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Control of Supply in Water Networks – Nonlinear case

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Glossary of terms and notations

Cf. = cited from
L = incidence matrix (network's connectivities and flow direction)
Q (Q^L, Q^R) = internal flow (linear, residual)
q = external inlet/outlet
H (H^L, H^R, H^m) = water head (linear, residual, measured)
H_0 = reference water head
ε (ε^L, ε^R, ε^m) = pressure head (linear, residual, measured)
K = characteristic of a branch
l = length of a branch
R = hydraulic compliance
R′ = flow coefficient determining the opening degree of outlet valves
ε^0 = virtual distortion (design variable)
$H^{@\varepsilon^0=1}$ = water head at unit distortion (influence vector)
D = influence matrix in terms of water head
$\tilde{\varepsilon}$ = limit pressure head for non-linear characteristic
$\tilde{Q}$ = limit flow for non-linear characteristic
Δ = step length in optimisation process
γ = controlled inlet intensity

Abstract

Methodology (based on so-called Dynamic Virtual Distortion Method) of control of water supply in water networks is presented. Minimization of supply pressure in inlets to the network, subject to inequality constraints imposed on outlet pressure (in chosen nodes) is discussed. Taking advantage of pre-computed influence vectors, the real-time control strategy can be realised with small computational effort and therefore, can be managed with use of

hardware-based controllers. Linear constitutive relation (water flow vs. pressure head) has been assumed in order to develop the first step of the methodology. However, generalisation of the proposed approach for a piece-wise-linear case is proposed as the second step.

1 Introduction

A software tool for signal processing in control of supply in water networks is presented. It is assumed that the water head in the network's nodes in a distance of controlled inlet can be measured and also that the inlet water head can be modified in real time in a controlled way. Then, making use of the analytical network model of this installation and using presented below, VDM technique, the control of water supply can be performed.

The problem of management of water sources is more and more important in the world scale specially in Yemen which undergoes a water crisis. Therefore, there is requirement for an automatic water supply control. The proposed approach is based on continuous observation of the pressure distribution in nodes of the water network. Having a reliable (verified versus field tests) numerical model of the network and its responses for determined inlet and outlet conditions, any modifications to the normal network response (pressure distribution) can be detected. Then, applying proposed bellow numerical procedure, the correction of water supply can be determined.

The proposed methodology is based on the so-called *Virtual Distortion Method* (VDM) approach, applicable also in the problem of damage identification through monitoring of piezo-generated elastic wave propagation (Ref. 2). This technique (called *Piezodiagnostics*) is focused on efficient numerical performance of inverse, non-linear, dynamic analysis. The crucial point of the concept is pre-computing of structural responses for locally generated impulse loadings by unit virtual distortions (similar to local heat impulses). These responses stored in the so-called *influence matrix* allow composition of all possible linear combinations of the influence of local non-linearities (due to defect) on final structural response. Then, using a gradient-based optimization technique, the intensities of unknown, distributed virtual distortions (modelling local defects) can be tuned to minimize the distance between the computed final structural response and the measured one.

2 Definitions and linear analysis

Let me describe the *network analysis* [cf.1] based approach to modelling of water systems using oriented *graf* of small example shown in Fig.1.1, with topology defined by the following incidence matrix:

$$L=\begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \qquad (2.1)$$

where rows correspond to the network's nodes while columns correspond to the branches.

Defining the following quantities describing the state of the water network:

H – the vector of water head in network's nodes

ε - the vector of pressure head in network's branches

Q – the vector of water flow in network's branches

R – the vector of hydraulic compliance in network's branches (depends on pipes' cross-sections, length, material, etc.)

the following equations governing the water distribution can be formulated:

- equilibrium of inlets and outlets for nodes:

$$\mathbf{L}\ \mathbf{Q} = \mathbf{q} \qquad (2.2)$$

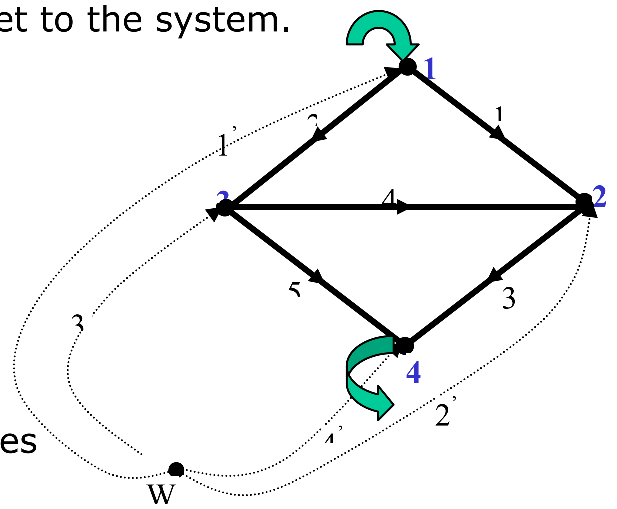
- continuity equation for the network's branches:

$$\mathbf{L}^{\mathbf{T}}\ \mathbf{H} = \varepsilon \qquad (2.3)$$

- constitutive relation governing local flow in branches

$$\mathbf{Q}^{2} = \mathbf{R}\ \varepsilon \qquad (2.4)$$

where **q** denotes external inlet to the system.



