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FOAM DRAINAGE AND WETTING- A NUMERICAL INVESTIGATION

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ABSTRACT

Dry foam can be assumed to be essentially a network of solid tubes, called Plateau borders (PB) that can expand or contract in response to liquid content. The flat films that span from tube to tube can be assumed to contain no fluid. Foam can be wetted by wash water delivered on its top boundary, or drained by allowing the liquid to move downward in response to gravity. These processes are governed by nonlinear equations that in special cases accept exact solutions. Here we discuss numerical solutions that satisfy realistic boundary conditions.

INTRODUCTION

In many industrial processes, foams develop over large tanks that contain liquids. In some cases foams are desirable, while in others they are detrimental to the engineering process. One of the industrial processes that the present authors are studying is the flotation of minerals. Bubbles are generated in a stirring tank and mixed with ground product. Ore particles are then attached to the bubbles and float to the top, where they form foam. One of the most important applications is coal cleaning and separation. This specific coal process is efficient because coal particles are hydrophobic and thus attach to the bubbles, and the aggregates float to the top to form froth. Understanding and prediction of foam processes is necessary in order to design efficient machines.

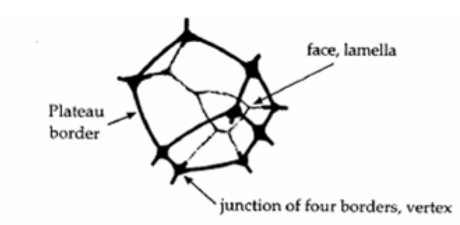

Figure 1. A single cell with its associated Plateau borders.

Rising bubbles that reach the pulp/froth interface quickly form a geometrical foam structure made up of polyhedra. Within just a few bubble diameters above the liquid foam interface, foam is practically dry. Dry foam can be defined as the state of foam where the sides of the polyhedra are flat films with very small thickness. These films are called lamellae, and the edges of the polyhedra are tubes with tricuspid cross-sections called Plateau borders (PB). A typical polyhedron cell is shown in Fig. 1. In dry foam, the liquid is contained mostly in the Plateau borders. Small amounts of surfactants are enough to arrest the fluid velocity on the walls of the lamellae and the PBs. A dry foam structure is thus equivalent to a complex network of solid pipes, except that these pipes can expand and contract depending on the wetness of the foam. Some authors (Leonard and Lemlich, 1965; Verbist *et al*. 1996) neglect the viscous losses along the junctions and arrive at simple equations that govern foam drainage. More advanced modeling has also been achieved (Neethling *et al*, 2002; Koehler *et al*, 2000) taking into account losses in the junctions. But most of these solutions have been limited to ideal situations that neglect the rising of the foam and its coarsening. Analytical solution also exist that meet idealized initial and/or boundary conditions. The aim of the present paper is to derive more general governing equations for foam draining and drying, and present numerical results for realistic situations.

ANALYTICAL MODELING

Conservation of mass of the fluid yields a simple relationship between the cross-section of the Plateau borders (PB), A, and the mean velocity in a PB, u:

$$\frac{\partial A}{\partial t}+\frac{\partial}{\partial x}(Au)=0 \quad (1)$$

where x is the distance in the vertical direction with origin at the free surface of the foam column. This is based on the assumption of uniform distribution of bubble sizes in the vertical direction, x. If the velocity is related to hydrostatic pressure gradients and laminar viscous pressure drop through the PB, one can arrive at the following nonlinear equation (Verbist *et al*.,1996):

$$\frac{\partial A}{\partial t}+\frac{1}{\eta}\frac{\partial}{\partial x}(\rho g A^2-\frac{C\gamma}{2}\sqrt{A}\frac{\partial A}{\partial x})=0 \quad (2)$$

where C is a constant determined by the shape of the PB cross section (=0.4 for the current work), η is the viscosity, g is the gravitational acceleration, γ is the surface tension and ρ is the density of the fluid. Using the following nondimensional quantities among which $\eta*$ is the effective viscosity approximately 150 times larger than η (Verbist *et al*.,1996),

$$x=\xi x_0 \qquad A=\alpha x_0^2 \qquad t=\tau t_0$$
$$x_0=\sqrt{C\gamma/\rho g} \qquad t_0=\eta^*/\sqrt{C\gamma\rho g} \quad (3)$$

one arrives at the equation :

$$\frac{\partial \alpha}{\partial \tau}+\frac{\partial}{\partial \xi}(\alpha^2-\frac{\sqrt{\alpha}}{2}\frac{\partial \alpha}{\partial \xi})=0 \quad (4)$$

This is essentially the Berger's equation. It is parabolic in nature, but if the last term is small, it accepts wave-like solutions. Its character is similar to boundary-layer equations and can be solved numerically by methods established many years ago (Telionis, 1981).

