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## DEVIATION OF UNDER-EXPANSION SONIC FLOW NEAR THE WALL

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### ABSTRACT

This paper is described deviation and shock cell of under-expansion sonic flow experiment and calculation on two-dimensional model of an air cap in the paint spray gun. The flow is observed by Schlieren system and the mechanism of shock cell and deviation of flow are calculated with the combination of expansion and compression wave theory. The shock cells in the under-expansion sonic flow can be prospected by the numerical method. The deviation angle of under-expansion sonic flow for the centerline of the spray gun can be determined by the theoretical method that is introduced by one of the authors. The deviation angle relates to the shape of nozzle exit, offset ratio and the difference of pressure around the under-expansion sonic flow. The length of the shock cell is linearly increasing with nozzle pressure and the radius of curvature of the sonic flow is controlled by the offset ratio. The location of interaction of two sonic flows in a spray gun can be easily controlled.

### INTRODUCTION

The original model of the air spray gun manufactured in the U.S. in 1907. After that, the production of the air gun increased with the development of automobile industry since 1920. The technology of air spray gun has dramatically developed by the industrialization of the lowest viscosity of nitro-cellulose and the solvent. The spray gun introduced to Japan in 1923, after three years, IWATA tried manufacture of the spray gun. The production of the spray gun has been continued through the two generations. The coating spray machine are produced 200,000 in a year, this is the most popular and multiple use as the industrial coating machine. The spray gun is consisted with the three main nozzles. One is paint nozzle located at center of spray gun. The second is main air

circular nozzle surrounding the central paint nozzle. The third is supplement nozzles, 5 to 10, using for modification of spray pattern. One of authors has and it systematically studied the basic characteristic of high-speed flow has been solved by observation and experiments. The wake mechanism of the interaction and the confluence zone of

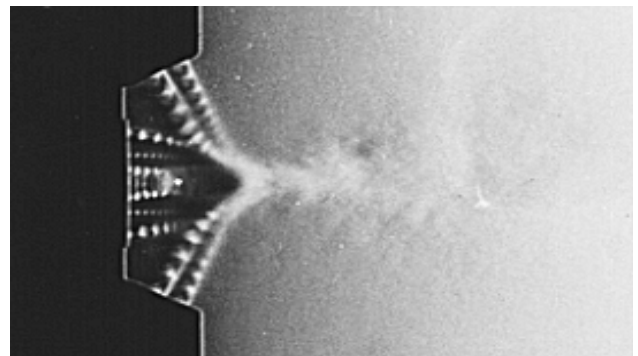


Fig. Photograph showing the under-expansion sonic flow in the actual paint spray air gun

two flows have been studied. The deformation and minimization (atomization) of particle of paint by the force of under-expansion sonic flow has been cleared in the study<sup>(1)</sup>.

This study is focused on the deviation and shock cell of under-expansion flow relates to the nozzle and the shape of wall. A virtual boundary that is the centerline of a three-dimensional paint nozzle was treated as a solid wall. The wall has two shapes; straight and curved. The experiment was carried out using two-dimensional models of the spray gun. The air, except the paint nozzle, is ejected from the air nozzle, as the under-expansion sonic flow is involved shock cell, as

shows in Fig.1. As well known, the shock cell consisted of expansion and compression wave.

This paper is described the experimental and theoretical analyses of the deviation and shock cell of under-expansion sonic flow for the straight and curved wall and effect of the offset ratio.

**EXPERIMENTAL NOZZLE AND NAMBERING OF CELL**

Fig.2 shows two-dimensional and experimental nozzle of air spray gun. The nozzle is consisted of two different active nozzles. The one is a nozzle for the paint ejection nozzle ( ) located at the center of spray gun. The other is an air nozzle (I) a main nozzle in the spray gun. In general, the configuration of an actual spray gun is symmetrical with the centerline of gun, but this model is the upper half of it. The centerline of this experimental model is replaced with the straight wall or the curved wall. The model is enlarged fifteen times of the actual spray gun and three plates compose it. The first plate is straight or curved wall. The second one is middle plate that divides in the paint nozzle and air nozzle. It can be moved to give the offset for the face of the air nozzle exit. The third one is upper part of the air nozzle. The thickness of these plates is 10 mm. They are sandwiched with Pyrex glass, 8mm in thickness, to make the two-dimensional spray gun nozzle. The exit width of two nozzles is w=6mm each.

