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# FUNDAMENTAL ENERGY TRANSFER MECHANISM CAUSED IN ROTATING FLOW PASSAGE OF TURBOMACHINERY

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## ABSTRACT

Fluid particles rotational motion is constructed from two different kinds: One is the fluid particles appearance rotational motion around the center together with impeller blades and the other is the substantial rotational motion due to infinitely small but infinitely large number fluid particles portions rotational vortex motion around themselves at infinitely small rotational radiuses. It seems that formation of vertex motion, that is, production of pump head at off design flow rate is related to remaining time of fluid particles in the rotating flow passage of impeller blades.

## 1. INTRODUCTION

If energy is transferred from mechanical to hydraulic energy in the rotating flow passage of turbomachinery, it is supplied to fluid particle directly as its own hydraulic energy, and results to produce flow rate and pump head. This newly supplied or transferred hydraulic energy is constructed from two different kinds: mass weight flow rate $\rho gQ$ and pump head H [1]. Transferred energy rate at each flow rate Q is shown in H-Q curve. Therefore, appropriate description of pump head H is one of the most fundamental and important subjects among engineers and scientists to be performed in the study of turbomachinery.

Stepanoff, A. J. [2] considered that mechanical energy is supplied to fluid particles contained between two adjacent impeller blades at time t=0, therefore, energy change is caused on those fluid particles.

While Tanaka, T. [3] considered that mechanical energy is supplied to whole system. Then, hydraulic energy change has to be caused not only on the fluid particles contained between two adjacent impeller blades at time $t=0$, but also has to be caused on the fluid particles entering the impeller blades from the leading edge of impeller inlet in time interval $dt$.

They applied the conservation law on angular moment of momentum to those masses of liquids in the rotating flow passage of impeller blades and obtained theoretical equations represent the theoretical pump head. However, both of their results do not explain any interrelation between pump head and flow rate at all. In these overall viewpoints, theoretical pump head is investigated.

In this paper, conservation low of centrifugal force is introduced theoretically and it is applied to the fluid flow in the rotating flow passage of impeller blades and fundamental consideration is made on energy transfer mechanism. Energy transfer mechanism is overviewed theoretically and the interrelation between the pump head and the flow rate is discussed for various changes in flow rate in the practical operation of turbomachinery. Those results are reported in this paper.

## 2. INTRODUCTION OF CONSERVATION LAW ON CENTRIFUGAL FORCE

Let us consider fluid particles rotational motion in two dimensional flow fields in the rotating flow passage of centrifugal pump at zero flow rate. Fluid particle rotates together with impeller blades at constant angular velocity $\omega_o$. Fluid particle stated here therefore causes a forced vortex motion. Blade thickness is assumed zero. If we consider two arbitrary circular lines A and B around the rotational axis O and if they have the relation $r_{OA} < r_{OB}$, circular line B locates outside of circular line A. Then centrifugal force $F_{OB}$ at outside circular line B is given by

$$F_{OB} = \rho(\frac{U_{OB}^2}{r_{OB}}) = \rho\omega_O^2 \cdot r_{OB} \qquad (1)$$

Centrifugal force $F_{OA}$ at rotational radius $r_{OA}$ is given by

$$F_{OA} = \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho\omega_O^2 \cdot r_{OA} \qquad (2)$$

Then, difference of centrifugal forces $F_{OB}$ and $F_{OA}$ is given by equations (1) and (2) as follow,

$$F_{OB} - F_{OA} = \rho(\frac{U_{OB}^2}{r_{OB}}) - \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho\omega_O^2 \cdot (r_{OB} - r_{OA}) \qquad (3)$$

Rotational radiuses $r_{OA}$ and $r_{OB}$ have the relation

$r_{AB}$

$$r_{OB} - r_{OA} = r_{AB} \qquad (4)$$

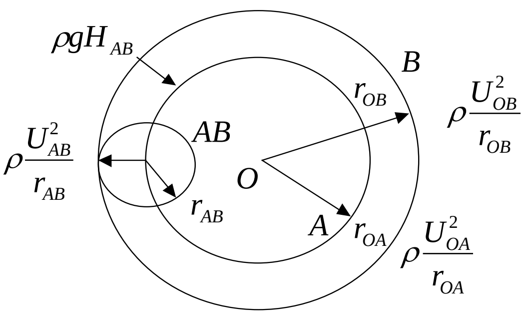


Fig.1 Illustration to show the conservation low on centrifugal force caused at rotating flow passage around center O at constant angular velocity $\omega_o$.