Fictitious branches

real branches

Fig.1.1 Oriented *graph* modelling a 2-loop water network

The constitutive relation (2.4) is non-linear. Nevertheless, let me assume temporarily linearity of this relation. Substituting Eqs. (2.4) and (2.3) to (2.2), the following formula can be obtained:

$$\mathbf{L\ R\ L^T\ H = q} \tag{2.5}$$

allowing determination of water head in nodal points as the response for determined inlets.
It will be demonstrated, that having numerical model of the water network and knowing water head distribution at nodes (measured in real time), the optimal water supply can be determined.
Generally, the above set of equations (2.5) can be expressed (including the outlet conditions) in the following forms:

$$\mathbf{(LRL^T - IR')} \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^* \end{bmatrix} = \begin{bmatrix} q \\ 0 \end{bmatrix} \tag{2.6a}$$

or:

$$\mathbf{(L\ R\ L^T - I\ H^*)} \begin{bmatrix} \mathbf{H} \\ \mathbf{R}' \end{bmatrix} = \begin{bmatrix} q \\ 0 \end{bmatrix} - \mathbf{L\ R\ L^T} \cdot \begin{bmatrix} 0 \\ \mathbf{H}^* \end{bmatrix} \tag{2.6b}$$

where H^* denotes water head in outlet nodes.

3 NON-LINEAR ANALYSIS

Analogously to the Virtual Distortion Method (VDM) applicable to the truss structures [cf.2] let me postulate that local modification of a network parameter can be introduced into the system through the *virtual distortion* ε^0 **,** incorporated into the formula (2.5):

$$\mathbf{LR(L^TH - \varepsilon^0) = q} \tag{3.1}$$

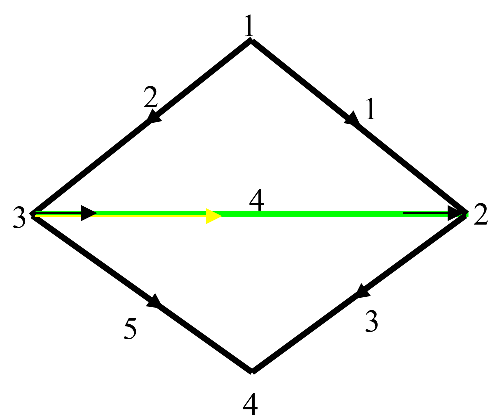


Fig. 3.1. Distortion simulating water flow (pressure head modification) in the branch No. 4

The influence of virtual distortions on the resultant flow redistribution can be calculated making use of the so-called *influence matrix* D_{ij} collecting *i* responses (row-wise) in terms of water heads $H_i^{@\varepsilon^0=1}$ induced in the network by imposing the unit virtual distortion $\varepsilon^o{}_j=1$ generated consecutively in each network branch *j*. Thus each *influence vector* $H_i^{@\varepsilon^0=1}$ can be calculated on the basis of the following equation obtained from Eq. (2.1):

$$\mathbf{L\,R\,L^T\,H}^{@\varepsilon^0=1} = \mathbf{q^*} + \mathbf{LRI} \tag{3.2}$$

The vector q* disregards the external inlet and outlet (the flow is now provided by the imposition of virtual distortion), and it accounts for water flaw distribution in the closed network (cf. Eq. (2.6)). There is a set of j^- (j^- the number of branches) equations (2.2) to be solved in order to create the full influence matrix D. Each time the right hand-side changes as the unit virtual distortion is applied to another branch. In practice this can be realised by applying a pair of inlets-outlets $L_{ik}\ R_{kj}\varepsilon_j^0$ corresponding to each branch (cf. Eq. (2.1)) – it is the so-called *compensative charge*.
So the parameter modification in the system is accounted for by superposing the so-called *linear response* of the original network and the so-called *residual response* due to imposition of the virtual distortion. Therefore, the resultant water head distribution can be expressed as:

$$H_i = H_i^L + H_i^R = H_i^L + \sum_j D_{ij}\varepsilon_j^0 \tag{3.3}$$

and the resultant water flow as:

$$Q_j = Q_j^L + Q_j^R = Q_j^L + R_j L_{ij}^T \sum_j \left(D_{ij} - \delta_{ij}\right)\varepsilon_j^0 \tag{3.4}$$