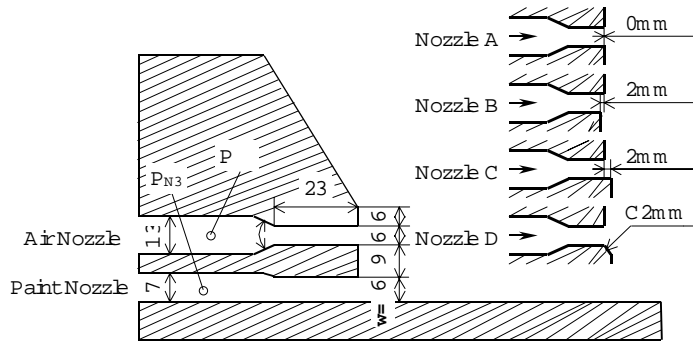


Fig.2 Configurations of two-dimensional half upper side spray gun and four different shapes nozzles.

The angle of deviation to the under-expansion sonic flow can be changed by the offset ratio or cut of nozzle edge. The value of offset ratio is changed 0.33 in this experiment. These configurations of nozzles are shown the upper side in Fig.2. The four different shapes of the nozzle are used in this study. They were named A B C and D. The nozzle A is normal configuration without offset. B is setback type nozzle and the plus-offset nozzle is C. D is cut the lower corner of the exit of the nozzle off around 45degree. The characteristics of the under-expansion sonic flow have been gotten by analysis of the Schlieren photographs. The pattern of outside sonic flow was observed with the path line of a drop-water and the results are used for evaluation of the deviation of the under-expansion

sonic flow. The behavior of corner vortex near the nozzle edge was visualized by smoke and it was also used for the inspection of the deviation of sonic flow.

As shown in Fig. 1, many shock cells are seen in these sonic flows. It can be said that almost of these flow in the spray gun are flowing under the condition of under-expansion. If the shape of cell is not symmetric, for example the shape is not lozenge; the sonic flow would be bent caused by the asymmetric shape nozzle or the unbalance pressure of outside of the flow. It is considered that the expansion or compression waves in the cell intersect each other. The geometric shapes of the shock cell are important due to analyze the deviation angle of the under expansion sonic flow.

Fig.3 represents the thematic diagram of sonic flow and it is unit shock cell in the two-dimensional sonic flow. The regions

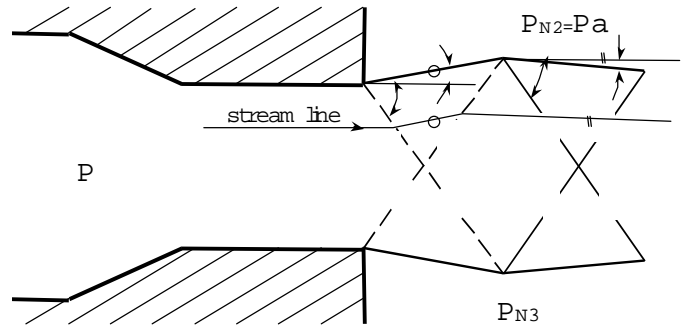


Fig.3 Definition of under-expansion flow pattern and region of shock cell.

of the shock cell are distinguished by number from 1 to 7. The pressure in each region is presented by  $p_1, p_2, p_3, p_4, p_5, p_6, p_7$ . The back pressure of air nozzle ( ) is  $p_{N1}$  and the upper pressure ( $p_{N2}$ ) of the sonic flow is always exposed atmospheric pressure ( $p_a$ ). The pressure  $p_2$  in the region 2 is equal  $p_{N2}$ , in the same way  $p_3 = p_{N3}$ . The deflection angle of an expansion wave, Mach wave angle and the compression wave angle denote  $\theta$ ,  $\alpha$ , and  $\beta$ , respectively. In Fig. 3, the broken lines a b c and d are expansion waves. The pressure  $p_1$  of region 1 is higher than region 2 or 3, since  $p_1 > p_2$  or  $p_3$ . The solid lines e f g and h show compression waves. The pressure  $p_2$  of the region 2 is equal to the pressure  $p_{N2}$ , because a side of region 2 is exposed to the atmospheric pressure. The pressure of region 3 is also equal to the pressure  $p_{N3}$ . The pressure  $p_5$  of region 5 is the atmospheric pressure. The pressure  $p_6$  of region 6 is the same to pressure of paint nozzle,  $p_{N3}$ . But, the pressure  $p_5$  and  $p_6$  are higher than  $p_4$  caused of the compression wave.