Verbist *et al*. (1996) derived an analytical solution of Eq. (4), based on the assumption that the profile has the form of a solitary wave. The wetting of foam then takes the form:

$$\alpha(\xi,\tau)=\begin{cases} v\tanh^2(\sqrt{v}[\xi-v\tau]) & \xi\le v\tau \\ 0 & \xi\ge v\tau \end{cases} \quad (5)$$

The wave front represents the fully wet foam and propagates along the distance from the free surface of the foam. But the nature of a propagating wave does not conform to a realistic initial condition. This is because initially foam could be either wet or dry, with area profile that could be nearly uniform. Here the area is proportional to the wetness ratio of foam, as described later in this paper.

EXTENSION OF THE ANALYTICAL MODEL

Equation (4) is based on the assumption that foam consists only of a network of tubes. This may be acceptable in the case of very dry foams where the sizes of junctions are negligible. But it may no longer be a good assumption if the foam contains more fluid. The fluid contained in the junctures, *i.e*. the vertices of the polyhedra and the corresponding friction losses must be considered. Whether losses through the vertices or PBs, dominates the drainage is still controversial (Leonard and Lemlich,1965; Verbist *et al*.,1996; Neethling *et al*.,2002; Koehler *et al*.,2000;2004) but Neethling *et al*.(2002) claim that both play important roles. Accounting for the losses through the vertices, a more generalized drainage equation is expressed in terms of liquid fraction ε as follows (Neethling *et al*., 2002).

$$\frac{\partial \varepsilon}{\partial t}+\nabla\left(\frac{\varepsilon\rho g}{k\eta}\right)+\nabla\left(\frac{\varepsilon\nabla(-P_{gas}+\gamma/r)}{k\eta}\right)=0 \quad (6)$$

where r is the radius of curvature of PB, P_{gas} is the gas pressure in a bubble cell, and k is the coefficient of pressure drop that includes both the PB viscous losses (C_{PB}) and vertex losses (C_V) , and the bubble radius (r_b) as:

$$k=\left(\begin{array}{l}\frac{3C_{PB}}{r^2}+\frac{4.178(C_V-0.418C_{PB})}{rr_b}(1-\varepsilon)^{1/3}\\ +\frac{6.806(C_V-1.588C_{PB})}{rr_b}(1-\varepsilon)^{2/3}\end{array}\right) \quad (7)$$

In order to recast this equation in terms of the Plateau border cross-sectional area, we noted its relationship to the liquid fraction:

$$\varepsilon=NA/S=N\alpha x_0^2/S \quad (8)$$

where N is the total number of PBs in the foam column, and S is the cross-sectional area of the column. Assuming that the temperature does not vary significantly in the froth, one obtains the following relation between the pressure and volume of bubbles (V_{b}):

$$pV_b=\text{constant}=(pV_b)_{top} \quad (9)$$

and thus we express Eq. (6) in terms of the liquid fraction:

$$\frac{\partial \varepsilon}{\partial t}+\frac{\partial}{\partial x}\left(\frac{\varepsilon\rho g}{k\mu}+\frac{\varepsilon\nabla(-(pV)_{top}/(\frac{4}{3}\pi r_b^3)+\gamma/r)}{k\mu}\right)=0 \quad (10)$$

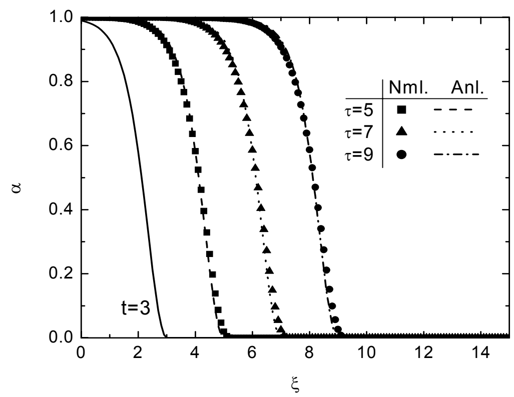

Figure 2. PB area profiles. Closed form solution (analytical) and numerical solution (symbols) of adding wash water to dry foam

It should be noted here that the pressure of gas inside a foam cell, neighboring a Plateau border is constant, and therefore does not produce a pressure gradient to drive the flow. But the pressure is dropping from cell to cell as we move up in the foam, and therefore these pressure changes will drive the flow through junctions.

