where, θ is the smoothing function taken as a Gaussian here, x and z are respectively the streamwise and spanwise directions. The wavelet coefficients at scale s are given by:

$$^{s}W_{1}u\ (x,z) = u \otimes {}^{s}\psi_{1}(x,z) = s\frac{\partial}{\partial x}(u \otimes {}^{s}\theta)(x,z)$$

$$^{s}W_{2}u\ (x,z) = u \otimes {}^{s}\psi_{2}(x,z) = s\frac{\partial}{\partial z}(u \otimes {}^{s}\theta)(x,z) \qquad (4)$$

The local extrema of $^{s}W_{i}\,u(x,z)$ correspond to the *SHARP SMOOTHED GRADIENTS* of the velocity field, or to the inflection points of the convolution $u \otimes {}^{s}\theta$.

Let us introduce now, $^{s}\widehat{W}_{1}u(x,z)$ and $^{s}\widehat{W}_{2}u(x,z)$ the DIRECTIONAL HILBERT TRANSFORMS of the wavelet coefficients. For the band-pass wavelet transforms at a given scale s, one can define a local amplitude and wavenumber through:

$$^{s}W_{i}\,u\ (x,z) = {}^{s}A_{i}\,u\ (x,z)\cos\left[\int {}^{s}k_{i}\,u(x,z)\,d\vec{x}\right] \qquad (5)$$

Where, the amplitude is:

$$^{s}A_{i}\,u(x,z) = \sqrt{{}^{s}W_{i}^{\ 2}\,u(x,z) + {}^{s}\widehat{W}_{i}^{\ 2}\,u(x,z)} \qquad (6)$$

The local wavenumber that minimize the space variations of the envelope is given by:

$$^{s}k_{i}\,u(x,z) = \frac{{}^{s}W_{i}\ \partial/\partial x_{i}\,{}^{s}\widehat{W}_{i} - {}^{s}\widehat{W}_{i}\ \partial/\partial x_{i}\,{}^{s}W_{i}}{{}^{s}A_{i}^{2}\,u(x,z)} \qquad (7)$$

The carrier wavenumber is defined by:

$$^{s}k_{c\,i}\,u(x,z) = \frac{E\left[{}^{s}A_{i}^{\ 2}\,u(x,z)\,k_{i}\,u(x,z)\right]}{E\left[{}^{s}A_{i}^{\ 2}\,u(x,z)\right]} \qquad (8)$$

with E standing for spatial averaging. With the introduction of the carrier frequency, one can define a local phase by:

$$^{s}W_{i}\,u(x,z) = {}^{s}A_{i}\,u(x,z)\cos\left[{}^{s}k_{ic}\,u(x,z)\,x_{i} + {}^{s}\Phi_{i}\,u(x,z)\right]$$

The constant phase zones correspond *to smoothly oscillating singularities*. They furthermore, coincide with ***the meandering zones of the streaks***, when the method is applied to the streamwise velocity fluctuations in the streamwise direction near the wall. The method described above clarifies the mechanism of the streak instability, thus the near wall vorticity generation mechanism as a function of scale.

Results

We present results obtained at $y^{+} = 20$. Fig. 2a and 2b shows the spatial distribution in the x-z plane of the wavelet coefficients defined by (4) i.e., respectively $^{s}W_{1}u\ (x,z)$ and $^{s}W_{2}u\ (x,z)$ as a function of the scale $s = 2^{m}$. It is seen that both coefficients are highly coherent at large scales (m=3 and 4). The small-scale intermittence is present in both coefficients but $^{s}W_{2}u\ (x,z)$ is more streaky and less intermittent than $^{s}W_{1}u\ (x,z)$. Note that $^{s}W_{1}u\ (x,z)$ is related to $\partial u/\partial x$, while $^{s}W_{2}u\ (x,z)$ is large when there are strong spanwise variations of the streamwise velocity ($\partial u/\partial z$), corresponding to the wall normal vorticity surrounding the streaks.