Then, equation (3) can be rewritten as follow:

$$F_{OB} - F_{OA} = \rho(\frac{U_{OB}^2}{r_{OB}}) - \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho\omega_O^2 \cdot r_{AB} \qquad (5)$$

In other words,

$$F_{OB} - F_{OA} = \rho(\frac{U_{OB}^2}{r_{OB}}) - \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho(\frac{U_{AB}^2}{r_{AB}}) \qquad (6)$$

Where,

$$U_{AB} = \omega_O r_{AB} \qquad (7)$$

Here, circular lines A and B could be recognized as the locus of real fluid particles rotational motion at the rotating flow passage of impeller blades. However, in case of circular line AB, fluid particles practical rotational motion along the circular line AB is seemed not possible to be really caused in the rotating flow passage of turbomachinery because circular line AB crosses substantial stream lines formed between A and B in the rotating flow passage of impeller blades. Therefore, it is not possible to recognize as the locus of real fluid particles rotational motion. This imaginary circular line AB touches internally with the outer circular line B and locates its center on the circular line A.

Nevertheless, from physical organization and make up of equation (7), although it is very hard to accept or realize from the viewpoint of physical movement it would be rather slightly possible to realize that physical parameter $U_{AB}$ corresponds to peripheral velocity of fluid particles along the imaginary circular line AB and imaginary circular line AB corresponds to the locus of fluid particles imaginary rotational motion at rotational radius $r_{AB}$ at angular velocity $\omega_o$.

Here, the things to be remained in mind are that this kind fluid particles physical movement may not be really caused in the practical rotational motion in the flow passage of turbomachinery. However, as shown later on, this imaginary strange physical technical term actually plays fairly important part when we consider energy transfer mechanism caused in the rotating flow passage of turbomachinery. In this situation and viewpoints, its physical meanings and characteristics are very important and significant.

Therefore, at this moment, let us try to remain these physical parameters and their relationships just as they are until their physical meanings and properties become clear later on at the end of this discussion.

Rotational radius $r_{AB}$ of imaginary circular line AB changes arbitrary in the region between center O and outside rotational radius $r_{OB}$ along the radial line. It is due to radius $r_{OA}$ of inside circular line A regardless whether the rotational radius $r_{OB}$ of outside circular line B is fixed at constant or not if we consider unit outside rotational radius $r_{OB}=1$. If the rotational radius $r_{OA}$ becomes large radial outward, rotational radius $r_{AB}$ becomes small. Imaginary circular line AB forms various kinds circular lines for the change in rotational radius $r_{OA}$ between center O and outside rotational radius $r_{OB}$. However, all rotational motions caused along those circular lines are caused under constant angular velocity $\omega_o$ equivalent to those of circular lines A and B at center O. This is very clear from equation (7).

Now, equation (6) can be represented as follow,

$$F_{OB} = F_{OA} + F_{AB} \tag{8}$$

Or

$$\rho(\frac{U_{OB}^2}{r_{OB}}) = \rho(\frac{U_{OA}^2}{r_{OA}}) + \rho(\frac{U_{AB}^2}{r_{AB}}) \tag{9}$$

In other words,

$$\rho\omega_O^2 \cdot r_{OB} = \rho\omega_O^2 \cdot [r_{OA} + r_{AB}] \tag{10}$$

Where

$$F_{AB} = \rho \cdot (\frac{U_{AB}^2}{r_{AB}}) = \rho\omega_O^2 \cdot r_{AB} \tag{11}$$

These state that the summation of magnitudes of centrifugal forces due to that ($F_{OA}$) of real rotational motion at rotational radius $r_{OA}$ along the real circular line A and that ($F_{AB}$) of imaginary rotational motion at rotational radius $r_{AB}$ along the imaginary circular line AB is always equivalent to that ($F_{OB}$) due to rotational motion at rotational radius $r_{OB}$ along the circular line B. While their angular velocities $\omega_o$ are equivalent among them. Rotational radius $r_{OA}$ is arbitrary along the radial line between center O and outside radius $r_{OB}$ in the rotating flow passage. And above interrelation is always satisfied regardless whether outside radius $r_{OB}$ is fixed at constant or not if we consider unit radius $r_{OB}=1$. Then, these equations could be called or recognized as *the Conservation Law of* Centrifugal *force due to rotational motion at constant angular velocity* $\omega_o$ [4].

In other viewpoints, magnitude of centrifugal force due to rotational motion along the outside circular line B is constructed from those of two different kinds rotational motions. One is that of centrifugal force due to rotational motion along the inside circular line A whose rotational axis locates at center O, which is equivalent to that of outside circular line B. And the other is that of imaginary centrifugal force due to imaginary rotational motion along the imaginary circular line AB.

