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NONLINEAR CONTROL OF A SELF-EXCITED SYSTEM

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ABSTRACT

The objective of the proposed work is to demonstrate the feasibility of the use of the nonlinear saturation-based control concept to suppress self-excited vibrations by means of active nonlinear feedback control. The authors use the van der Pol oscillator as the working model for a self-excited system. A saturation phenomenon is induced by tuning the frequency of the under-damped second-order absorber to one-half that of the primary system. The authors conclude that although we can achieve some level of performance with this control technique, we question its robustness due to the addition of rich dynamics introduced by the controller.

INTRODUCTION

The force acting on a vibrating system is usually external to the system and independent of its motion. However, there are systems for which the exciting force is a function of the state of the system, such as its displacement, velocity, and acceleration. Such systems are called self-excited oscillators because the force that sustains the motion is created or controlled by the motion itself. In many cases, a self-excited system possesses negative linear damping and positive nonlinear damping. A positive damping force is directed opposite to the velocity and a negative damping force is directed along the velocity.

Aero-elastic phenomena have emerged and threatened our ability to build safe and reliable aircraft. Perhaps the most frightening of these obstacles has been flutter, which continues to impress itself upon modern aircraft design. A variety of active control techniques to suppress vibrations due to flutter has been investigated, but the bulk of the methods [1-5] have implemented linear control strategies. In this paper, we describe a nonlinear strategy for controlling the self-excited oscillations of a flexible wing. Self-excited oscillations resulting from a form of negative damping occur in many physical systems. A number of textbooks deal with self-excited systems and the methods used to investigate them [6, 7]. Thus, it would be of much interest developing new ways to reduce the persistent oscillations of such systems.

Parametric vibrations are induced by a varying system parameter (e.g. stiffness). On the other hand, autoparametric vibration (self parametric) is characterized by an internal coupling involving at least two modes. From a mathematical point of view, this coupling is induced by nonlinear terms in the equations of motion of the combined system. Physically speaking, an autoparametric system consists of two parts: a main system and a secondary system. When the main system is externally excited, the secondary system is parametrically excited as a result of the variation of its stiffness with the response of the main system. In other words, a two-mode interaction occurs when the main system exhibits a forced response which, in turn, drives the secondary system into parametric resonance. In this case, energy is transferred from one part of the combined system to the other. This energy transfer depends on the type of nonlinearities and the damping forces. It can be partial or complete, depending on the system parameters. In the case of quadratic nonlinearities and free vibrations, the energy transfer is complete when the natural frequency of the main system is exactly twice the natural frequency of the secondary system. This case is referred to as a two-to-one internal resonance [7, 8]. In the case of forced vibrations, the response of the main system can be suppressed. When the main system is excited at a frequency near its natural frequency (i.e., primary resonance), the response of the main system will have the same frequency as the excitation. The response amplitude will linearly increase with the amplitude of excitation. However, after a critical value, the response of the main system saturates at a specific value and all of the additional energy from the exciter is channeled to the secondary system. This phenomenon is referred to as the saturation phenomenon and has been developed as a nonlinear control strategy [9-11]. A similar control strategy of self-excited vibrations was implemented by Verhulst [12] where he quenched undesirable vibrations by energy absorption through embedding the oscillator in an autoparametric system by coupling to a damped oscillator.

Hall et al. [13] applied numerically the nonlinear saturation phenomenon to suppress the flutter of a wing. They illustrated the concept by means of an example with a rather flexible, large-span wing of the type found on such vehicles as high endurance aircraft and sailplanes. They concluded that to control the amplitude response, they need to actively adapt the controller frequency because of the nonlinear nature of the systems.

In this paper, authors will evaluate the method of saturation-based control developed by Oueini et al. [11] in suppressing the large-amplitude vibrations of a van der Pol self-excited system. We will evaluate the feasibility of the control law for various parameter combinations.