**THEORETICAL CALCULATION OF SHOCK CELL**

The pressure  $p_1$  of region 1 is determined by the condition of isentropic flow. Where, assuming that the pressure  $p_{N1}$  is the stagnation pressure in the air nozzle. Therefore, the pressure  $p_1$  of region 1 is calculated as following,

$$p_1 = 0.528 p_{N1}$$

If the pressure  $p_1$  is greater than  $0.528 p_{N1}$ , the Mach number at the air nozzle (I) exit is just  $M_1=1$ , since  $\beta_1$  is equal to  $0^\circ$  and  $\beta_1=90^\circ$ . The pressure  $p_2$  in region is equal to pressure  $p_{N3}$ . Because, a side of the region is contacted with an atmosphere, therefore, the pressure  $p_2$  in the region is the same as the atmospheric pressure. Mach number  $M_2$  of the region is calculated under the condition of isentropic flow by the pressure ratio and  $M_2$ .

$$p_{N1}/p_2 = \{1 + (\gamma - 1)/2 M_2^2\}^{-(\gamma - 1)/(\gamma)}$$

The angle of expansion wave  $\beta_2$  is determined by the Prandtl-Meyer function (in degree) using of Mach number  $M_2$ . The Mach angle  $\beta_2$  is calculated from the Mach number  $M_2$ , as following,

$$\beta_2 = \sin^{-1}(1/M_2).$$

The values of  $p_3$ ,  $M_3$  and  $\beta_3$  in the region are also determined by the above calculation method. In this case,  $p_3$  is equal to  $p_{N3}$ , where  $p_{N3}$  is the pressure of paint nozzle.

The slipstream<sup>(2)</sup> can not be found in the region, therefore the flow in the region must be uniform. The flow in the region is parallel to the direction of deflection angle. Since, after the both flows through the region and have the same properties in the region. Mach number  $M_4$  is calculated by the following equation,

$$M_4 = \left\{ \frac{1 + (\gamma - 1)/2 M_2^2}{1 + (\gamma - 1)/2 M_3^2} \right\}^{1/(\gamma - 1)}$$

where  $\beta_2, \beta_3$  is function of  $M_2, M_3$ .

The waves a, b, c and d are expansion wave, the flow in region is isentropic, the pressure  $p_4$  is determined by the following equation,

$$p_4 = p_{N3} / \{1 + (\gamma - 1)/2 M_4^2\}^k; \quad k = (\gamma - 1)/(\gamma)$$

Where  $p_{N2}$  is outside pressure of the sonic flow, in this case, it is an atmospheric pressure. The angle of expansion wave  $\beta_4$  is,

$$\beta_4 = \sin^{-1}(1/M_4)$$

The expansion waves are expressed with the broken line c and d in Fig.3. The Mach angle  $\beta_4$

$$\beta_4 = \sin^{-1}(1/M_4)$$

The one side of region is contacted with atmosphere pressure,

$$p_5 = p_{N2}$$

Where the solid lines e, f, g and h are oblique shock waves, therefore the pressure  $p_5$  is increased than pressure  $p_4$ . The angle of oblique shock wave  $\beta_5$  is obtained as following equation,

$$p_5/p_4 = \{2 M_4^2 \sin^2 \beta_5 - (\gamma - 1)\} / (\gamma + 1)$$

The angle of flow through the oblique shock wave,  $\beta_5$  can be calculated by

$$\tan \beta_5 = \{2 \cot \beta_5 (M_4^2 \sin^2 \beta_5 - 1)\} / \{M_4^2 (\gamma + 2 \cos^2 \beta_5) + 2\}$$