Making use of the following substitution: $D^\varepsilon = L^T D$, the above relations take the form (cf. Eqs. (3.3), (3.4)):

$$\varepsilon_i = \varepsilon_i^L + \varepsilon_i^R = \varepsilon_i^L + \sum_j D_{ij}^\varepsilon \varepsilon_j^0 \tag{3.5}$$

$$Q_i = Q_i^L + Q_i^R = Q_i^L + R_i \sum_j \left(D_{ij}^\varepsilon - \delta_{ij}\right)\varepsilon_j^0 \tag{3.6}$$

Non-linearity of constitutive relations (cf. Eq. (2.4)) can be simulated through virtual distortions. To this end, let me assume that this relation is approximated through a piece-wise-linear function, for example composed of two pieces (see Fig. 3.2). The algorithm for non-linear analysis of water networks is analogous in this case to progressive collapse analysis of elasto-plastic truss structures (cf. [2], [4]), where sequence of overloaded (i.e. exceeding the yield stress limit due to increasing load intensity) elements should be determined and the corresponding sequence of "growing" sets of linear equations solved.

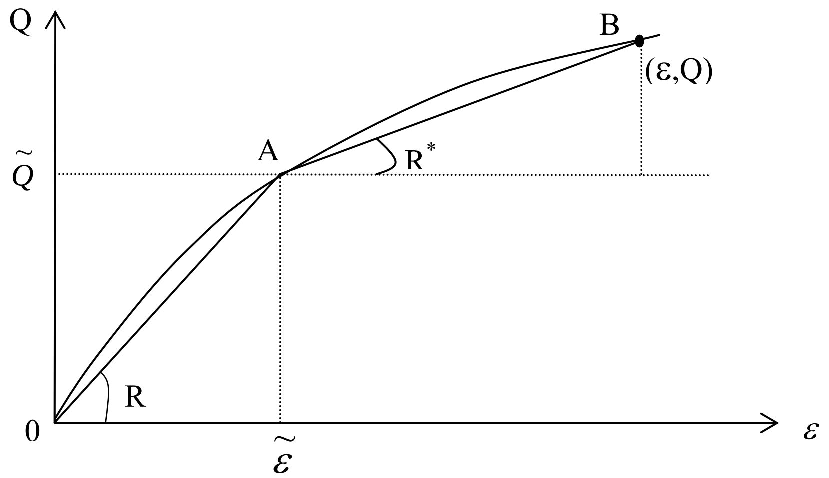


Fig. 3.2 Piece-wise-linear approximation of the non-linear constitutive relation

By analogy to structural mechanics, the conditions for simulation of non-linear behaviour of network branches (see Fig. 3.2) take the following form (describing line AB):

$$Q_i - \widetilde{Q}_i = R_i^*\left(\varepsilon_i - \widetilde{\varepsilon}_i\right) \tag{3.7}$$

Denoting $\gamma_i = \dfrac{R_i^*}{R_i}$ and substituting Eqs. (3.5), (3.6) to Eq. (3.7), the following set of linear equations is obtained:

$$\sum_l \left(\left(1-\gamma_k\right)D_{kl}^{\varepsilon} - \delta_{kl}\right)\varepsilon_l^0 = -\left(1-\gamma_k\right)\left(\varepsilon_k^L - \widetilde{\varepsilon}_k\right) \tag{3.8}$$

where ε_l^0 denotes virtual distortions modelling nonlinear behaviour in branch " j ".
The set (3.8) should be solved with respect to the unknown virtual distortions ε_l^0 where the indices *k*, *l* run through the branches of non-linear characteristics only. More accurate approximation of non-linearities requires application of more piece-wise linear sections and therefore leads to increase of virtual distortion components ε_l^0 to be determined.

4 CONTROL OF WATER SUPPLY

4.1 LINEAR CASE

Let me discuss the *control* of supply in water networks. The objective is to minimise the water supply, keeping the pressure in all outlets above some limit value. Assuming that the water head in outlet nodes can be monitored in real time, specially programmed controller can adapt (through a feedback procedure) the inlet intensity to meet the minimum supply condition. The aim of the following analysis is to determine the basis for the controller operation.

In the case of low water head (below the imposed limit value) in any of the outlets, the controller provokes an increase of the inlet to assure the right water head level. Contrary, in the case of water head higher than the limit value in all outlets, the controller provokes a reduction of the inlet in order to meet the limit water head value in at least one outlet.

The problem of active control of the inlet intensity $q_1^* = \gamma\, q_1$ (where γ denotes the controlled inlet intensity) can be formulated as follows.

$$\min \gamma \tag{4.1}$$

subject to constraints (2.6a or 2.6b) and the following conditions requiring the water head in the outlet joints to be not smaller than some minimal admissible value H':

$$\mathbf{H} \geq H^* \tag{4.2}$$

Let me now discuss the above problem using the network example, illustrated in Fig.4.1. We assume that the water head H_3^m and H_4^m of nodes No.3 and No.4 were previously measured and we have to minimize inlet $\gamma\, q_1$ in order to keep values H_3 and H_4 not lower than H'.