NOMENCLATURE

a, b	response amplitude of plant and controller
x, y	generalized coordinates of the plant and controller
$\dot{x}$, $\dot{y}$	generalized coordinates first derivative with time
$\ddot{x}$, $\ddot{y}$	generalized coordinates second derivative with time
A, B	arbitrary complex-valued functions of T_1
D_n	$\partial/\partial T_n$
α, β	phase response of plant and controller
δ_1, δ_2	coefficients for energy exchange between the oscillator and absorber
ε	small bookkeeping parameter to express the nearness of ω_1 to $2\omega_2$
μ_1	negative linear damping factor of the system
μ_3	positive nonlinear damping factor of the system
μ_3	positive linear damping factor of the controller
ω_1, ω_2	natural frequencies of the system and controller

MATHEMATICAL MODEL

Consider the van-der-Pol oscillator [6] coupled with a saturation-based controller [9] we obtain the following system:

$$\ddot{x} + \omega_1^2 x = \mu_1 \dot{x} - \mu_3 x^2 \dot{x} + \delta_1 y^2 \tag{1}$$

$$\ddot{y} + \omega_2^2 y = -\mu_c \dot{y} + \delta_2 xy \tag{2}$$

Here, x and y denote the generalized coordinates of the oscillator and absorber respectively, ω_1 and ω_2 are their linear undamped natural frequencies. The overdot represents differentiation with respect to time t. For energy dissipation oscillator, we incorporate negative linear damping term: $\mu_1 \dot{x}$, and positive nonlinear damping term: $\mu_3 x^2 \dot{x}$. For the absorber, we incorporate a positive linear damping term: $\mu_c \dot{y}$. Finally, the two quadratic coupling terms: $\delta_1 y^2$ and $\delta_2 xy$ are necessary for the energy exchange between the oscillator and absorber to take place. The authors will study these equations using both numerical and analytical techniques. To begin the analytic studies, we must first scale the governing equations. The scaled equations are

$$\ddot{x} + \omega_1^2 x = \varepsilon\mu_1 \dot{x} - \varepsilon\mu_3 x^2 \dot{x} + \varepsilon\delta_1 y^2 \tag{3}$$

$$\ddot{y} + \omega_2^2 y = -\varepsilon\mu_c \dot{y} + \varepsilon\delta_2 xy \tag{4}$$

where ε is a small bookkeeping parameter. To express the nearness of ω_1 to $2\omega_2$, we write

$$\omega_1 = 2\omega_2 + \varepsilon\sigma \tag{5}$$

We attack the problem using the method of multiple scales, Nayfeh [1], and assume that x and y can be approximated by the following expansions:

$$x \approx x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1)$$

$$y \approx y_0(T_0, T_1) + \varepsilon y_1(T_0, T_1)$$

Where $T_0 = t$ and $T_1 = \varepsilon t$ Substituting the above expansions into the scaled equations and equating coefficients of like powers of ε, we obtain the following systems of differential equations that are solved in cascade:

$O(1)$:

$$D_0^2 x_0 + \omega_1^2 x_0 = 0 \tag{6}$$

$$D_0^2 y_0 + \omega_1^2 y_0 = 0 \tag{7}$$

$O(\varepsilon)$:

$$D_0^2 x_1 = -2D_0 D_1 x_0 + \mu_1 D_0 x_0 - \mu_3 x_0^2 D_0 x_0 + \delta_1 y_0^2 \tag{8}$$

$$D_0^2 y_1 = -2D_0 D_1 y_0 - \mu_c D_0 y_0 x_0 + \delta_2 x_0 y_0 \tag{9}$$

where $D_n = \dfrac{\partial}{\partial T_n}$.