The Mach number  $M_5$  in the region is determined from the next equation, here  $M_4, \beta_5$  and  $\beta_5$  have been calculated.

$$M_5 = \left\{ \frac{2 + (\gamma - 1)(M_4 \sin \beta_5)^2}{2 (M_4 \sin \beta_5)^2 - (\gamma - 1)} \right\}^{1/2} / \sin(\beta_5 - \beta_4)$$

The one side region is exposed to the paint nozzle pressure, which is closed between the sonic flow and the straight wall, since the pressure  $p_6$  equal to  $p_{N3}$ ,

$$p_6 = p_{N3}$$

The angle of  $\beta_6$  can be calculated with  $M_4$  and  $p_4$ , moreover the angle of oblique shock wave  $\beta_6$  and Mach number  $M_6$  are determined by the same procedure as same as the region.

The two sides of region, g and h, are compression wave. The flow through the two side g and h goes to the region. The two regions, and, these flow directions, these pressures and these Mach numbers must be equalized each other values at the region, because the region is required does not generate the slipstream.

The wave angle  $\beta_7$  at the region against the direction of the flow in the region and the inclination angle  $\beta_7$  which is the wave angle for the flow direction in the region are given for the following equations as the appropriate values. After that, the approximately values of the each deflection angle  $\beta_7$  and  $\beta_7$  are determined to be calculate by following equations,

$$\tan \beta_7 = 2 \cot \beta_7 (M_5^2 \sin^2 \beta_7 - 1) / \{M_5^2 (\gamma + 2 \cos^2 \beta_7) + 2\}$$

And

$$\tan \beta_7 = 2 \cot \beta_7 (M_6^2 \sin^2 \beta_7 - 1) / \{M_6^2 (\gamma + 2 \cos^2 \beta_7) + 2\}$$

Moreover, the pressure  $p_7$  and  $p_7$  are calculated by the using above these two values.

$$p_7 = p_5 \{2 (M_5 \sin \beta_7)^2 - (\gamma - 1)\} / (\gamma + 1)$$

And

$$p_7 = p_6 \{2 (M_6 \sin \beta_7)^2 - (\gamma - 1)\} / (\gamma + 1)$$

These two values must be equalized at the region. If these values are not the same each other, the preliminary assumption values of wave angle are somewhat changed and the above calculation is repeated until it will be equalize,  $p_7 = p_7$ .

If these two pressures are become the same,  $p_7 = p_7$ , the Mach number  $M_7$  in the region can be determined by following equation,

$$M_7 = \left\{ \frac{2 + (\gamma - 1) M_5^2 \sin^2 \beta_7}{2 (M_5 \sin \beta_7)^2 - (\gamma - 1)} \right\}^{1/2} / \sin(\beta_7 - \beta_4)$$

These repeatable calculations are continued until they will be convergent in the property at the final region. As mentioned above, the calculation procedure of the one unit cell was described. At next article, the curvature of under-expansion sonic flows is mentions.

## CURVATURE OF UNDER-EXPANSION SONIC FLOW

The radius of curvature of under-expansion sonic flow can be calculated with the law of the centrifugal force and the equation of momentum. The centrifugal force ( $F_c$ ) is defined as following equation

$$F_c = m R_j \omega^2 = m v$$

Where  $m$  is unit time mass flow,  $R_j$  radius of curvature, angular velocity and  $v$  tangential velocity. The mass flow is

$$m = \{2 / (\gamma + 1)\}^{(\gamma + 1)/(\gamma - 1)} A_n / R T_{N1}$$

Where cross-section area of the air nozzle is  $A$ ,  $R$  is the gas constant and  $T_{N1}$  is temperature of air in the nozzle. The tangential velocity  $v$  is calculated by the next expression,

$$v = [2 R T_{N1} / (\gamma - 1) \{1 - (p_3/p_{N1})^{(\gamma - 1)/\gamma}\}]^{1/2}$$

Thus the centrifugal force is obtained from following form.

$$F_c = p_{N1} A_e \left\{ \frac{2}{(+)^{(-1)}} \frac{2}{(-)} \right\} \left\{ 1 - \left( \frac{p_3}{p_{N1}} \right)^{(-1)} \right\}$$