Equation (10) is more general than the equation presented in Neethling *et al.*(2002)'s work because it accounts for variations in the cell size, expressed in terms of the bubble volume V_b or the bubble radius r_b. To non-dimensionalize Eq. (10), one can follow the same approach with Verbist *et al.*(1996)'s as shown in Eq.(3) and introduce more of its

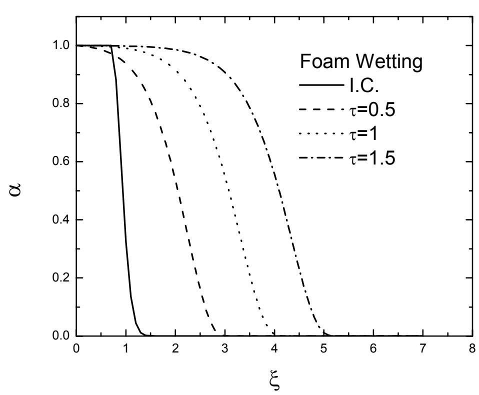

Figure 3. PB area profiles. Numerical solution of foam wetting with a uniform initial profile

characteristic parameters such as pressure $\Pi=\mu / t_0$ and surface tension $\Gamma= \mu x_0 / t_0$. Then all terms in the equation can be non-dimensionalized with following relations:

$$\begin{aligned} &A = \alpha x_0^2 \quad && k = \kappa \cdot C_V / x_0^2 \quad && x = \xi x_0, \\ &r = \delta \cdot x_0 && r_b = \beta \cdot x_0 && \\ &p = p^* \Pi && \gamma = \gamma^* \Gamma && t = \tau t_0 \end{aligned} \tag{11}$$

Also from the definitions of the liquid fraction and Bond number, *Bo*,

$$\varepsilon = \frac{4CN/\pi\gamma}{d^2 \rho g}\alpha = \frac{4CN/\pi}{Bo}\alpha \tag{12}$$

in which d is the diameter of the channel. Substituting all of those terms in Eq. (10), one finally obtains:

$$\frac{\partial \alpha}{\partial \tau} + f_1 \frac{\partial}{\partial \xi}\left(\frac{\alpha}{\kappa} + \frac{\alpha}{\kappa}\frac{\partial}{\partial \xi}\left(\frac{-C_1}{\beta^3} + \frac{1}{C\delta}\right)\right) = 0 \tag{13}$$

Where, f_1 is a constant defined as $150/C_v$. The other constant C_1 is determined by fluid properties ρ, γ and the value of pV at the top of the foam.

Eq. (10) can be simplified further if one can neglects, for very dry foams, the drainage effect by plateau border junctions (C_V), and assumes monodisperse bubbles. Hence the non-dimensional form of the drainage equation reduces to

$$\frac{\partial \alpha}{\partial \tau} + f_2 \frac{\partial}{\partial \xi}\left(\alpha^2 - \frac{\sqrt{\alpha}}{2}\frac{\partial \alpha}{\partial \xi}\right) = 0 \tag{14}$$

Except for the numerical factor , f_2, this is in the same form with the drainage models in the literature that consider only dry

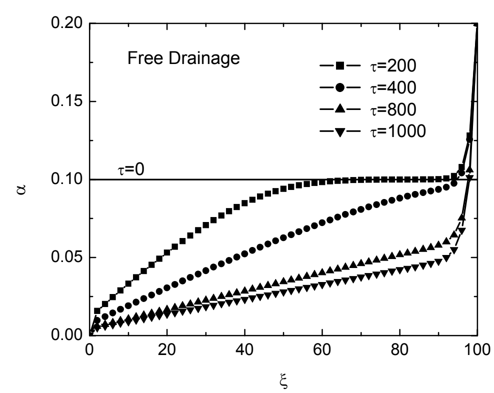

Figure 4. PB area profiles. Numerical solution of foam drainage.

foam with uniform bubble sizes.