In this case the corresponding set of equation (2.5) looks as follows:

$$\begin{bmatrix} R_1 + R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1 + R_3 + R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2 + R_4 + R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3 + R_5 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1^* \\ 0 \\ q_3 \\ q_4 \end{bmatrix} \tag{4.3}$$

and inequality constraints take the form:

$$H_3 \geq H^*, \qquad H_4 \geq H^* \tag{4.4}$$

where:

$$q_3 = R_3^{'}(H_0 - H_3)$$

$$q_4 = R_4^{'}(H_0 - H_4), \qquad q_1^{*} = \gamma q_1 \quad R = \frac{K^2}{l}, \qquad (4.5)$$

and:
K- the characteristic of an element, l - the element's length,
H – denotes the water head in a node, q – denotes the flow in a branch.

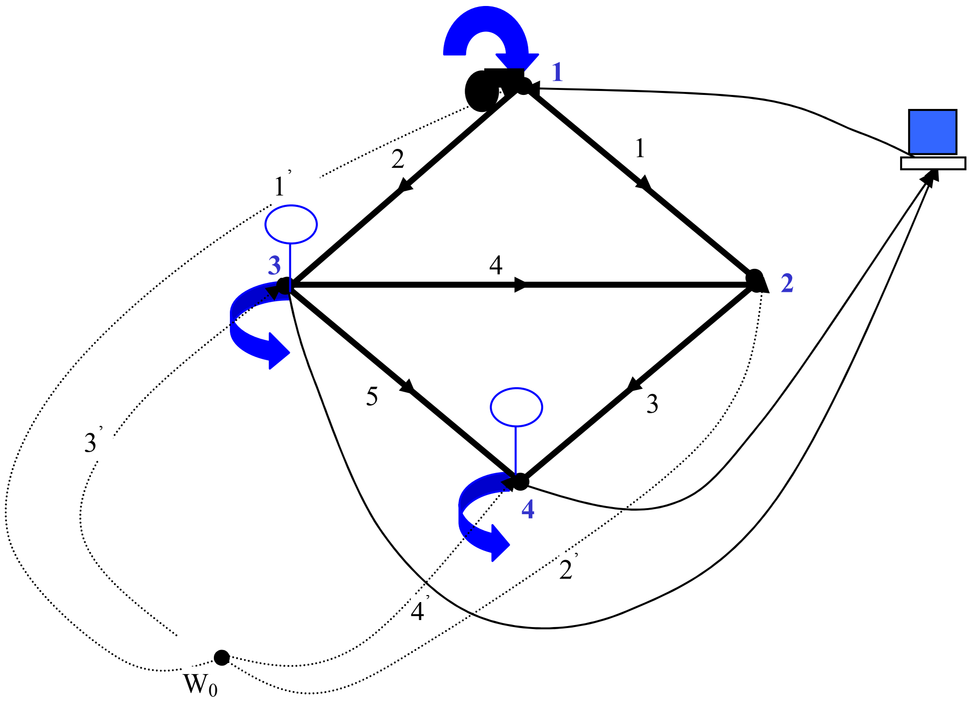


Fig. 4.1 The concept of water supply control

Substituting Eqs. (4.5) to (4.3), the corresponding set of equations takes the following form (2.6c):

$$\begin{bmatrix} R_1+R_2 & -R_1 & 0.0 & 0.0 \\ -R_1 & R_1+R_3+R_4 & 0.0 & 0.0 \\ -R_2 & -R_4 & H_3 & 0.0 \\ 0.0 & -R_3 & 0.0 & H_4 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ R_3^{'} \\ R_4^{'} \end{bmatrix} = \begin{bmatrix} q_1^{*} \\ 0.0 \\ -(R_2+R_4+R_5)H_3 \\ -(R_3+R_5)H_4 \end{bmatrix} \qquad (4.6)$$

allowing determination of coefficients $R_3^{'}$ and $R_4^{'}$denoting the intensity of the outlet valve opening. On the other hand, the water heads H_1^L, H_2^L determined from (4.6) together with the measured ones H_3^m, H_4^m define the water network response to the inlet q_1 and the opening coefficients $R_3^{'}, R_4^{'}$.

It was assumed that the network is supplied only through the node No.1 (inlet with intensity q_1) and the outlets are through the nodes No.3 and No 4. $R_2' = 0$, which means, that the outlet in the node No. 2 vanishes.
The inlet control problem (4.1) - (4.5) (linear programming with convex domain of admissible solutions) leads to the solution located on the boundary (4.4) and can be determined via the following requirement:

$$\text{Inf}\,(H_3, H_4) = H^* \qquad (4.7)$$

Where $H_3 = \gamma H_3^m, H_4 = \gamma H_4^m$ and γ is the scaling coefficient modifying the inlet intensity q_1. Therefore, the control parameter γ takes the value:

$$\gamma = \frac{H^*}{\inf\,(H_3^m, H_4^m)} \qquad (4.8)$$

The problem is not so simple if we have more than one inlet or we take into account retention tanks built into the network.