The constant phase zones of the streamwise wavelet coefficients $^{s}W_{1}u\ (x,z)$ are shown in Fig. 3 versus the scale. In these zones, the wavelet coefficients (singularities) smoothly oscillate according to:

$$^{s}W_{x}\,(x,z) = {}^{s}A_{x}\,(x,z)\cos\left({}^{s}k_{c\,x}\ x + \Phi_{x}\right)$$

where $\Phi_{x} = Constant$. Fig. 3 clearly demonstrates that the meandering of the streaks is present *at each scale*. It is striking to note that the streamwise constant phase zones occupy 25 % of the space even at small scales (Fig.4). There are no significant constant phase zones in the spanwise wavelet coefficients $^{s}W_{2}u\ (x,z)$. Therefore, the streak instability mechanism is basically streamwise.

The carrier frequency shown in Fig. 5 has to be interpreted as the preferred frequency of the related instabilities. It is seen that the ratio k_{c}/k is constant in a large range of scale. The flatness factor of the amplitudes of the singularities is shown in Fig. 6. The spanwise singularities *are not intermittent nor are the constant phase zones* (Fig. 7). The intermittence is significantly large in the varying phase zones. The streamwise constant phase zones have large amplitudes and are dynamically important in the generation mechanism, with only slight flatness as shown in Fig. 7. These zones are highly coherent at each scale.

CONCLUSION AND CURRENT WORK

*Significant constant phase zones AT EACH scale for the streamwise and ONLY streamwise gradients of the streamwise velocity components:

*These zones correspond to smoothly oscillating singularities.

*Their amplitudes are large.

*They ARE NOT INTERMITTENT.

*MEANDERING of the streaks is present at small scales

* The small-scale intermittence increases significantly when they are suppressed.

*There are identifiable regular oscillating zones in $\partial u/\partial x$ at small scales.

*The spanwise $\partial u / \partial z$ singularities are significantly less intermittent.

The study will be extended to the whole wall layer for all velocity components. Multifractal analysis will be applied to the experimental data in the constant phase zones. The Lipschitz measure of regularity of the singularities will be determined.

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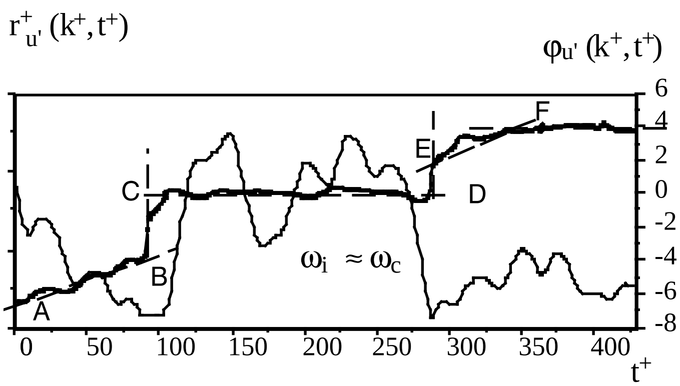

Figure 1 **Samples of the instantaneous amplitude and phase in radians of the Haar wavelet coefficients of the fluctuating streamwise velocity at y^+=10 vs. time .The scale parameter of the wavelet transform is $k^+ = 0.057$ in wall units.**

CD: Constant phase zone wherein the instantaneous frequency is equal to the carrier frequency: Singularities are smoothly oscillating due to the meandering of the streaks.

AB: The phase increases while the amplitude decreases: Apparition of small-scale structures.

EF: The phase and amplitude increase simultaneously: Active but smaller structures.

DE: Phase jump.