The solution to $O(1)$ problem is given by

$$x_0 = A(T_1) e^{i\omega_1 T_0} + \bar{A} e^{i\omega_1 T_0} \tag{10}$$

$$y_0 = B(T_1) e^{i\omega_2 T_0} + \bar{B} e^{i\omega_2 T_0} \tag{11}$$

where $A(T_1)$ and $B(T_1)$ are arbitrary complex-valued functions of T_1. Substituting this solution into equations (8) and (9), we obtain

$$D_0^2 x_1 + \omega_1^2 x_1 = \left(-2i\,\omega_1 D_1 A + i\,\mu_1\omega_1 A - i\,\mu_3\omega_1 A^2\bar{A} + \delta_1 B^2 e^{-i\sigma T_1}\right)e^{i\omega_1 T_0} + NST + cc \tag{12}$$

$$D_0^2 y_1 + \omega_1^2 y_1 = \left(-2i\,\omega_2 D_1 B - i\,\mu_c\omega_2 B + \delta_2 A\bar{B}e^{i\sigma T_1}\right)e^{i\omega_2 T_0} + NST + cc \tag{13}$$

where *NST* stands for the nonsecular terms and *cc* stands for the complex conjugate of the preceding terms. In order to obtain a uniform expansion up to first order, the terms in brackets must vanish. This condition yields the modulation equations for the complex-valued amplitudes A and B:

$$2i\,\omega_1 A' = i\,\mu_1\omega_1 A - i\,\mu_3\omega_1 A^2\bar{A} + \delta_1 B^2 e^{-i\sigma T_1} \tag{14}$$

$$2i\,\omega_2 B' = -i\,\mu_c\omega_2 B + \delta_2 A\bar{B}e^{i\sigma T_1} \tag{15}$$

where primes are derivatives with respect to the slow time scale T_1. Expressing equations (14) and (15) in terms of polar coordinates as

$$A = \frac{1}{2}ae^{i\alpha} \tag{16}$$

$$B = \frac{1}{2}be^{i\beta} \tag{17}$$

where a, b, α and β are unknown real-valued functions of T_1. Substituting these equations for A and B into equations (16) and (17), we obtain

$$i\,\omega_1\left(ia\alpha' + a'\right)e^{i\alpha} = \frac{1}{2}i\,\mu_1\omega_1 ae^{i\alpha} - \frac{1}{8}i\,\mu_3\omega_1 a^3 e^{i\alpha} + \frac{1}{4}\delta_1 b^2 e^{2i\beta}e^{-i\sigma T_1} \tag{18}$$

$$i\,\omega_2\left(ib\beta' + b'\right)e^{i\beta} = -\frac{1}{2}i\,\mu_c\omega_2 be^{i\beta} + \frac{1}{4}\delta_2 abe^{i(\alpha-\beta)}e^{i\sigma T_1} \tag{19}$$

Separating real and imaginary parts in equations (18) and (19), we obtain the following system of real-valued modulation equations.

$$a' = \frac{1}{2}\mu_1 a - \frac{1}{8}\mu_3 a^3 - \frac{\delta_1 b^2}{4\omega_1}\sin\left(\sigma T_1 + \alpha - 2\beta\right) \tag{20}$$

$$b' = -\frac{1}{2}\mu_c b + \frac{\delta_2 ab}{4\omega_2}\sin\left(\sigma T_1 + \alpha - 2\beta\right) \tag{21}$$

$$a\alpha' = -\frac{\delta_1 b^2}{4\omega_1}\cos\left(\sigma T_1 + \alpha - 2\beta\right) \tag{22}$$

$$b\beta' = -\frac{\delta_2 ab}{4\omega_2}\cos\left(\sigma T_1 + \alpha - 2\beta\right) \tag{23}$$

These modulation equations represent a non-autonomous system, to obtain an equivalent autonomous system, we define

$$\gamma = \sigma T_1 + \alpha - 2\beta \tag{24}$$

Thus the modulation equations are reduced from a system of four-first order differential equations into the following set of three equations

$$a' = \frac{1}{2}\mu_1 a - \frac{1}{8}\mu_3 a^3 - \frac{\delta_1 b^2}{4\omega_1}\sin\gamma \tag{25}$$

$$b' = -\frac{1}{2}\mu_c b + \frac{\delta_2 ab}{4\omega_2}\sin\gamma \tag{26}$$

$$ab\gamma' = ab\sigma + \left(\frac{\delta_2 a^2}{2\omega_2} - \frac{\delta_1 b^2}{4\omega_1}\right)b\cos\gamma \tag{27}$$