In other expression, radius $r_A$ of inside circular line A and radius $r_{AB}$ of imaginary circular line AB construct radius $r_B$ of outside circular line B, and the fluid particles angular velocities $\omega_o$, that is, the rotational speeds along the circular lines A, AB, and B are remained the same.

## 3. IF CIRCULAR LINE Y LOCATES BETWEEN CIRCULAR LINES A AND B

Let us consider a circular line Y in addition to circular lines A and B. If their rotational radiuses have the relation $r_{OA}<r_{OY}<r_{OB}$, circular line Y locates between circular lines A and B. Then, centrifugal force $F_{OB}$ at the outside circular line B can be written as follow,

$$F_{OB} = F_{OA} + F_{AY} + F_{YB} \tag{12}$$

Or

$$\rho(\frac{U_{OB}^2}{r_{OB}}) = \rho(\frac{U_{OA}^2}{r_{OA}} + \frac{U_{AY}^2}{r_{AY}} + \frac{U_{YB}^2}{r_{YB}}) \tag{13}$$

In other words,

$$\rho\omega_O^2 \cdot r_{OB} = \rho\omega_O^2 \cdot (r_{OA} + r_{AY} + r_{YB}) \tag{14}$$

In the same manner, let us consider infinitely large number circular lines C, D, ... X, and Y between circular lines A and B. Then fluid particles geometrical rotational radiuses, $r_{AC}$, $r_{CD}$, ..., $r_{XY}$, and $r_{YB}$, which correspond to rotational radius $r_{AB}$ of imaginary circular line AB in Fig.1, become infinitely small. This indicates that infinitely small but infinitely large number fluid particles are rotating at infinitely small rotational radiuses $r_{AC}$, $r_{CD}$, ..., $r_{XY}$, and $r_{YB}$ along the infinitely small circular lines AC, CD, ..., XY, and YB at angular velocity $\omega_o$ in the region between $r_{OA}$ and $r_{OB}$ along the radial line.

Furthermore, it is very clear from geometrical formation that the summation of magnitudes of those imaginary centrifugal forces due to infinitely small but infinitely large number fluid particles rotational motions between radiuses $r_{OA}$ and $r_{OB}$ along the radial line O-B and the magnitude of imaginary centrifugal force $F_{AB}$ due to imaginary rotational motion along the imaginary circular line AB are equivalent.

This indicates that their magnitude do not change by the number of circular lines between them. That is,

$$F_{AB} = F_{AC} + F_{CD} + \cdots + F_{XY} + F_{YB} \tag{15}$$

Or

$$\rho(\frac{U_{AB}^2}{r_{AB}}) = \rho(\frac{U_{AC}^2}{r_{AC}} + \frac{U_{CD}^2}{r_{CD}} + \cdots + \frac{U_{XY}^2}{r_{XY}} + \frac{U_{YB}^2}{r_{YB}}) \tag{16}$$

In other words,

$$\rho\omega_O^2 \cdot r_{AB} = \rho\omega_O^2 \cdot (r_{AC} + r_{CD} + \cdots r_{XY} + r_{YB}) \tag{17}$$

These indicate that if outside rotational radius $r_{OB}$ is fixed at constant or if we consider the unit radius $r_{OB}$=1, total magnitude of centrifugal force $F_{OB}$ at outside rotational radius $r_{OB}$ does not changes at all by the number of circular lines inside of it.

## 4. PHYSICAL PROPERTY OF CIRCULAR LINE AB

If we consider infinitely large number circular lines C, D, ... X, and Y between circular lines A and B, fluid particles geometrical rotational radiuses $r_{AC}$, $r_{CD}$, ..., $r_{XY}$, and $r_{YB}$, which corresponds to imaginary rotational radius $r_{AB}$ along the circular line AB in Fig.1, become infinitely small. This indicates that infinitely small but infinitely large number fluid particles may distribute in the rotating flow passage between circular lines A and B along the radial line O-B and they might really cause imaginary rotational motions at infinitely small rotational radiuses $r_{AC}$, $r_{CD}$, ..., $r_{XY}$, and $r_{YB}$ equivalent to that caused at rotational radius $r_{AB}$ along the imaginary circular line AB in Fig.1.

Whilst, the total magnitude of imaginary centrifugal forces $F_{AB}$ as the summation of imaginary centrifugal forces $F_{AC}$, $F_{CD}$, ..., $F_{XY}$, $F_{YB}$ due to infinitely small but infinitely large number fluid particles imaginary rotational motions at infinitely small rotational radiuses $r_{AC}$, $r_{CD}$, ..., $r_{XY}$, $r_{YB}$ along the imaginary circular lines AC, CD, ..., XY, YB at angular velocities $\omega_o$ and the magnitude of imaginary centrifugal force $F_{AB}$ due to imaginary circular line AB at rotational radius $r_{AB}$ at angular velocity $\omega_o$ are equivalent in the rotating flow passage between circular lines A and B along the radial line O-B.