On the other hand, the sonic flow would be bend caused by the pressure difference of the outside and inside of it as shown Fig. 3. The force  $F_p$  accompany with the pressure difference is defined as following,

$$F_p = (p_3 - p_{N1}) R_j w$$

Where  $w$  is width of nozzle. Solutions of above two equations are the same each other, so that the radius of curvature  $R_j$  of the sonic flow can be found from following expression.

$$R_j = \left( \frac{p_{N1} A_e}{w} \right) \left\{ \frac{2}{(+)} \right\}^{(+1)(-1)} \frac{2}{(-)} \left\{ 1 - \left( \frac{p_3}{p_{N1}} \right)^{(-1)} \right\} / (p_{N3} - p_{N2})$$

The characteristics of the under-expansion sonic flow in this study can be prospected by the above whole equations.

### OBSERVATION OF UNDER-EXPANSION SONIC FLOW

Fig.4 showing the typical Schlieren photograph of under-expansion flow in the two-dimensional air and paint nozzles of the spray gun. The flow is almost the same one as shown in Fig.1. It can be seen the shock cells, which are associated with the expansion and compression waves. The pitch of the cell is somewhat decreasing as going to down-stream, the width of sonic flow is slightly expanded but is not so much. The length of the under-expansion sonic flow is about 10 times the width of the nozzle. The end of the flow is suddenly expanded toward the straight wall and the cells are diffused at the location.

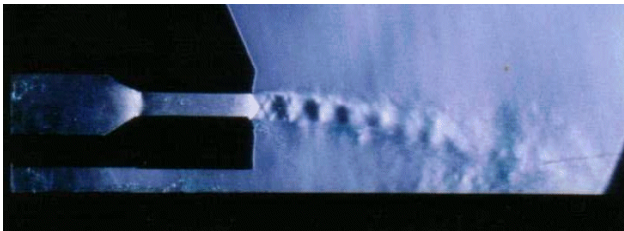


Fig.4 Typical Schlieren photograph showing under-expansion sonic flow in the two-dimensional air nozzle

The under-expansion sonic flow in the actual spray gun is almost straight-line as shown in Fig.1. But, it can be seen that the flow of the central nozzle in the spray gun is slightly bent inward to the centerline of the gun. For the flow in the model nozzle, which is fifteen times the size of the actual nozzle, the flow pattern is almost similar to the pattern in the actual nozzle.

The under-expansion sonic flow is bent caused by the surrounding pressure difference and the suction effects accompany with the air ejection. Of cause, the sonic flow from the nozzle is also affected by the corner vortex formed at the space between the paint and air nozzle. The vortex was observed with the visualization by the drop water.

If the deviation of sonic flow could be controlled by the adjustment of nozzle shape, it is useful for atomize the spray paint. Since, to know the bending mechanism and the control parameters of the under-expansion sonic flow is more

important to consider the best configuration and nozzle arrangement. It is required to produce the suitable spray gun.

The sonic flow would be mainly bent by the pressure difference. Therefore, the relations between pressure  $P_{N1}$  and  $P_{N2}$  for the changing of nozzle shape, as shown in Fig.2, is conducted. The results are shown in Fig.5. It can be seen that the characteristics of the pressure  $P_{N2}$  for the variation of pressure  $P_{N1}$  is divided in two groups for the nozzle shape. The one is A, B and D, the other is C. The nozzle C is given the plus offset (+0.33). The pressure reduction ratio ( $p_{N2}/p_{N1}$ ) of the nozzle A, B and D are -0.0253 respectively. The nozzle C is -0.0485. When the nozzle has positive offset, the bending of the sonic flow becomes large a little, because the pressure of the space between the paint and air nozzle becomes lower. At that time the spray paint is much discharged from the paint nozzle caused by the strong suction effect. This phenomenon is often seen on the actual spray gun. It is recognized that the sonic flow would be easily bent by the change of the offset ratio.