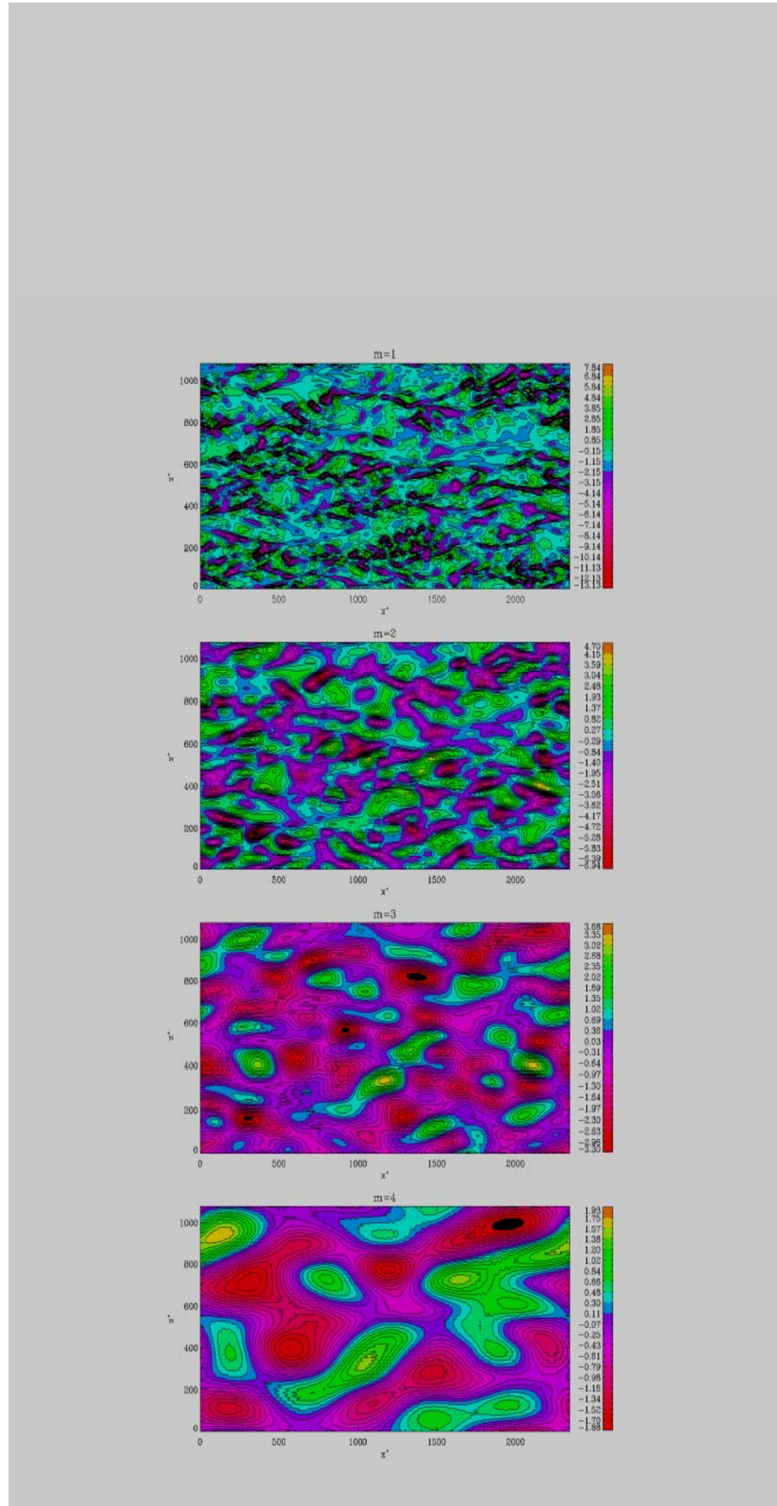

Figure2a Streamwise wavelet coefficient $^sW_1(x,z)$ versus scale (m=1 is the smallest scale).