In other words infinitely small but infinitely large number fluid particles cause imaginary rotational motion at infinitely small rotational radiuses at angular velocities $\omega_o$ in the rotating flow passage between $r_{OA}$ and $r_{OB}$ and result to cause centrifugal force equivalent to the magnitude of imaginary centrifugal force $F_{AB}$ due to imaginary rotational motion at rotational radius $r_{AB}$ along the imaginary circular line AB at angular velocity $\omega_o$.

This indicates that the magnitude of centrifugal forces due to infinitely small but infinitely large number fluid particles rotational motions corresponds to the magnitude of imaginary centrifugal force due to imaginary rotational motion at rotational radius $r_{AB}$ along the imaginary circular line AB at angular velocity $\omega_o$.

While the magnitude of imaginary centrifugal force of imaginary rotational motion at rotational radius $r_{AB}$ along the imaginary circular line AB at angular velocity $\omega_o$ could be recognized as substantial in the practical operation of centrifugal pump. However physical meanings of imaginary rotational motion along the imaginary circular line AB are not clear yet. These might be clear from equations (15), (16) and (17).

In other viewpoints, it could be said that infinitely small but infinitely large number fluid particles rotational motions are really caused at angular velocity $\omega_o$ by the impelling action of impeller blades. And they are distributed between $r_{OA}$ and $r_{OB}$ along the radial line in the rotating flow passage, and the magnitude of imaginary centrifugal force $F_{AB}$ produced is really substantial in the practical operation of turbomachinery.

This indicates that fluid particles rotational motion around the axis of rotation caused in the rotating flow passage of turbomachinery is constructed from two different kinds: the rotational motion around the axis of rotation together with impeller blades and the rotational motion of infinitely small but infinitely large number fluid particles portions rotational vortex motions around themselves at infinitely small rotational radiuses. This theoretical discovery on the latter fluid particles rotational motion is very important to consider the energy transfer mechanism caused in the rotating flow passage of turbomachinery. Because most of our discussion has been made based upon the former fluid particles rotational motion up-to-date.

Anyhow, these indicate that although fluid particles rotational motion along the circular line AB was estimated in the previous discussion that it may not be possible to be really caused in the rotating flow passage of turbomachinery from the physical viewpoints and they are discussed as an imaginary physical parameter in the following discussion. However now it could be really recognized that these estimations and treatments were not adequate.

To tell the truth, substantial magnitude of imaginary centrifugal force $F_{AB}$ equivalent to that explained by imaginary circular line AB is really caused in the form of infinitely small but infinitely large number fluid particles rotational motions in the rotating flow passage. In other words, the summation of magnitudes of imaginary centrifugal forces caused by the infinitely small but infinitely large number fluid particles vortex motions around themselves are really caused in the region between $r_{OA}$ and $r_{OB}$ along the radial line, and their magnitudes are equivalent to that of imaginary centrifugal force $F_{AB}$ constructed geometrically at rotational radius $r_{AB}$ along the imaginary circular line AB.

From these viewpoints it could be said that fluid particles, which are forced to rotate together with impeller blades around the axis of rotation in the rotating flow passage, may not only rotate together with impeller blades around the axis of rotation at center O, but also cause additional fluid particles own rotational vortex motion around themselves at infinitely small rotational radiuses.

In addition, all those rotational motions are caused at equivalent angular velocities to those of rotational motions along the circular lines A and B at center O in the rotating flow passage of turbomachinery. In other words, the angular velocities of infinitely small but infinitely large number fluid particles vortex motion around themselves are caused at an equivalent magnitude of angular velocities to that of impeller blades angular velocity at the axis of rotation.

## 5. PHYSICAL MEANING OF CIRCULAR LINE AB

Now it is very clear that the magnitude of imaginary centrifugal force $F_{AB}$ caused by the fluid particles rotational motion along imaginary circular line AB at rotational radius $r_{AB}$ is equivalent to the summation of magnitudes of centrifugal forces caused by the infinitely small but infinitely large number fluid particles portions rotational vortex motions around themselves at infinitely small rotational radiuses at angular velocity $\omega_o$ along the radial line in the rotating flow passage.