The equilibrium solution (fixed points) are obtained by setting $a' = b' = \gamma' = 0$, leading to

$$\frac{1}{2}\mu_1 a - \frac{1}{8}\mu_3 a^3 - \frac{\delta_1 b^2}{4\omega_1}\sin\gamma = 0 \tag{28}$$

$$-\frac{1}{2}\mu_c b + \frac{\delta_2 ab}{4\omega_2}\sin\gamma = 0 \tag{29}$$

$$ab\sigma + \left(\frac{\delta_2 a^2}{2\omega_2} - \frac{\delta_1 b^2}{4\omega_1}\right)b\cos\gamma = 0 \tag{30}$$

From equations (28-30), we obtain the following simplified equations for a, b and γ which are the response amplitudes of the plant, controller and a measure of the phase between the responses, respectively.

$$\begin{aligned}&\delta_2^2\mu_3^2\left(a^2\right)^3 - 4\left[\omega_2^2\mu_c^2\mu_3^2 + 2\delta_2^2\left(\mu_1-\mu_c\right)\mu_3\right]\left(a^2\right)^2 + \\ &16\left(\mu_1-\mu_c\right)\left[\delta_2^2\left(\mu_1-\mu_c\right) + 2\omega_2^2\mu_c^2\mu_3\right]a^2 - \\ &4\omega_2^2\mu_c^2\left[16\left(\mu_1-\mu_c\right)^2 + 64\sigma^2\right] = 0\end{aligned} \tag{31}$$

$$b^2 = \frac{\omega_1\delta 2}{\omega_2\delta_1\mu_c}\left(\mu_1 - \frac{1}{4}\mu_3 a^2\right)a^2 \tag{32}$$

$$\sin\gamma = \frac{2\mu_c\omega_2}{\delta_2 a} \tag{33}$$

Authors note that equation (31) is a cubic equation in a^2 for the nontrivial solutions a. Hence, we expect up to three real solutions for the plant response excluding the trivial one. Numerically, we can determine the roots of the cubic equation which are the a values, then we can solve for b and γ from equations (32-33).

Also, we can try and solve equations (28-30) analytically. Here, to minimize the math, the authors consider only the case in which the controller is perfectly tuned, (i.e. $\sigma = 0$). Again, there are two primary cases, the controller is deactivated, $b = 0$, or the controller is activated, $b \neq 0$. When the controller is activated, equation (29) requires that $a \neq 0$ when $\mu_c \neq 0$. When $\sigma = 0$, the four possible solutions are

i	$a = 0$	$b = 0$	γ arbitrary
ii	$a = \left(\dfrac{4\mu_1}{\mu_3}\right)^{\frac{1}{2}}$	$b = 0$	γ arbitrary
iii	$a = \dfrac{2\omega_2\mu_c}{\delta_2}$	$b = \left[\dfrac{\omega_1\omega_2\mu_c\left(4\mu_1 - \mu_3 a^2\right)}{\delta_1\delta_2}\right]^{\frac{1}{2}}$	$\gamma = \dfrac{\pi}{2}$
iv	$a = \left[\dfrac{4\mu_1 - 8\mu_c}{\mu_3}\right]^{\frac{1}{2}}$	$b = \left[\dfrac{2\omega_1\delta_2}{\omega_2\delta_1}\right]^{\frac{1}{2}} a$	$\gamma = \sin^{-1}\left(\dfrac{2\omega_2\mu_c}{\delta_2 a}\right)$

Solution (*i*) is the trivial solution, solution (*ii*) corresponds to the uncontrolled response, solution (*iii*) is the saturated solution, and solution (*iv*) is another controlled solution. Solution (*iii*) is identified as a saturated solution because the amplitude of the plant response a is independent of the original system parameters ω_1, μ_1, and μ_3, therefore, it will not change even as the nonlinear damping μ_3, for example is varied. The amplitude a of solution (*iii*) is specified by the controller damping μ_c and the controller feedback δ_2, and thus, can be made as small as desired. The additional energy that would normally be dissipated by the plant response is channeled into the controller through the two-to-one internal resonance (i.e., through the quadratic coupling terms).