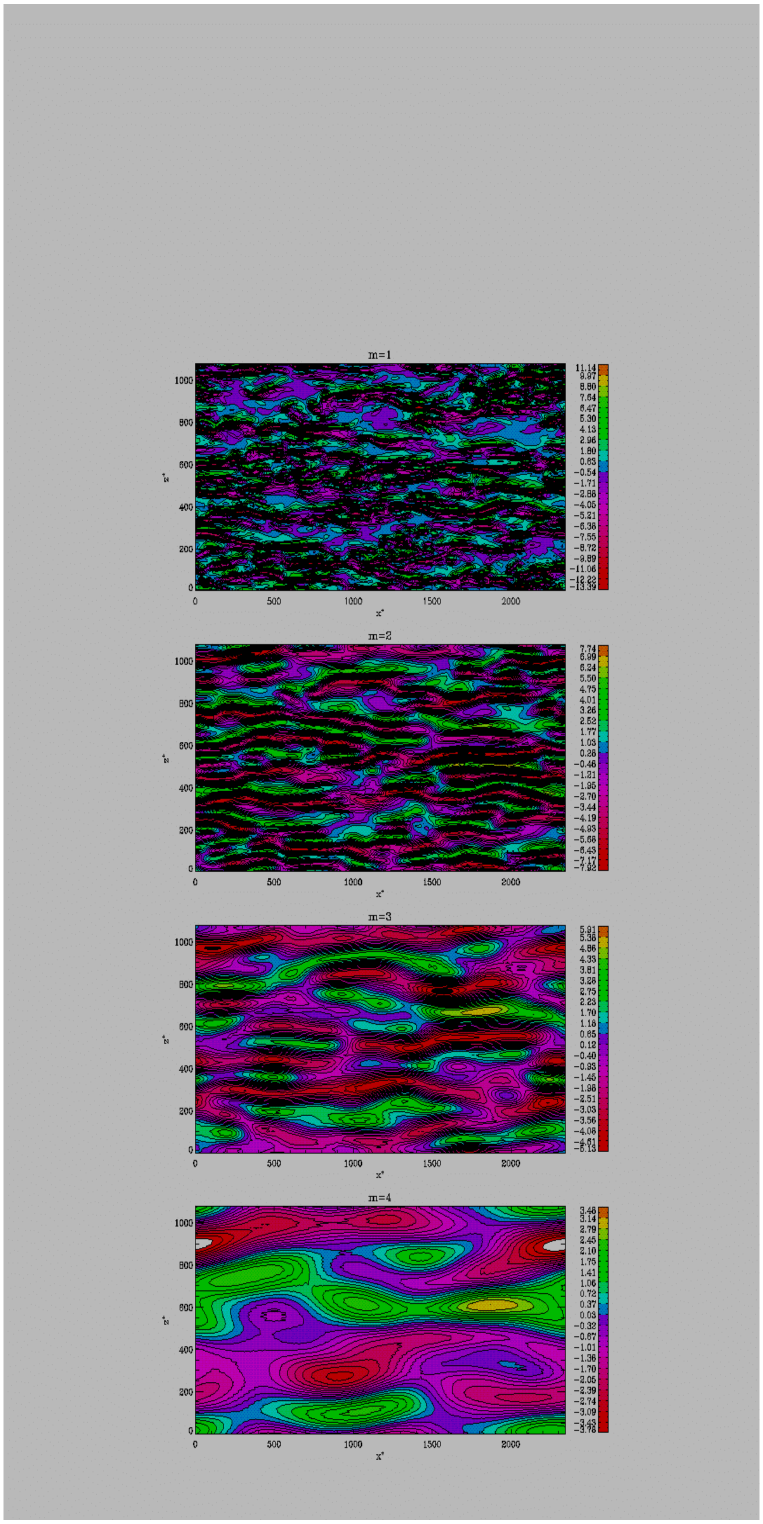

Figure 2b **Spanwise wavelet coefficient $^{s}W_{2}(x,z)$ is less intermittent than $^{s}W_{1}(x,z)$.**