4.2 NON-LINEAR CASE

The problem of active control of the inlet intensity $q_1^* = \gamma q_1$ (where γ denotes the controlled inlet intensity) can be now generalised for the non-linear case:

$$\min \gamma \qquad (4.9)$$

subject to constraints (2.6a with distortions modelling non-linearity) and (3.8):

$$\mathbf{L\,R\,(\,L^T\,H - \beta^0\,) - I\,R'\,H} = \begin{bmatrix} q^* \\ \mathbf{0} \end{bmatrix} \qquad (4.10)$$

$$\mathbf{R\,(\,L^T\,H - \beta^0\,)} - \tilde{Q} = R^*(L^T H - \tilde{\varepsilon}) \qquad (4.11)$$

where:

$$\mathbf{L}^T\,\mathbf{H} = \mathbf{L}^T\,\mathbf{H}^L + \mathbf{L}^T\,\mathbf{D}\,\beta^o \qquad (4.12)$$

and:

$$\mathbf{H} \geq H^* \qquad (4.13)$$

Substituting (4.12) to (4.10) and (4.11) one can get:

$$(\mathbf{L}\,\mathbf{R}\,\mathbf{L}^{\mathbf{T}}-\mathbf{I}\,\mathbf{R}')\,\mathbf{H}^{\mathbf{L}}+(\mathbf{L}\,\mathbf{R}\,\mathbf{L}^{\mathbf{T}}-\mathbf{I}\,\mathbf{R}')\mathbf{D}^{\mathbf{H}}\boldsymbol{\beta}^{\mathbf{0}}=\begin{bmatrix} q^{*} \\ 0 \end{bmatrix} \tag{4.14}$$

$$(1-\alpha)\,\mathbf{R}\,\mathbf{L}^{\mathbf{T}}\mathbf{H}^{\mathbf{L}}+(\mathbf{1}-\boldsymbol{\alpha})\mathbf{R}\,\mathbf{L}^{\mathbf{T}}\mathbf{D}^{\mathbf{H}}\boldsymbol{\beta}^{\mathbf{0}}=(1-\alpha)\;\mathbf{R}\,\tilde{\varepsilon} \tag{4.15}$$

The following quantities can be determined from the above set of equations: H^L (except of $H^L_{OUT}=H^m$), β^0 and R'.

For the example discussed above (Fig. 4.1) these conditions lead to the following set of equations:

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}\begin{bmatrix} H_1 \\ H_2 \\ \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \\ \beta_5^0 \\ R'_3 \\ R'_4 \end{bmatrix}=\begin{bmatrix} q_1+R_2H_3^m \\ R_4H_3^m+R_3H_4^m \\ -(R_2+R_4+R_5)H_3^m+R_5H_4^m \\ R_5H_3^m+(R_3+R_5)H_4^m \\ \theta R_1\tilde{\varepsilon} \\ \theta R_2(H_3^m+\tilde{\varepsilon}) \\ \theta R_3(H_4^m+\tilde{\varepsilon}) \\ -\theta R_4(H_3^m-\tilde{\varepsilon}) \\ -\theta R_5(H_3^m-H_4^m-\tilde{\varepsilon}) \end{bmatrix} \tag{4.16}$$

in which:

$$A=\begin{bmatrix} R_{1,2} & -R_1 & R_{1,2}D_{11}-R_1D_{21}-R_2D_{31} \\ -R_1 & R_{1,3,4} & -R_1D_{11}+R_{1,3,4}D_{21}-R_4D_{31}-R_3D_{31} \\ -R_2 & -R_4 & -R_2D_{11}-R_4D_{21}+R_{2,4,5}D_{31}-R_5D_{41} \\ 0 & -R_3 & -R_3D_{21}-R_5D_{31}+R_{3,5}D_{41} \\ \theta R_1 & -\theta R_1 & \theta R_1(D_{11}-D_{21}) \end{bmatrix},$$

$$B=\begin{bmatrix} R_{1,2}D_{12}-R_1D_{22}-R_2D_{32} & R_{1,2}D_{13}-R_1D_{23}-R_2D_{33} & R_{1,2}D_{14}-R_1D_{24}-R_2D_{34} \\ -R_1D_{12}+R_{1,3,4}D_{22}-R_4D_{32}-R_3D_{42} & -R_1D_{13}+R_{1,3,4}D_{23}-R_4D_{33}-R_3D_{43} & -R_1D_{14}+R_{1,3,4}D_{34}-R_3D_{44} \\ -R_2D_{12}-R_4D_{22}+R_{2,4,5}D_{32}-R_5D_{42} & -R_2D_{13}-R_4D_{23}+R_{2,4,5}D_{33}-R_5D_{43} & -R_2D_{13}-R_4D_{23}+R_{2,4,5}D_{33}-R_5D_{43} \\ -R_3D_{22}-R_5D_{32}+R_{3,5}D_{42} & -R_3D_{23}-R_5D_{33}+R_{3,5}D_{43} & -R_3D_{14}-R_5D_{34}+R_{3,5}D_{44} \\ \theta R_1(D_{21}-D_{22}) & \theta R_1(D_{13}-D_{23}) & \theta R_1(D_{14}-D_{24}) \end{bmatrix},$$