The range of an actual supply air pressure,  $p_{N1}$ , is from 0.15MPa.g. to 0.3MPa.g. In this experiment and the calculation,

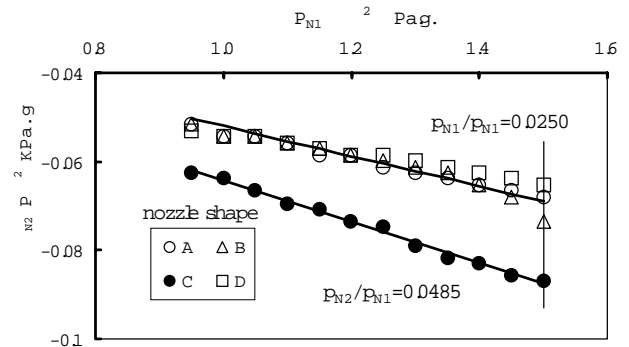


Fig.5 The effect of nozzle offset ratio for the pressure drop at the inside of the sonic flow relates to the bend.

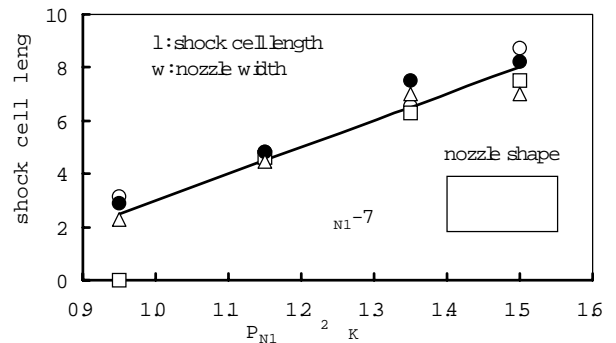


Fig.6 The variation of shock cell length for the nozzle pressure ( $p_{N1}$ ) with the four different shape nozzles. the air pressure  $p_{N1}$  is almost conducted under 0.15MPa.g. The pressure condition is shown by vertical dotted line in Fig.5.

Fig.6 is shown the variation of shock cell length ( $l/w$ ) for the nozzle pressure ( $p_{N1}$ ) with the four different shape nozzles. The length is linearly increased with increasing nozzle

pressure. From this result, the attachment point on the straight wall of the sonic flow can be prospected by the formula:  $l/w=10p_{N1}^{-7}$ .

**CALCULATION OF SHOCK CELL AND CURVATURE**

Fig.7 shows formation of shock cell in the sonic flow gotten by calculation. The upper side pressure ratio  $p_{N2}/p_N$  is 0.407 and lower  $p_{N3}/p_{N1}$  is 0.383. The Mach number of  $M_1$  is unit,  $M_2$  is 1.21 and  $M_3$  is 1.26. These region and are surrounded the expansion waves, the inclination of the waves a, b c and d are different each other caused by the pressure

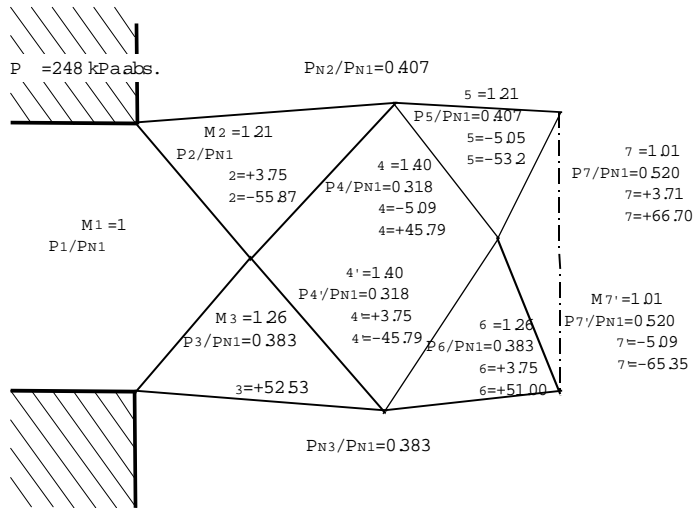


Fig.7 Showing shock cell shape and character values each region from the results based on theoretical equations.