Both controlled solutions exist only in certain regions of the parameter space. Solution (*iii*) exists when $\mu_3 < \mu_{32}$ (condition 1) where

$$\mu_{32} = \frac{\mu_1\delta_2^2}{\omega_2^2\mu_c^2}$$

and solution (*iv*) exists only when $\mu_1 > 2\mu_c$ (condition 2). The authors study the influence of both μ_3 and μ_c on the response behavior of the system.

STABILITY ANALYSIS

To determine the stability of the solutions, we introduce a small perturbation to the fixed points as

$$a = a_0 + \delta a \qquad (34)$$

$$b = b_0 + \delta b \qquad (35)$$

$$\gamma = \gamma_0 + \delta\gamma \qquad (36)$$

Then we substitute the above expansions into equations (25-27), linearize with respect to the perturbations δa, δb, and $\delta\gamma$, and obtain the following equations that describe the evolution of the perturbations:

$$\delta a' = \left[\frac{1}{2}\mu_1 - \frac{3}{8}\mu_3 a_0^2\right]\delta a - \left[\frac{2\delta_1}{4\omega_1}b_0\sin\gamma_0\right]\delta b - \left[\frac{\delta_1}{4\omega_1}b_0^2\cos\gamma_0\right]\delta\gamma \qquad (37)$$

$$\delta b' = \left[\frac{\delta_2}{4\omega_2}b_0\sin\gamma_0\right]\delta a - \left[\frac{1}{2}\mu_c - \frac{\delta_2}{4\omega_2}a_0\sin\gamma_0\right]\delta b - \left[\frac{\delta_2}{4\omega_2}a_0 b_0\cos\gamma_0\right]\delta\gamma \qquad (38)$$

$$\begin{aligned} a_0 b_0\left(\delta\gamma\right)' &= \left[\left(\sigma + \frac{4\delta_2}{4\omega_2}a_0\cos\gamma_0\right)b_0\right]\delta a \\ &+ \left[a_0\sigma + \left(\frac{\delta_2}{2\omega_2}a_0^2 - \frac{3\delta_1}{4\omega_1}b_0^2\right)\cos\gamma_0\right]\delta b \\ &- \left[\left(\frac{\delta_2}{2\omega_2}a_0^2 - \frac{\delta_1}{4\omega_1}b_0^2\right)b_0\sin\gamma_0\right]\delta\gamma \end{aligned} \qquad (39)$$

We determine the stability of the four solutions by substituting each into equations (37-39) and calculating whether the perturbations grow or decay in time. For solution (*i*) we find that

$$(\delta a)' = \frac{1}{2}\mu_1\delta a$$

$$(\delta b)' = -\frac{1}{2}\mu_c\delta b$$

and hence the solution is unstable because a perturbation to $a = 0$ will grow. Next, we consider solution (*ii*):

$$(\delta a)' = -\mu_1\delta a$$

$$(\delta b)' = -\left[\frac{1}{2}\mu_c - \frac{\delta_2}{4\omega_2}\left(\frac{4\mu_1}{\mu_3}\right)^{\frac{1}{2}}\sin\gamma_0\right]\delta b$$

$$0 = \frac{2\delta_2}{\omega_2}\left(\frac{\mu_1}{\mu_3}\right)\cos\gamma_0\delta b$$

but $\delta b \neq 0$, therefore the third equation indicates that $\cos\gamma_0 = 0$ and we have

$$(\delta a)' = -\mu_1\delta a \qquad (40)$$

$$(\delta b)' = -\left[\frac{1}{2}\mu_c - \frac{\delta_2}{4\omega_2}\left(\frac{4\mu_1}{\mu_3}\right)^{\frac{1}{2}}\right]\delta b \qquad (41)$$

Solution (*ii*) is stable when $\mu_3 > \mu_{32}$

As before, the authors determine the stability of the other solutions but instead of analytically, numerically by calculating the eigen-values of equations (37-39) [14]. Since some equations does not uncouple and we obtain a cubic polynomial for the eigenvalues that is difficult to solve in closed form; therefore, we resort to numerical techniques. If the real part of every eigenvalue is negative, the corresponding equilibrium solution is asymptotically stable. If the real part of any of the eigenvalues is positive, the corresponding equilibrium solution is unstable.