$$C=\begin{bmatrix} R_{1,2}D_{15}-R_1D_{25}-R_2D_{35} & 0 & 0 \\ -R_1D_{15}+R_{1,3,4}D_{25}-R_4D_{35}-R_3D_{45} & 0 & 0 \\ -R_1D_{15}-R_4D_{25}+R_{2,4,5}D_{35}-R_5D_{45} & -D_{31}-D_{32}-D_{33}-D_{34}-D_{35}+H_3^m & 0 \\ -R_3D_{25}-R_5D_{35}+R_{3,5}D_{45} & 0 & -D_{41}-D_{42}-D_{43}-D_{44}-D_{45}+H_4^m \\ \theta R_1(D_{15}-D_{25}) & 0 & 0 \end{bmatrix},$$

$$D=\begin{bmatrix} \theta R_2 & 0 & -\theta R_2(D_{11}-D_{31}) \\ 0 & \theta R_3 & -\theta R_3(D_{21}-D_{41}) \\ 0 & -\theta R_1 & -\theta R_4(D_{21}-D_{31}) \\ 0 & 0 & -\theta R_5(D_{31}-D_{41}) \end{bmatrix},$$

$$E=\begin{bmatrix} -\theta R_2(D_{12}-D_{32}) & -\theta R_2(D_{13}-D_{33}) & -\theta R_2(D_{14}-D_{34}) \\ -\theta R_3(D_{22}-D_{42}) & -\theta R_3(D_{23}-D_{43}) & -\theta R_3(D_{24}-D_{44}) \\ -\theta R_4(D_{22}-D_{32}) & -\theta R_4(D_{23}-D_{33}) & -\theta R_4(D_{24}-D_{34}) \\ -\theta R_5(D_{32}-D_{42}) & -\theta R_5(D_{33}-D_{43}) & -\theta R_5(D_{34}-D_{44}) \end{bmatrix},$$

and:

$$F=\begin{bmatrix} \theta R_2(D_{15}-D_{35}) & 0 & 0 \\ \theta R_3(D_{25}-D_{45}) & 0 & 0 \\ \theta R_4(D_{25}-D_{35}) & 0 & 0 \\ \theta R_5(D_{35}-D_{45}) & 0 & 0 \end{bmatrix}$$

as well as,

$\theta = (1+\alpha)$,
$R_{1,2} = R_1 + R_2$,
$R_{1,3,4} = R_1 + R_3 + R_4$,
$R_{2,4,5} = R_2 + R_4 + R_5$,

As it was in the linear case (cf. 4.7) let us determine the scaling coefficient γ from the following requirement:

$$\gamma = \frac{H^*}{\inf\,(H_{out})} \qquad (4.17)$$

where H_{out} denotes the water head in the outlet nodes.

The above control rule can be realised via close-loop control hardware reacting actively in real time, or via open-loop system, determining the control parameter γ on the base of our numerical model of the network. To this end, the following algorithm has to be realised.

4.3 General Algorithm

In the case of non-linear constitutive relation we have to perform the following steps:

1- We assume that the water head in the outlet nodes has been previously measured and the number of overflowed (nonlinear) branches equal to zero, as well as, small parameter δ.

2- Determine water head distribution H^L, R'_{OUT} and virtual distortion β^0 modelling non-linearity from (4.16). Then, search for maximally overflowed branch "k" $(Q \rangle \tilde{Q})$. If there is new overflowed branch, incorporate this branch to the set S, other ways go to the following step.

3- Determine the lowest H_{out} and solve the conditions (4.18) and (4.19) for γ and β^0.

4- Next, we have to check the condition (4.20), if it is satisfied then we have to stop, if it is not satisfied then we have to modify the inlet incrementally and as a result, distortions are generated and the non-linear domain expands. We keep applying this procedure until the condition (4.20) is satisfied.

$$\begin{cases} S = \{0\} \text{ - set of overflowed branches (nonlinear)}, \\ \delta \text{ - small parameter}, \\ H^m \text{ – H measured in OUT nodes}, \end{cases}$$