difference,  $p_{N2}$  and  $p_{N3}$ . Therefore, the centerline of sonic flow slightly inclines toward the straight wall side. The region is associated with the two expansion waves, c and d, and two sides of the downstream, e and f, which are compression waves. Since, the flow in region is must be satisfied the condition of non slip flow and it must be uniform. The Mach number is  $M_4 = 1.40$  and the pressure ratio  $p_4/p_{N1}$  is 0.318. The region and are formed by four compression waves respectively, and the pressure in these regions are equal to the outside pressure of the sonic flow. These Mach numbers are different each other. Their Mach numbers are  $M_5 = 1.21$  and  $M_6 = 1.26$ . The Mach number in region is  $M_7 = 1.01$  and the pressure is  $p_7/p_{N1} = 0.985$ . It can be said these values are approximately the same as the values of the region. Because the sonic flow is frictionless and these values of the cells can be gotten by the calculation based on the previous expressions.

The deviation angle ( ) of the sonic flow for the nozzle center-line at the region is  $-2.7^\circ$ . This value is the result of calculation for the normal shape nozzle and the pressure difference between the upper side and the lower side of sonic flow is zero. It can be concluded that the way of this calculation can be applied to the sonic flow bent by the pressure difference around its flow.

Fig. 8 is shown the deformation of the shock cell and the deviation angle of the under-expansion sonic flow for the systematic changing of the shape of nozzle exit. The solid line is sketched from the Schlieren photographs. The dotted line is drawn base on the calculation values of the same procedure as shown in Fig.7. The nozzle pressure in each case is kept  $p_{N1} = 150 \text{ kPa.g.}$  The pressure of upper and lower sides of the sonic flow has slight difference. From these results, it can be easily recognized that the shape of shock cell is deformed from the triangle or lozenge to the irregular form by the unsymmetrical configuration of the nozzle exit

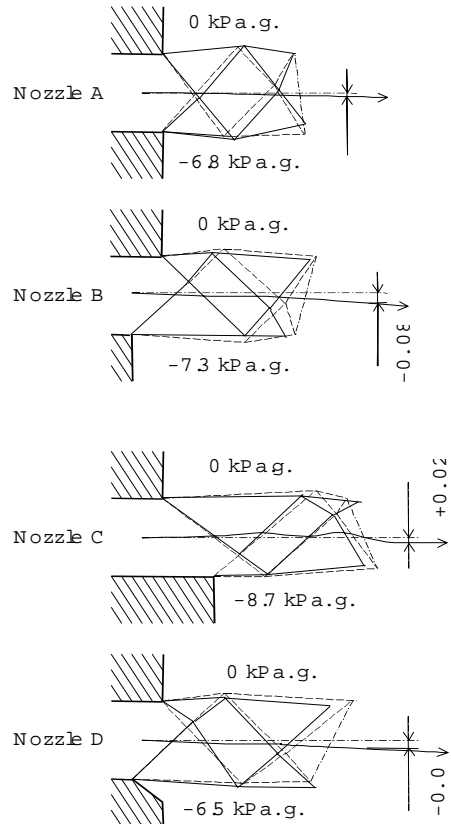


Fig.8 Deformation of shock cell for the difference of nozzle shape by the results of the calculation and Schlieren method.

The outside pressure of the flow  $p_{N2}$  is maintained constant at the atmospheric pressure for the experiment and the calculation. The pressure  $p_{N3}$  in the paint nozzle is a little varied in the region  $-6.5 \text{ kPa.g.} > p_{N3} > -8.7 \text{ kPa.g.}$ . The pressure difference is not so large. Since, it can be considered that the deviation of flow is mainly depended on the nozzle shape or the offset ratio. The inclination angle of first expansion waves for the nozzle centerline is decreasing with the increasing of pressure ratio  $p_{N2}/p_{N1}$  and the results of calculation almost coincided with the observation of the flow visualization. From this, it is clearly said that the negative offset and corner cut nozzle presented the same effect to the deviation angle.

Fig.9 shows the variation of the shape cell in the case of no pressure difference both sides of sonic flow.

