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# LAMINAR AND TURBULENT FLOW CHARCTERISTICS OF NON-NEWTONIAN HOMOGENEOUS SLURRIES

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## ABSTRACT

In many situations, hydraulic transport is considered a competitive means of solids transportation because it can bring several advantages. Compared to a mechanical transport, the hydraulic transport is dust free, demands less labor, and enables full automation for operation. So, it may be one of the most economic and safe transport methods. To keep hydraulic transport economically attractive, it is required to transport slurries at high solids concentrations. Therefore it is required to have enough information on the flow behavior of dense slurries in pipelines. For homogeneous slurries, when increasing the solids content the mixture can behave as non-Newtonian fluids.

The aim of this paper is experimentally investigate the flow characteristics of homogeneous slurries that showed non-Newtonian behavior. So, the experimental measurements of kaolin slurries in a pipeline test loop have been studied and compared with the theoretical predictive approaches. The slurry volumetric concentrations ranged from $C_v$ = 2.8% to 22.6%. Both laminar and turbulent flow regime were obtained.

## INTRODUCTION

Hydraulic transport has been a progressive technology for conveying a large quantity of solid materials in the form of slurries. The economical system of pipelining prefers the dense slurries, since hydraulic transport of dense hydro-mixtures can bring several advantages. Compared to a mechanical transport, the pipeline one has a minimal pollution, it demands substantially less space, makes possible full automation and needs a minimum of operating staff, (1). Some applications, such as transport of minerals in undeveloped terrain, the pipeline could well provide the shortest route and other forms of transport may not be possible. Partially processed coal slurry from the mines to electrical generating stations is transported through long pipelines using high-head pumps. The drinking water treatment process requires continuous disposal of the sludge produced by water clarification and filters backwash processes by long pipelines. There is considerable temptation to classify slurries rigidly to settling and non-settling as a result of behavior of a static sample of the slurry.

Settling slurry flows are not axially symmetrical and a concentration gradient or density gradient form in the pipe during horizontal flow.

Non-settling slurries, whose particles settle very slowly, consist of fine particles, which are likely to be uniformly distributed over the pipe cross-section. Their flows appear to be homogeneous, and when increasing solids content, the mixture can no longer be regarded as two individual components. The resulting fluid could have non-Newtonian characteristics implying a more complicated relation between the shear stress and shear rate. Non-Newtonian fluids are distinguished from Newtonian fluids in that the viscosity is dependent upon the rate at which the fluid is sheared; hence the use of a single viscosity is no longer appropriate. Instead, empirical relations are fitted to the rheological measurements. The rheological properties of that fluids can not be calculated from first principles. When mathematical relations are used to approximate the experimentally determined rheograms, they are known as rheological models, and these models can be employed to drive relations linking the pressure gradient along a pipe to the discharge. However, such relations are approximations to the actual behavior of the fluid and should not be used outside the range of conditions (particularly shear rates) for which they were determined.

The aim of this paper is experimentally investigate the flow characteristics of homogeneous slurries that showed non-Newtonian behavior. So, the experimental measurements of kaolin slurries at different concentrations, in both laminar and turbulent flow regime, have been studied and compared with the theoretical predictive approaches.

## NOMENCLATURE

| | | |
|---|---|---|
| $b$ | proportional parameter | (kg/m.s] |
| $C_v$ | volumetric concentration | [-] |
| $C_\mu$ | characteristic parameter | [kg/m.s] |
| D | pipe internal diameter | [m] |
| $d_{50}$ | mass median diameter | [μm] |
| $f$ | Fanning friction factor | [-] |
| $i$ | hydraulic gradient of a slurry | [m water/m pipe] |
| $i_w$ | hydraulic gradient of water at the same velocity | [m water/m pipe] |
| $k$ | consistency index | [$\text{kgm}^{-1}\text{s}^{n-2}$] |
| $n$ | flow behavior index | [-] |
| Re | Reynolds number | [-] |
| $\text{Re}_2$ | Reynolds number defined by Slatter & Lazarus | [-] |
| $S_m$ | mixture specific gravity | [-] |
| $u$ | local fluid velocity | [m/s] |
| $U_*$ | shear velocity $(\tau_w/\rho)^{0.5}$ | [m/s] |
| $V$ | mean velocity | [m/s] |
| $y$ | vertical distance in a pipeline cross-section | [m] |
| $\gamma$ | shear rate | [1/s] |
| $\gamma_w$ | wall shear rate | [1/s] |
| $\mu$ | dynamic viscosity | [kg/ms] |
| $\rho$ | density | [$\text{kg/m}^3$] |
| $\tau$ | shear stress | [$\text{kg/ms}^2$] |
| $\tau_w$ | wall shear stress | [$\text{kg/ms}^2$] |
| $\tau_Y$ | yield stress | [$\text{kg/ms}^2$] |