Figs. 1-2 present the effect of detuning the absorber's frequency away from one-half the excitation frequency on the amplitude response of both the oscillator and the absorber for two different gains. The dashed lines denote unstable equilibrium solutions and the solid lines denote stable equilibrium solutions. For both the oscillator and absorber responses, there exists one response that corresponds to the trivial solution and another response that is close to the trivial solution. Both of these responses are unstable. The third and fourth amplitude responses are similar but one is the mirror image of the other. By slowly increasing the value of the frequency σ from -8 to 8 the authors encounter supercritical pitchfork bifurcation *PF*1 at about σ = -4 for the third solution. Then we encounter a reverse pitchfork bifurcation *PF*2 for the fourth solution. For the cases of *PF*1 and *PF*2, we have one branch of stable fixed points on one side of the bifurcation that continues to be unstable due to the bifurcation and another created stable branch whose amplitude of response monotonically decreases in the case of the oscillator and increases for the case of the absorber. Thus it is clear that only at σ = 0 there is only one stable solution while for any other σ there are two stable solutions that coexist and can lead into jumps in the amplitude response of both the oscillator and the absorber. It is clear from this conclusion that this is not a robust way of control since there might be lots of disturbances that can change the value of σ and thus leading to unpredictable response. Fig. 2 shows the same behavior for the amplitude responses of the oscillator and the absorber as the feedback gain increases to a value of five instead of one. The only difference is on the response of the absorber, which is reduced by a factor proportional to the gain. Thus no real improvements in the oscillator response occur which is what we want. Thus the choice of gain can be decided upon based on the practical implementation of the absorber circuit and the actuator used.

Next, the authors examine the influence of the nonlinear damping μ_3 on the system response. Figs. 3-4 show the four different solutions that were determined analytically earlier for the case of σ = 0. Here again solid lines represent stable solutions while dashed lines represent unstable ones. It is clear from the figures that only solution (*iv*) is the stable one and that as the nonlinear damping coefficient increases, the response amplitude decreases for both the system and the absorber. The only difference between Figs. 3-4 is the value of the damping ratio ζ. We can see that the response amplitude of Fig. 4 is less than that of Fig. 3 which is expected because there is less energy needed (due to lower value of ζ) for the flow of energy between the two modes to occur.

Next, Plots of the fixed-point solutions as a function of μ_c are shown in Fig. 5. The shaded region represents the region where the limit-cycle solutions exist. Again reverse Pitchfork bifurcations exist at μ_c of about 4.4 after which only solution (*ii*) exists and thus obeying condition (1). Also solution (*iv*) exists only for values of μ_c < 0.6 and thus abiding by condition (2).