RESULTS AND DISCUSSION

Equations (4) and (10) are parabolic in character. They can therefore be solved numerically by marching in time. The vertical dimensionless space co-ordinate, ξ, is given the value of 0.0 at the top of the foam column, and assumes positive values downward. The boundary conditions on the dimensionless PB area for the problem of wetting a dry foam from the top are α= 1.0 at the top and 0.0 away from the top. The closed-form analytical solution given by Eq. (5) is unrealistic, because it cannot satisfy a physically meaningful initial condition. It represents a wave that for all times has the same profile in space. A realistic initial condition would be a uniform very small distribution of α, representing dry foam. On the other hand, this analytic solution is valuable as a validation tool for any numerical solution.

In order to compare the numerical results with the solution given by Eq. (5), we imposed as an initial condition the analytical profile evaluated at a dimensionless time of 3. We then allowed the numerical solution to proceed in time. The comparison of the analytical solution with the corresponding numerical results at later times is shown in Fig. 2. The analytical and the numerical results are in very good agreement, having a uniformly wet foam on the top, α=1 and dry beneath the top, α=0.

A more realistic initial condition would require a uniformly dry foam that would be represented by a nearly constant small value of α. We carried out numerical calculations with such an initial condition. The results are displayed in Fig. 3. We observe that the solution quickly takes again the form of a propagating wave, consistent with the analytical solution.

We then considered the problem of draining mildly wet foam. The initial condition imposed is a uniform value of α=0.1. The solution is presented in Fig. 4. We now observe that this solution does not have the wave-like character. The area α, or equivalently the wetness ratio, as represented by Eq. (8) quickly assumes the value of zero at the top, and gradually the foam drains, as α decreases from the top down. We formulated similar numerical solutions of Eq. (12) but we limited these

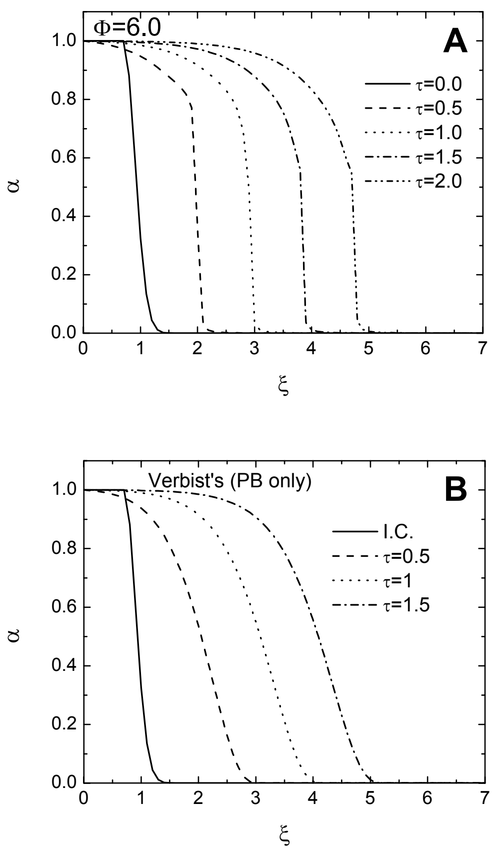

Figure 5. A) The propagation of wetness simulation from Eq. (14). C_{PB}=30, C_V=5. B) The wetness profile by Verbist *et al.* (1996)

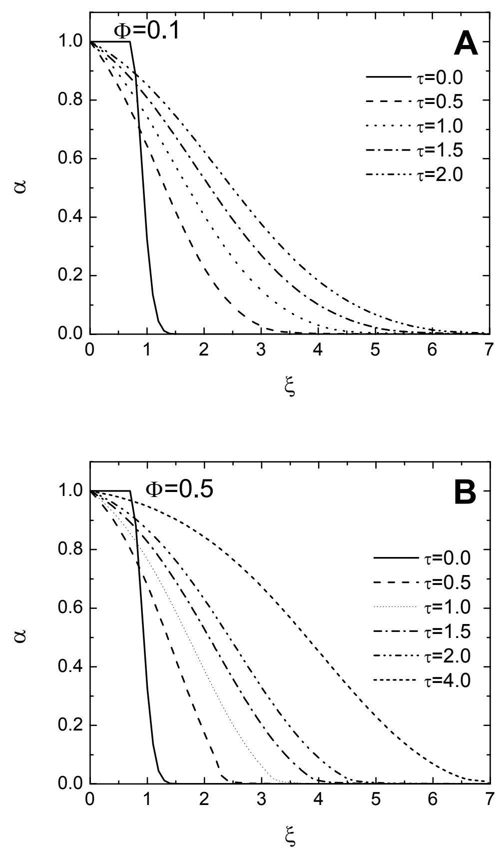

Figure 6. Propagation of the wetness with different C_V dominance with f_1=5. A) Φ=0.1 B) Φ=0.5