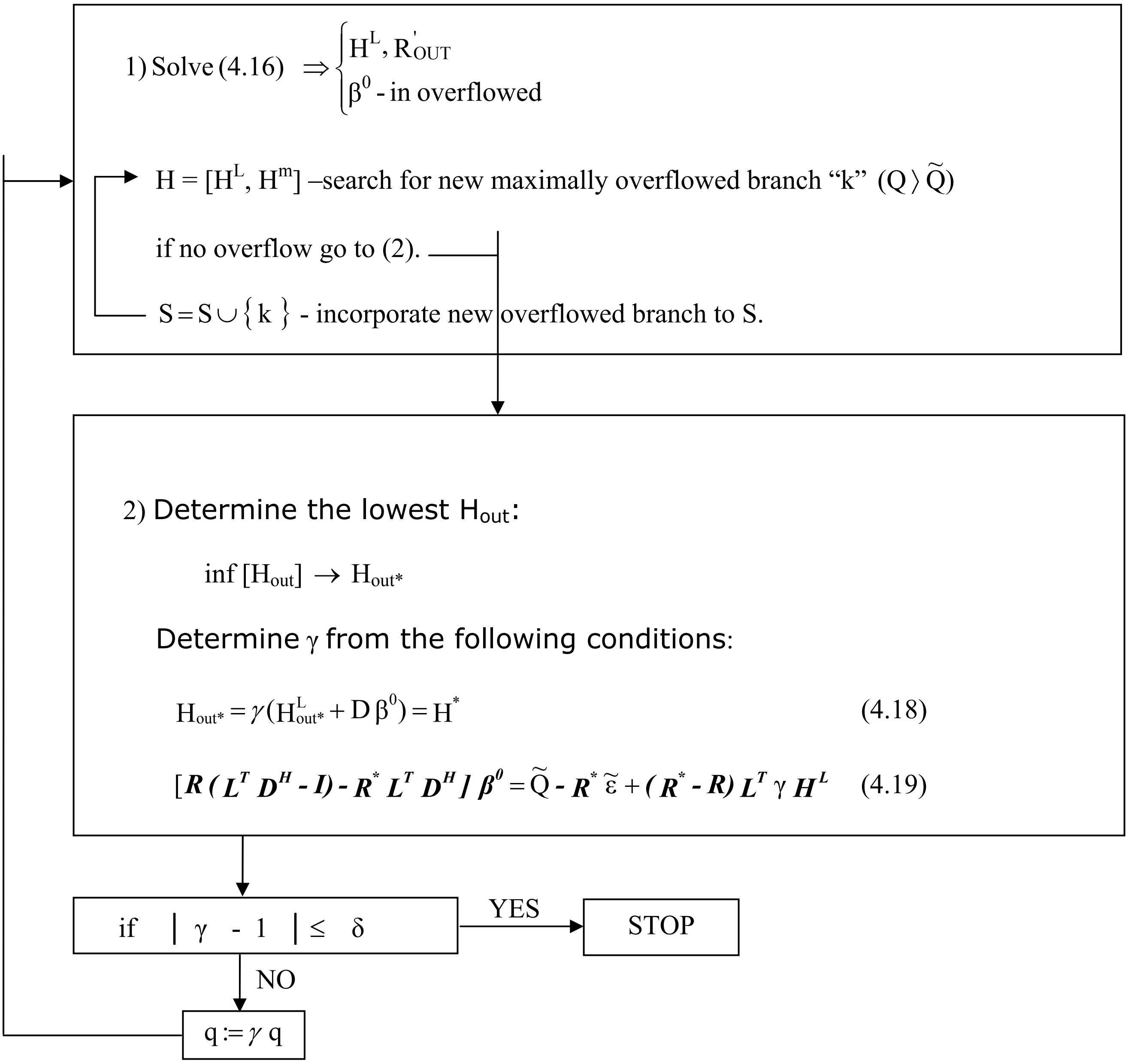


Table 4.1. Algorithm for WATNET-C non-linear control

Let me finally demonstrate the result of active control on the basis of the example discussed for the linear case, however, with non-linear properties determined by parameters $\tilde{\varepsilon} = 0.8\text{m}$, (see section 3).

4.4 Numerical example:

Two-loop network:

Let me demonstrate the result of active control for the network shown in Fig. 4.1.

Assuming the following data, ($K1=K2=K3=K4=K5=0.061 m^3/s$, $l_1 = l_2 = l_3 = l_5 = 5.000$ m, $l_4=7.071$ m, $q_1=0.005\ m^3/sec$, $H_0 = 0.0$ and having the following measurements: $H_3^m = 0.0015m$ and $H_4^m = 0.0005m$ of nodes No.3 and No.4, the water head distribution for non-linear constitutive relation ($\alpha =$ 0.5 and $\bar{\varepsilon} = 3.0$) looks as follows:

$$H = [0.8980 \quad 0.3320 \quad 0.0015 \quad 0.0005]^T$$

while the opening coefficients in the outlet nodes are: $R_3' = R_4' = 0.5$.