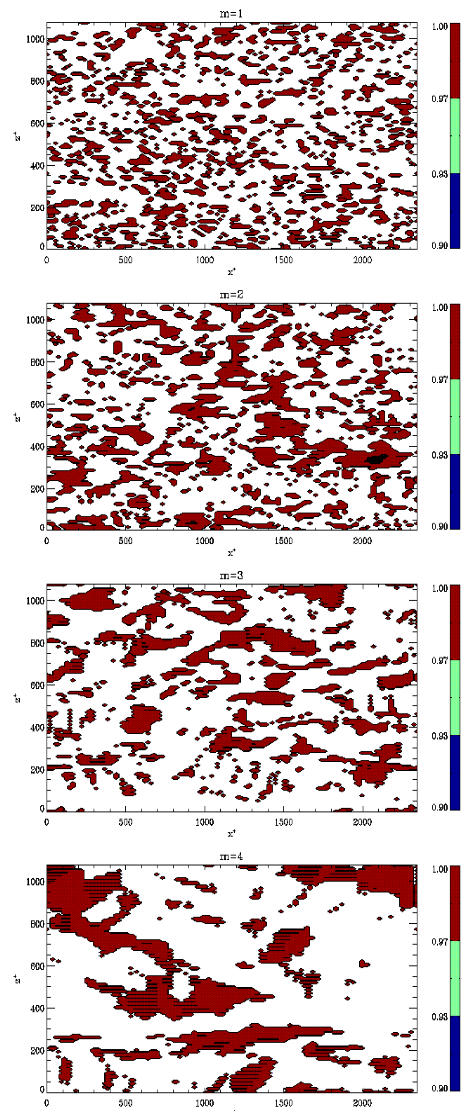

Figure 3. Constant phase zones (shown by red) of the streamwise wavelet coefficients.

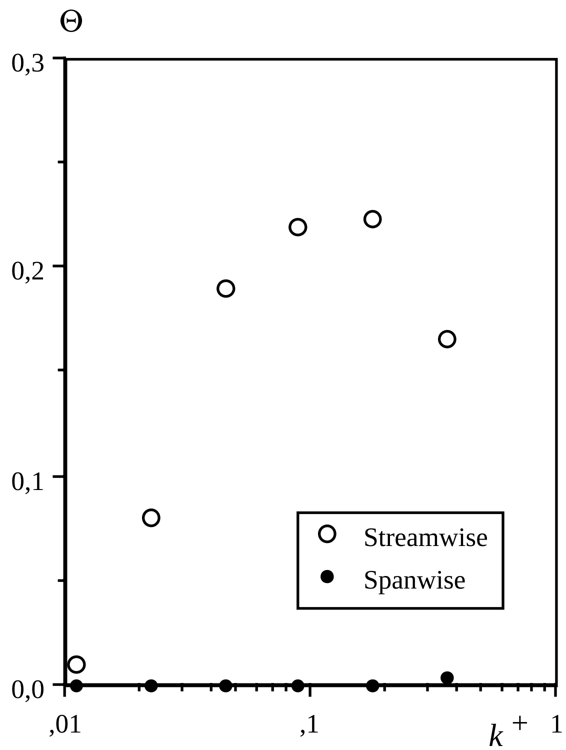

Figure 4 Relative space occupation of the constant phase zones wherein the singularities oscillate smoothly.