## RHEOLOGICAL CHARACTERISTICS AND FLOW BEHAVIOR

The simplest rheological model is the Newtonian model with a single rheological parameter, the viscosity $\mu$. The viscosity is constant at a given temperature and pressure. It is represented by:

$$\tau = \mu\,\gamma = \mu\,\frac{du}{dy} \tag{1}$$

For a non-Newtonian fluid, there is no single value of viscosity, but it is a function of the rate at which the fluid is sheared. The yielded pseudo-homogeneous model (Herschel-Bulkley equation) is often approximates the behavior of wide range homogeneous non-Newtonian slurries. It is given by:

$$\tau = \tau_Y + k\gamma^n \tag{2}$$

One of the most important practical problems connected with non-Newtonian fluid flow in pipes is the reliable prediction of the pressure drop. For the laminar flow the problem is well established as the relation between the pressure drop and mean velocity can be derived by integration of the representative rheological model, e.g. for the Herschel-Bulkley rheological model the relation will be:

$$V = \frac{nD}{2\tau_w^{\,3}\,k^{1/n}}\left(\tau_w - \tau_Y\right)^{\frac{n+1}{n}}\left[\frac{\left(\tau_w-\tau_Y\right)^2}{1+3n} + \frac{2\tau_Y\left(\tau_w-\tau_Y\right)}{1+2n} + \frac{\tau_Y^{\,2}}{1+n}\right] \tag{3}$$

However, the prediction of the pressure losses in the turbulent regime stills one of the difficult theoretical and practical problems. This difficulty results from that the turbulent flow behavior could be found independent on the laminar rheological characteristics and it is similar to Newtonian turbulent flow behavior, see El-Nahhas et al. (2 and 3). For turbulent flow of non-Newtonian suspensions several models were derived, based on slurry rheological parameters. Torrance (4) model has been derived for non-Newtonian flow in pipes described by the yield pseudoplastic rheological model. A predictive model introduced by Wilson & Thomas (5) and Thomas & Wilson (6) is based on the hypothesis that the thickness of the viscous sub-layer would increase in non-Newtonian fluids. Slatter (7) model based upon the concept of particle roughness turbulence effect. Xu et al. (8) and Bartosik et al. (9) showed the conditions under which the Wilson and Thomas model succeeds or fails. El-Nahhas (10) and El-Nahhas et al (2) showed that the models of Torrance, Wilson & Thomas and Slatter could be successfully used for the turbulent flow prediction, however rheological parameters should be determined from experimental turbulent flow data. It could be obviously found that the rheological parameters that well fit the laminar behavior are not suitable for turbulent flow predictions by the different models.

El-Nahhas (10) studied the similarity of the turbulent flow behavior of non-Newtonian slurries with that of Newtonians using the logarithmic relation of Newtonian liquids that has been applied to fit the experimental turbulent flow data of the non-Newtonian slurries tested. The Newtonian viscosity, $\mu$, which is meaningless for the non-Newtonian fluids, has been replaced by a characteristic shear-independent parameter $C_\mu$. The logarithmic equation has been rewritten as:

$$\frac{V}{U_*} = 2.5\ln\left(\frac{U_*\,D\,\rho}{C_\mu}\right) \tag{4}$$

El-Nahhas and Vlasak (11) presented comparative plots showing that the experimental turbulent flow data for non-Newtonian (natural and peptized) kaolin slurries are fitted very well by equation 4. They showed that the single input parameter, $C_\mu$ encapsulates the effect of the solids presence and is direct proportional to the solids concentration regardless the change in the rheological characteristics and laminar behavior caused by peptizing effect.

El-Nahhas and Mostafa (12) suggested an equation to express the proportionality of the parameter $C_\mu$ on the solids concentration (or mixture density).