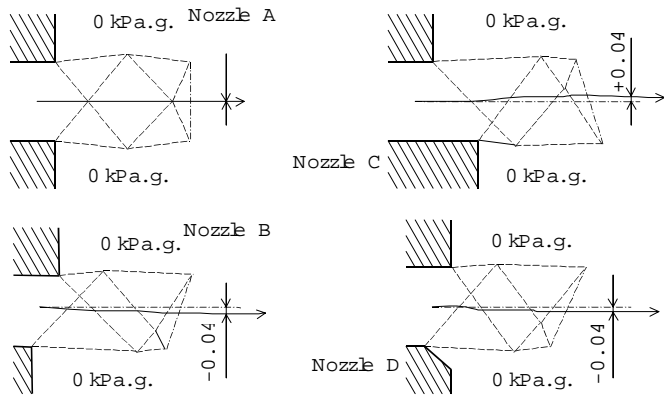


Fig.9. Variation of cell configuration by changing the nozzle shapes for no pressure difference both sides of sonic flow.

It can be seen that the displacement ( $\delta = y/w$ ) of the sonic flow for the nozzle centerline of the nozzle B and D are negative, but C is opposite. In this case, the intensity of expansion wave on both sides of nozzle edge is different each other. Therefore, the shape of cell becomes asymmetric and the flow is bent.

The deviation angle( $\theta$ ) and the values of theoretical ( $R_{th}$ ) and actual ( $R_{act}$ ) of the radius of curvature are represented in Table 1. As above mentioned the deviation of sonic flow is increased with the pressure difference between  $p_{N2}$  and  $p_{N3}$ . But, in this case, it can be seen that the sonic flow is bent without the pressure difference.

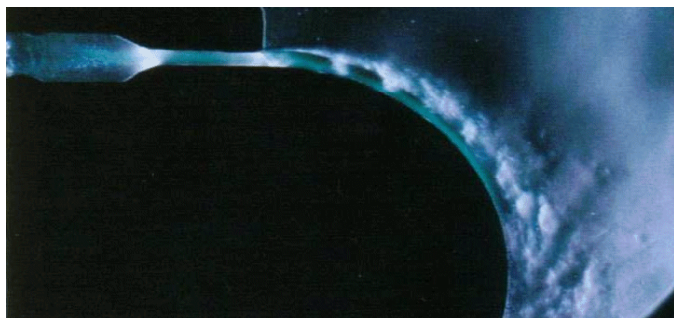


Fig.10 Schlieren photograph of the under-expansion sonic flow along the curved wall without separation until 70 degree from the nozzle edge

Fig.10 shows the under-expansion sonic flow along the curved wall. Through this experiment it would be said that, the sonic flow is bent by the change of the offset ratio or the curved wall. The application of the flow bent by the curved wall leaves for further more studies.

Table 1. Deviation angle and Radius of Curvature.

	A	B	C	D
	-2.7	-2.9	-3.5	-2.5 deg.
$R_{th}$	190	178	152	200 mm
$R_{act}$	126	116	74	126 mm

**CONCLUDING REMARKS**

Based on a fundamental study of under-expansion sonic flow with two-dimensional upper half spray gun, which has straight and curved walls, the following general conclusions and comments can be made:

- (1) The air of spray gun is under-expansion sonic flow and forms the shock cells.
- (2) The geometric shape of the shock cell can be prospected by the theoretical method.
- (3) The deviation of under-expansion sonic flow varies with the changing of the offset ratio or the pressure difference around the sonic flow, because it takes place unsymmetrical waves at the exit of nozzle.
- (4) The radius of curvature of under-expansion sonic flow can be calculated by the theoretical method, which is based on the centrifugal force and the pressure difference between the upper and lower sides of the sonic flow.
- (5) The gravity or suction type spray gun, when the air nozzle has an offset, which is positive offset ratio, the paint flow rate increases compared with the normal type. This can be estimated from the deviation of the sonic flow, which is bent toward the outside direction. Therefore, the region near the exit edge of the nozzle becomes lower pressure and then the suction area is enlarged.

It is considered that the narrowness of paint particle or the irregularity of paint mass flow is related to the deviation angle of the sonic flow and the geometrical configuration of the shock cell. To improve the performance of spray gun and to get better quality of the paint surface by avoiding of its uneven or blotch, in the future, it would be highly required continuous study of the complex of under-expansion sonic flow concerning to the nozzle or wall shape.

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