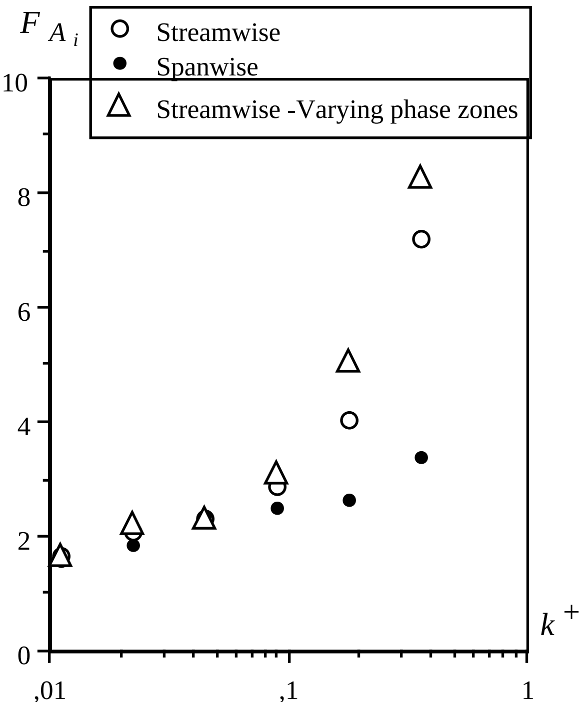

Figure 6 **Flatness factor of the amplitudes related to the singularities.**

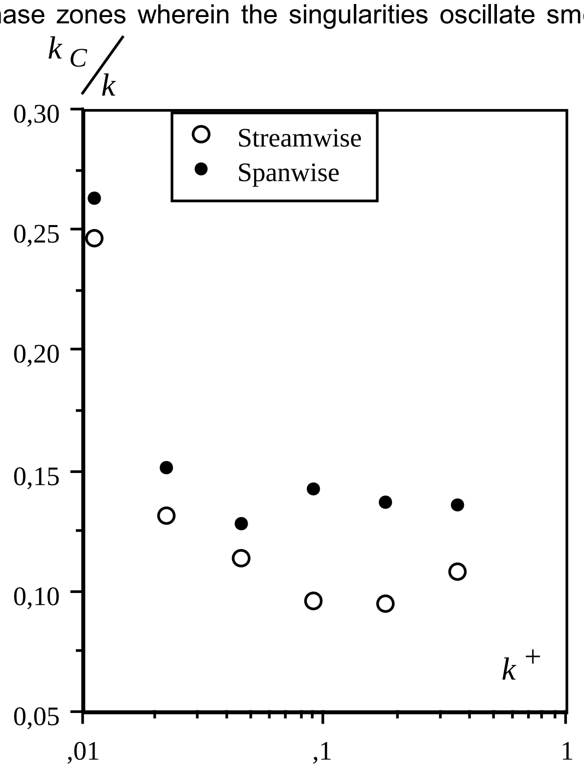

Figure 5 **Carrier wavenumber as a function of scale (wavenumber).**

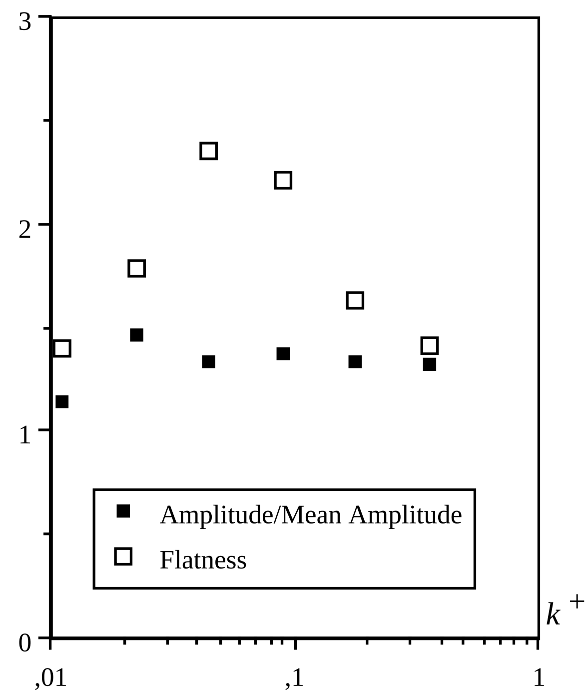

Figure 7 Amplitude and flatness of the constant phase zones of streamwise singularities.