Reynolds number, which is used mainly for predicting the transition between laminar and turbulent regimes of Newtonian fluids, faced a problem when applying for non-Newtonians requiring a value of the viscosity. Several approaches were suggested by researchers to determine a Reynolds number formula suitable for non-Newtonians, see El-Nahhas et al. (13). Among these researchers, Slatter & Lazarus (14) who proposed an approach to derive a Reynolds number ($\mathrm{Re}_2$) from the fundamental assumption taking into account the presence of yield stress, as:

$$\mathrm{Re}_2 = \frac{8\rho\,\mathrm{v}^2}{\tau_Y + k\left(\frac{8\mathrm{v}}{D}\right)^n} \qquad (5)$$

## EXPERIMENTAL FACILITY AND TESTED MATERIALS

The slurry flow behavior  can be obviously observed and described by several ways. The easiest way it could be established by analyzing the data measured by viscometry instruments (usually by rotational viscometers). By this way the result data could be used to create direct relation between the shear stress, $\tau_w$ and the shear rate, $\gamma$, i.e. the rheograms. More efficient method for describing the slurry flow behavior can be made by analyzing the experimental data measured by a pipeline test loop. The advantage of this method is the similarity of the tested slurry flow structure with that of the actual slurry pipeline. The pipeline test loop data in the laminar region could be used to create the rheograms of the tested slurries (2).

An open-loop recirculation pipeline system was employed for studying the slurry flow behavior . The slurry was forced by a screw pump driven by a variable-speed electric motor from an open storage tank to delivery pipe. The test section was located on the back branch of the pipeline and its length to diameter ratio exceeded 400. A stainless steel pipe of internal diameter 17.5 mm was used for measurements. The pipe was equipped with three pressure tappings connected through solids pods to differential Hottingger-Baldvin pressure transducers and the readings were monitored by a computer. At the downstream end of the test pipes a box divider was mounted that allows diversion of the discharge to a plastic container for weigh measuring. Since the divider arm was connected to an electric stopwatch, the mass flow rate was precisely measured. The slurry density and hence the volumetric concentration could also be determined. Calibration experiments with clear water, which was periodically run, showed that the pipe used in the test section behaved as a smooth pipe. For more details, see El-Nahhas (2).  The accuracy of differential pressure measurements is higher than 99% and that of flow rate and concentration measurements is higher than 95%.

The kaolin powder, of density 2549 kg/m$^3$ and median diameter $d_{50}$ = 2.8 $\mu$m, from Horni Briza (Czech Republic) was used for preparing a wide range of concentrations of homogeneous slurries. The mineralogy and chemical composition of this kaolin, which considerably influences the flow behavior  of its suspensions, could be found in reference (10). Kaolin slurries of eight different concentrations, beginning with $C_v$ = 2.8% up to the maximum possible concentration of $C_v$ = 22.6%, were prepared and tested in both laminar and turbulent regimes.

## DISCUSSIONS OF THE RESULTS

### 1. Rheological Characteristics

Previous studies, e.g. (2 and 3), showed that the lowest two concentrations (2.8%, 5.6%) slurries have wholly turbulent flow indicating Newtonian behavior. The slurries of volumetric concentration 8.9% and higher showed non-Newtonian behavior represented satisfactorily by the Herschel-Bulkley model. Figure 1 shows the rheograms of kaolin dense slurries. These rheograms have been obtained from the pipeline test loop experimental data in the laminar regime converted (by the Rabinowitsch-Moony method) to wall shear stress versus wall shear rate relations. Just the rheograms are performed, the rheological characteristic parameters ($\tau_Y$, $k$ and $n$) could be determined.