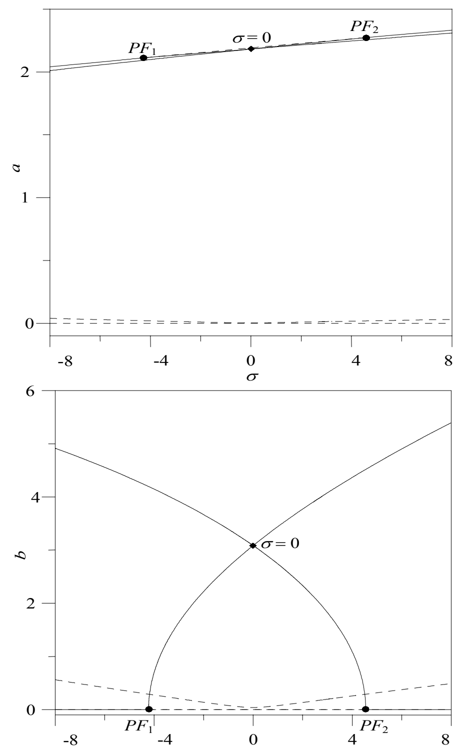

Fig. 1: Fixed-point solutions and their stability as a function of the frequency detuning σ when $g = 1$, $\omega_2 = 30$, $\delta_1 = 4g\omega_1$, $\delta_2 = 4\omega_2$, $\mu_1 = 2\zeta\omega_1$, $\zeta = 0.01$, $\mu_3 = 1$, and μ_c=0.005. Solid lines represent stable solutions, dashed lines represent unstable solutions.

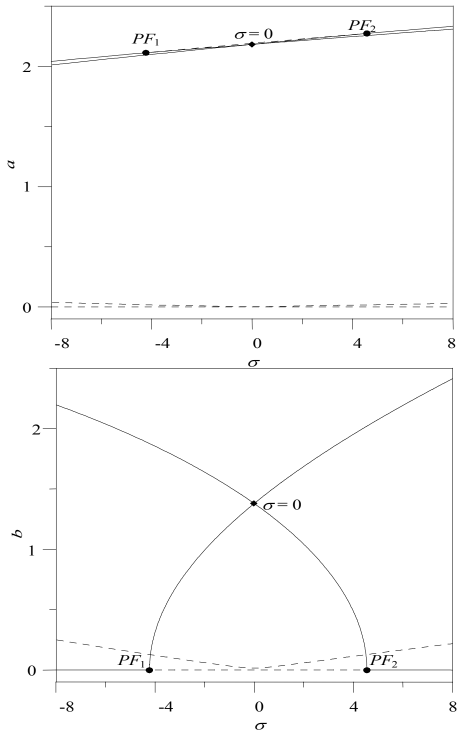

Fig. 2: Fixed-point solutions and their stability as a function of the frequency detuning σ when $g = 5$, $\omega_2 = 30$, $\delta_1 = 4g\omega_1$, $\delta_2 = 4\omega_2$, $\mu_1 = 2\zeta\omega_1$, $\zeta = 0.01$, $\mu_3 = 1$, and $\mu_c = 0.005$. Solid lines represent stable solutions, dashed lines represent unstable solutions.

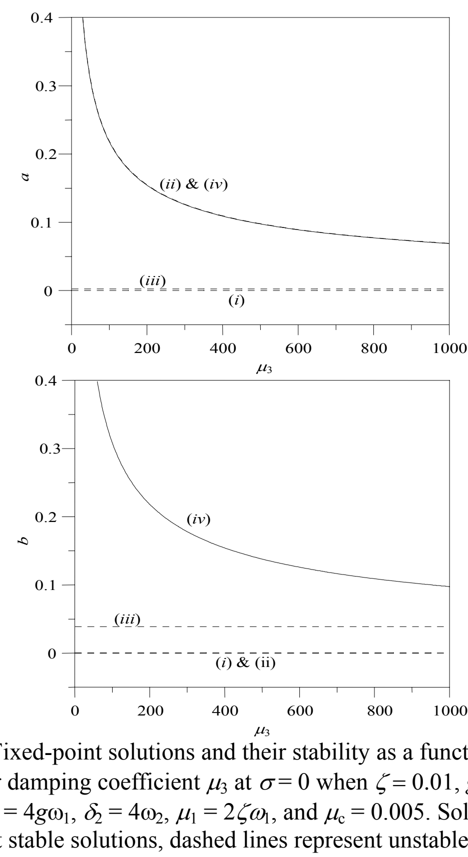

Fig. 3: Fixed-point solutions and their stability as a function of the nonlinear damping coefficient μ_3 at $\sigma = 0$ when $\zeta = 0.01$, $g = 1$, $\omega_2 = 30$, $\delta_1 = 4g\omega_1$, $\delta_2 = 4\omega_2$, $\mu_1 = 2\zeta\omega_1$, and $\mu_c = 0.005$. Solid lines represent stable solutions, dashed lines represent unstable solutions.