solutions to monodispersed foam, *i.e*. we assumed that β does not vary with space.

$$\frac{\partial \alpha}{\partial \tau}+f_1\frac{\partial}{\partial \xi}\left(\frac{\alpha}{\kappa}+\frac{\alpha}{\kappa}\frac{\partial}{\partial \xi}\left(\frac{1}{C\delta}\right)\right)=0 \qquad (15)$$

where κ is a nondimensional expression of k given by Eq. (7).

$$\kappa=\begin{pmatrix}\dfrac{3\Phi}{\delta^2}+\dfrac{4.178(1-0.418\Phi)}{\delta\beta}(1-\varepsilon)^{1/3}\\ +\dfrac{6.806(1-1.588\Phi)}{\beta^2}(1-\varepsilon)^{2/3}\end{pmatrix} \qquad (16)$$

Here, Φ is defined as the ratio of the pressure drop coefficient in the PB and in the vertex ($\Phi = C_{PB}/C_V$). One should note here that κ, as proportional to δ^{-2}, and this can become infinitely large when δ gets small. Accordingly, to avoid numerical singularities, we assume the minimum δ to be order of 10^{-1}. This may be reasonable considering the fact that the maximum mineral particle diameter that can be recovered from the flotation froth does not usually exceed the order of 10^{-4}m (Do and Yoon, 2006). The value of δ can also be obtained from the following relation between r and ε by Neethling *et al*. (2002).

$$\varepsilon \approx 0.3316\left(\frac{r}{r_b}\right)^2(1-\varepsilon)^{2/3}+0.5402\left(\frac{r}{r_b}\right)^3(1-\varepsilon) \qquad (17)$$

Although Eq. (17) can be solved numerically as well, it can be simplified further in terms of δ and ε as follows.

$$r=\delta\cdot x_0 \approx \begin{bmatrix}\dfrac{\beta}{\sqrt{0.3316}}\sqrt{\varepsilon}+\dfrac{\beta}{3\sqrt{0.3316}}\varepsilon^{3/2}\\ -\dfrac{0.5402}{2(0.3316)^2}(\varepsilon-\varepsilon^2)\end{bmatrix}\cdot x_0 \qquad (18)$$

We adopted an initial profile similar to the one employed in our earlier calculations, *i.e.* fully wet foam at the top (*ε = 0)* and dry in the remaining regions of the froth. This is shown in the following figures as a solid line.

Simulations were run for cases with several values of Φ. Because the typical values of C_{PB} or C_V available from the literature are both equal to about 20~30 (Neethling et al., 2002) and to about 50 when a rigid surface is assumed (Leonard and Lemlich, 1965), it may not be realistic to have any cases of one being exceptionally larger than the other. In fact, the authors have found that when the ratio is getting larger than 5, and with C_V as small as 5, the wetness profile looked similar to the profile by Verbist *et al*. Φ values larger than 6, gave frequent errors that stopped the calculation. Therefore, the current work