In terms of pressure head the final result is the following:

$$\varepsilon = [0.566 \quad 0.897 \quad 0.332 \quad 0.331 \quad 0.001]$$

Let us assume that the minimum water head in the outlet nodes should be $H' = 0.0005m$, what correspond to the above case.

Case I: when $H_4 < H^*$

Now let me analyse another case of the above network, in which measured water heads are as follow $H_3^m = 0.00094m$ and $H_4^m = 0.00025m$. The water head distribution for all nodes (non linear case) takes the following form in this case

$$H = [0.89800 \quad 0.33200 \quad 0.00094 \quad 0.000247]^T$$

While the opening coefficients are $R_3' = 0.80$, $R_4' = 1.00$. Having $H_{min} = H_4 < H^*$, the optimization procedure has to be applied in order to determine the required inlet intensity $\gamma = 1.06$, what leads to the following results:

$$H = [2.06600 \quad 0.68400 \quad 0.00099 \quad 0.00050]^T$$

and the virtual distortions modelling non-linearity:

$$\beta^0 = [0.103 \quad 0.326 \quad 0.000 \quad 0.000 \quad 0.000]^T$$

Development of the corresponding distortions is demonstrated in Fig. 4.2.

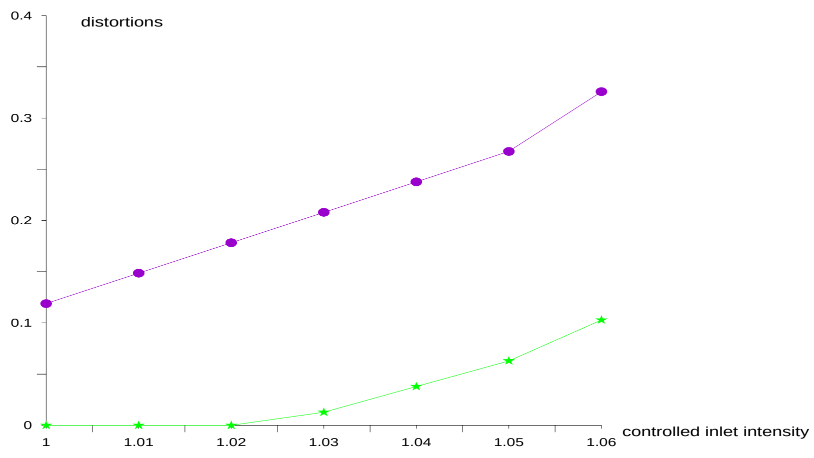


Fig. 4.2 Development of virtual distortions

Case II: when $H_4 > H^*$

Now let me analyse another case of the above network, in which the measured water heads are as follow: H_3^m =0.00746m and H_4^m =0.00127m. In this case the water head distribution for all nodes (non linear case) looks as follows:

$$H = [0.90300 \quad 0.33600 \quad 0.00746 \quad 0.00127]^T$$

While the opening coefficients are $R_3' = 0.20$, $R_4' = 0.10$. Having $H_{min} = H_4 > H'$ the optimisation procedure has to be applied in order to determine the required inlet intensity $\gamma = 0.84$, what leads to the following results:

$$H = [0.34700 \quad 0.12900 \quad 0.00104 \quad 0.00050]^T$$

Development of the corresponding distortions is demonstrated in Fig. 4.3.

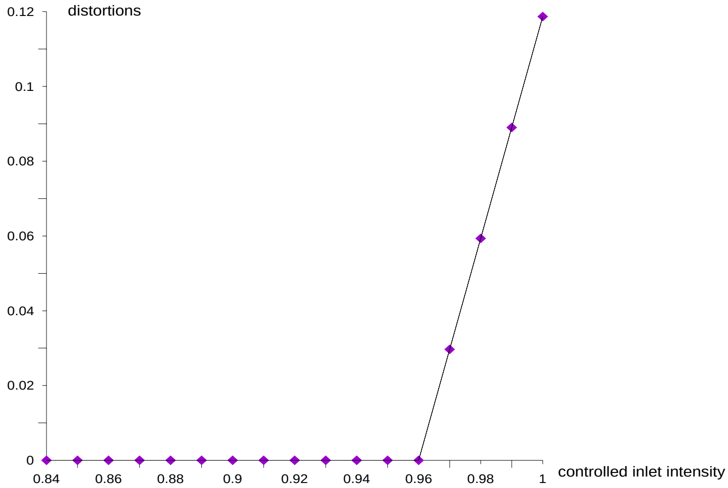


Fig. 4.3 Development of virtual distortions

5 Conclusions

1- Modelling the non-linearity of constitutive relation through the *Virtual Distortion Method*, assuming piece-wise linear characteristic, describing water flow in the considered networks.

2. Formulation and solution of the problem of optimal control of inlet intensity in water networks, based on continuous observation of the water head distribution in the outlet nodes.
3. Development of the algorithm for WATNET-C for the control of supply in water networks realising the problem mentioned in the point 2.

6 References

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