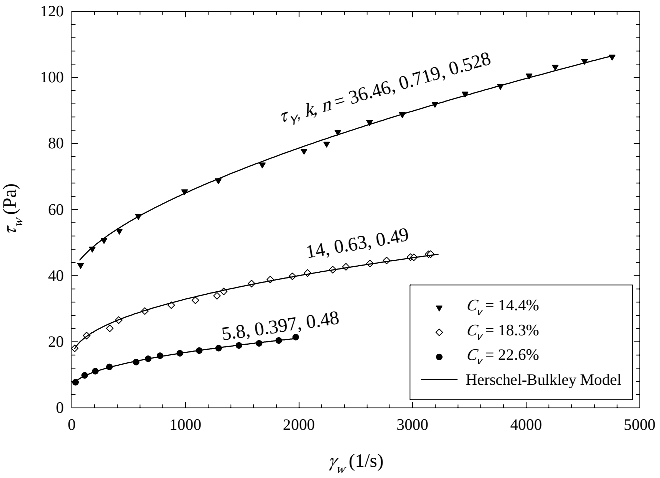

Figure 1 Rheograms of high-concentrated kaolin slurries based on the pipeline test loop experimental data

## 2. Flow Behavior

The resistance curve is a relation between the hydraulic gradient, $i$, and the mean velocity, $V$. For non-Newtonian slurries the resistance curve is a considerable characteristic means to describe the flow behavior entirely in both laminar and turbulent regimes. The laminar flow behavior can be observed obviously at the lower velocities for dense slurries. The laminar flow regime is characterized by the low-slope flat curve in the $V$-$i$ plots. Therefore, the excess head loss caused by the presence of solids, also called the solids effect ($i - i_w$), decreases as the mean velocity increases, as shown in Figures 2 and 3. At the end of the laminar regime, specially for dense slurries, it can be noticed that the solids effect could have a negative values i.e. the drag reduction described by Wilson & Thomas (5) is verified. As the mean velocity increases, the turbulent flow occurs having a steep $V$-$i$ relation and increasing in the solids effect. The mean velocity, at which this behavior transition occurs, increases as the slurry solids volumetric concentration increases and its identification is of great importance in pipeline design at which the flow behavior change fundamentally. It could be noticed also that at very low velocity ranges the solids effect increases with increasing the velocity, this could be due to the excess pressure losses for overcoming the yield stresses.

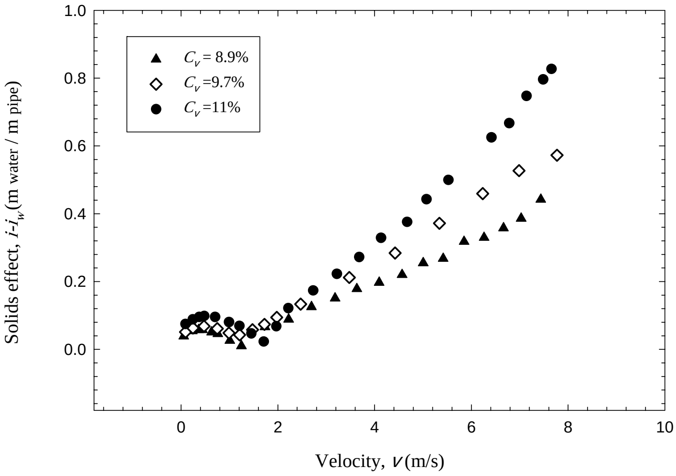

Figure 2 Flow behavior of low-concentrated slurries

The laminar flow prediction could be confidently made by the relation between the flow mean velocity and the pressure drop (or alternatively wall shear stress) obtained by the successive integration of the constitutive rheological model, equation (3). El-Nahhas (10 ) and El-Nahhas et al. (3) proved that there is a good agreement between the experimental measurements and the theoretical prediction for the laminar flow by equation (3). In the present study the laminar experimental data have been used to determine the Fanning friction factor, $f$, ($f = 2\tau_w/\rho v^2$) and Reynolds number $Re_2$ introduced by Slatter and Lazarus (14), in which the presence of yield stress has been taken into account. Figure 4 presents the relation between Fanning friction factor, $f$, and Reynolds number $Re_2$ for dense slurries.