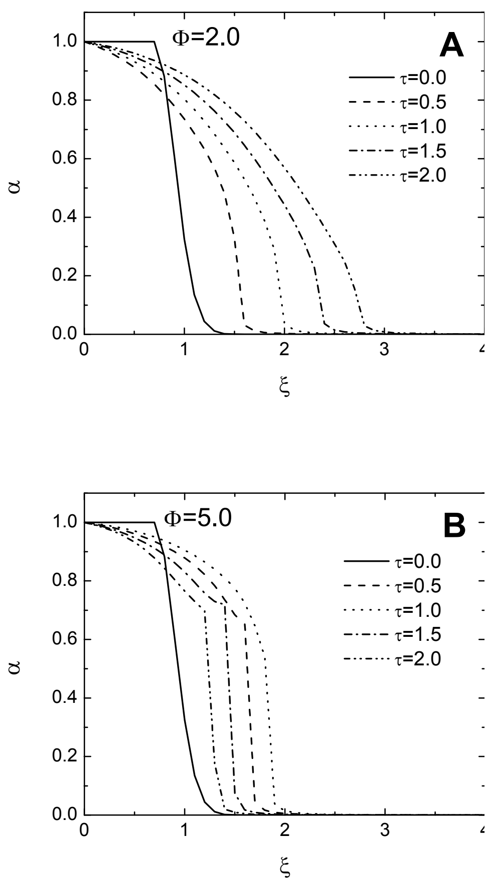

Figure 7. Propagation of wetness with different C_{PB} dominance with f_1=5. A) Φ=2.0 B) Φ=5.0

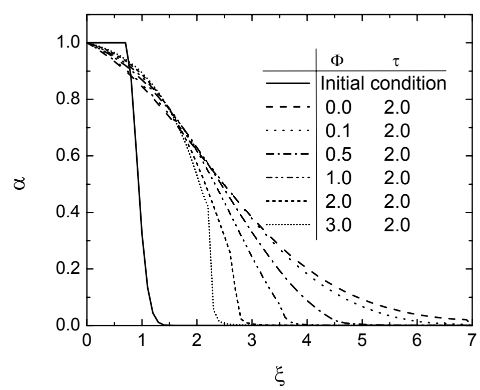

Figure 8. Wetness profile change according to C_{PB}-C_V ratio change. Given f_1=5, the results for the same instance (Φ=2) are shown.

mostly shows the results of Φ values of order 10^0 or closer. N_{PB} was assumed to be 30 when the monodisperse bubbles have a diameter of 5 mm and the foam column has a diameter of 3.4 cm. Also in general, simulation results seemed to have no dependence on grid size. For example, sample calculations for finer grid with $\Delta\xi$=0.05 and $\Delta\tau$=0.0005, did not deviate much from calculations with $\Delta\xi$=0.1 and $\Delta\tau$=0.001. Small deviations were found at the tails of the profile, but they were all less than 7%, which is insignificant.

In Fig.5 we compare the results of the current model with Verbist *et al*. (1996). Given the values of C_{PB}=30 and C_V=5, the current model in Fig. 5A propagates in a similar manner with the Verbist's model in Fig. 5B. Because the model by Verbist *et al.*, neglects the drainage effect by the PB junctions (vertices), this suggests that for given f_1 one can find any values as a criterion to ignore the PB junction effects or not.

Figure.6 shows the case when the drainage is dominated more by the vertices. Given C_V=30, the results also show wave-like profiles but more tilted back. In both frames of Figs. 6, the wetness in the relatively dryer region (lower foam) proceeds more like in Fig. 5B, while relatively wetter region (upper foam) marches are retarded. The slowing down of the drainage in the lower foam with increasing Φ indicates that the contribution from C_V is more significant than C_{PB} in the relatively dryer foam region.

In like manner, Figs. 7A and 7B show the wetness profile with different values of Φs when the effect by C_{PB} is larger than C_V. With higher value of Φ as shown in Fig. 7B, the wetness profile marches slower than the one with smaller Φ value in the lower foam region which is relatively dryer. This indicates that the role of vertices in drainage may be more effective than the role of PBs when it comes to the drainage through already dry foams. This supports what was observed in the simulation results of Fig. 5.

Finally, Fig. 8 shows the wetness profile for varying Φ values for the same instances. As Φ increases, the profile resembles more its initial wetness profile. The same was discussed in comparing the current model with the Verbist *et al.*(1996)'s model in Figs. 2. The junction where all profiles with different Φs meet reads approximately ξ=1.7. Then the drainage at ξ>1.7 is more likely to dependent on C_V and vice versa. This figure, however, helps us determine in what position the distinction between 'PB drainage' and 'Vertex drainage' should be made at a certain instance τ.

CONCLUSIONS

Analytical models are discussed and modified accordingly to simulate the wetting and drainage of nearly dry foam. The contribution to the process of viscous effects through foam vertices and Plateau borders are considered. The results indicate that wetting of foam has a wave-like character, which is emerging quickly, even if the initial condition is incompatible with the wave profile. On the other hand, draining follows a different path, with a slope of the wetness profile decreasing with time.

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