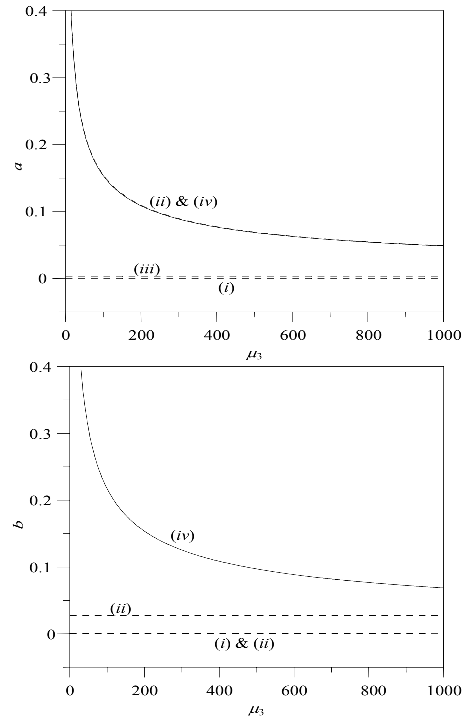

Fig. 4: Fixed-point solutions and their stability as a function of the nonlinear damping coefficient μ_3 at $\sigma = 0$ when $\zeta = 0.005$, $g = 1$, $\omega_2 = 30$, $\delta_1 = 4g\omega_1$, $\delta_2 = 4\omega_2$, $\mu_1 = 2\zeta\omega_1$, and $\mu_c = 0.005$. Solid lines represent stable solutions, dashed lines represent unstable solutions.

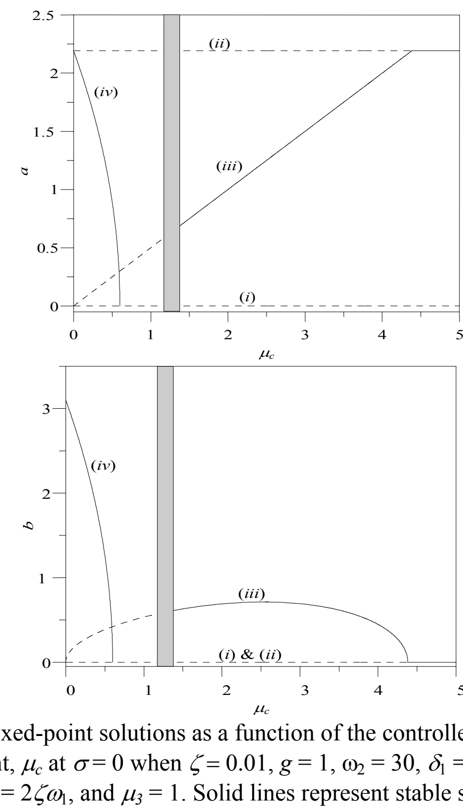

Fig. 5: Fixed-point solutions as a function of the controller damping coefficient, μ_c at $\sigma = 0$ when $\zeta = 0.01$, $g = 1$, $\omega_2 = 30$, $\delta_1 = 4g\omega_1$, $\delta_2 = 4\omega_2$, $\mu_1 = 2\zeta\omega_1$, and $\mu_3 = 1$. Solid lines represent stable solutions, dashed lines represent unstable solutions, and the shaded area is the region of Hopf bifurcated solutions.

CONCLUSION

From this analysis of the nonlinear feedback control implementation, we conclude that it is possible to obtain system responses that are independent of the original system parameters when the controller is exactly tuned to one-half the natural frequency of the plant. This is the saturated solution. Second, the amplitude of the controlled response is never larger than the amplitude of the uncontrolled system, although the improvement is sometimes small. Third, the addition of a saturation-based controller to a self-excited system may add an abundance of interesting dynamics to the system which might question the robustness of that control strategy.

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