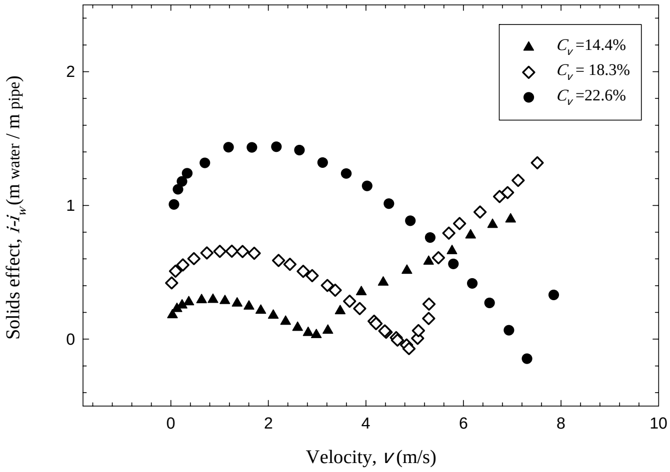

Figure 3 Flow behavior of high-concentrated slurries

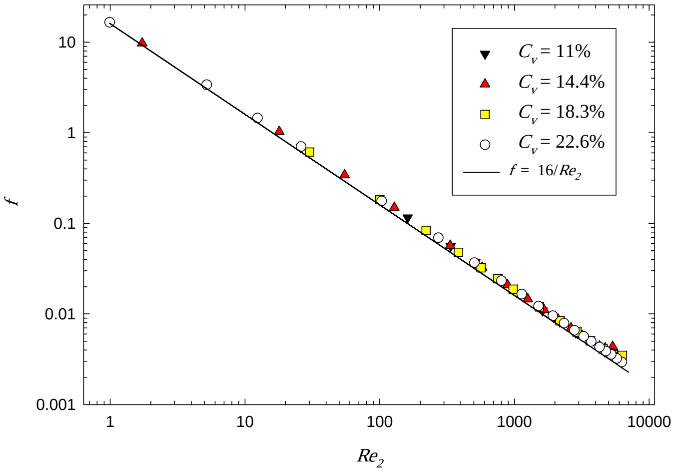

Figure 4 Friction factor for laminar flow of non-Newtonian slurries

It could be noticed that the friction factor of dense non-Newtonian slurries in the laminar regime is related with the Reynolds number in the same way as for Newtonian fluid. The laminar relation of Newtonian fluids

($f$ = 16 / $Re_2$) is approximately valid for all slurries. It could also revealed that the Reynolds number $Re_2$ defined by Slatter (14) is representative for non-Newtonians.

In turbulent flow, the group $(\tau_w/\rho)^{0.5}$ which is a basic parameter known as the shear velocity, $U_*$, having the dimension of velocity, is convenient to express the head losses. Figure 5 shows the shear velocity versus the mean velocity relations of the dense slurries in the turbulent regime.

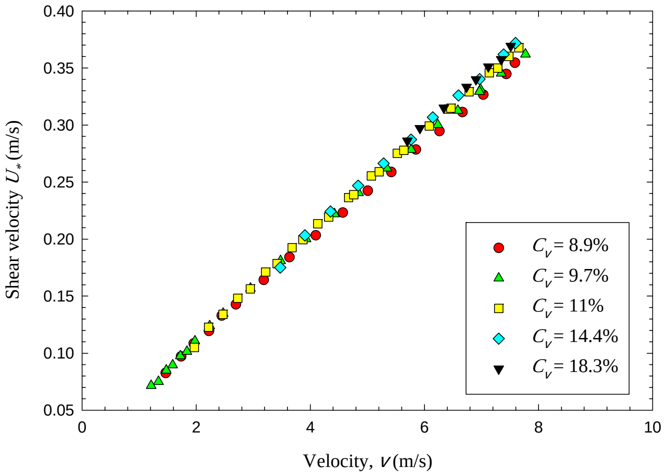

Figure 5 flow behavior for turbulent flow of non-Newtonian slurries

## CONCLUSIONS

The non-Newtonian slurry flow characteristics can be obviously observed and described by analyzing the experimental data measured by a pipeline test loop. The pipeline test loop data in the laminar regime could be used to create the rheograms of slurries. Tested slurries of volumetric concentration 8.9% and higher showed non-Newtonian behavior represented satisfactorily by the Herschel-Bulkley model.

At the laminar flow regime, the solids effect ($i$ - $i_w$), decreases as the mean velocity increases. At the end of the laminar regime, specially for dense slurries, the solids effect could have a negative values meaning that a drag reduction is verified. The laminar/turbulent transition velocity that could be noticed when the solids effect reaches its minimum value an begins to increase.

The friction factor of dense non-Newtonian slurries in the laminar regime is related with the Reynolds number (represented by $Re_2$ defined by Slatter (14)) in the same way as for Newtonian fluid.

The shear velocity versus the mean velocity plots are useful to characterize the turbulent flow behavior.

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