Therefore, geometrical rotational radius $r_{AB}$ to cause imaginary centrifugal force along the circular line AB due to imaginary rotational motion is to tell the truth equivalent to summation of those of imaginary circular lines formed by infinitely small but infinitely large number fluid particles in the imaginary rotational motion. Geometrical rotational radius $r_{AB}$ is the radial blade length between the impeller outlet and inlet in the practical operation of the turbomachinery. This indicates that radial blade length is strongly interrelated with the radial distribution of the infinitely small but infinitely large number fluid particles portions own rotational vortex motions around themselves at infinitely small rotational radiuses. In other words radial blade length is strongly related to substantial pump head produced in the rotating flow passage of turbomachinery.

In these viewpoints it could be recognized that imaginary circular line AB might be the geometrical hypothetical description or the locus line which is appeared accidentally or following the fundamental energy transfer mechanism in visible way to show total amount of fluid particles substantial rotational vortex motion at angular velocity $\omega_o$.

If the rotational center of imaginary circular line AB on the circular line A changes its location radial inward along the radius line from the location of rotational radius $r_{OB}$ at outside circular line B toward the rotational center O, imaginary centrifugal force $F_{AB}$ changes its magnitude. Magnitude of imaginary centrifugal force $F_{AB}$ becomes large if the location of rotational center moves radial inward from the surface of outside circular line B toward the center O. If it locates on the surface of circular line B, both the rotational radius $r_{AB}$ and the magnitude of imaginary centrifugal force $F_{AB}$ become the minimum (zero). If it becomes large and reaches the center O, both of them become the maximum, which are equivalent to those of rotational radius $r_{OB}$ and centrifugal force $F_{OB}$.

## 6. OVERVIEW OF THEORETICAL RESULTS

It is very clear from previous discussions that in case of fluid particles rotational motion, two different kinds rotational motions are caused in the rotating flow passage of impeller blades. One is the rotational motion around the axis of rotation and the other is the infinitely small but infinitely large number fluid particles portions rotational vortex motions around themselves at infinitely small rotational radiuses. From these it could be said that fluid particles may not only cause rotational motion around the axis of rotation together with impeller blades, but also causes fluid particles own rotational vortex motion around themselves in the rotating flow passage of impeller blades.

This theoretical result could be expanded to the idea that if a fluid particle turns its flow direction at a curved flow passage, it may cause a centrifugal force, then fluid particle may not only cause its flow directions change along the curved flow passage, but also causes a rotational vortex motion around itself. In other words, wherever if the fluid particle causes a centrifugal force due to rotational motion around the axis of rotation or the flow directions change due to the curved flow passage, fluid particle may cause two different kinds fluid particles movements: rotational motion around the axis of rotation or the flow directions change and the infinitely small but infinitely large number fluid particles portions rotational vortex motions around themselves at infinitely small rotational radiuses.

These indicate that pressure head raise due to fluid particles rotational motion around the axis of rotation and that due to flow directions change at curved flow passage may not be determined only by the fluid particles rotational motion around the axis of rotation or the geometrical flow directions change along the curved flow passage, but also by the fluid particles portions own rotational vortex motion around themselves.

From these viewpoints, it could be said that the fluid particles rotational motion around the axis of rotation and the flow directions change along the curved flow passage might be constructed from two different kinds rotational motions: One is the apparent rotational motion around the axis of rotation. And the other is the substantial rotational motion due to the infinitely small but infinitely large number fluid particles portions own rotational vortex motion around themselves at infinitely small rotational radiuses. These two different kinds fluid particles movements might be caused wherever fluid particle causes rotational motion around the axis of rotation or the flow directions change along the curved flow passage. Then fluid particles rotational motion around the axis of rotation at angular velocity $\omega_o$ and the flow directions change along the curved flow passage could be explained by those two different kinds fluid particles rotational motions. Especially, the latter fluid particles rotational motion due to infinitely small but infinitely large number fluid particles portions rotational vortex motion around themselves at infinitely small rotational radiuses is recognized as the substantial rotational motion. Then it could be recognized as the pure source of production of pump head in the rotating flow passage of turbomachinery. And as the same, infinitely small but infinitely large number fluid particles portions rotational vortex motions around themselves at a curved flow passage might be the pure source to cause the pressure head rise in the curved flow passage.

This result of discussion indicates that the fluid particles rotational motion around the axis of rotation and the flow directions change along the curved flow passage cannot be discussed without considering the physical condition on infinitely small but infinitely large number fluid particles portions rotational vortex motions around themselves. Hence, as a matter of fact, it could be said that energy transfer mechanism caused in the rotating flow passage of turbomachinery cannot be discussed in detail without the application of vortex theory to the infinitely small but infinitely large number fluid particles portions own rotational vortex motions around themselves at infinitely small rotational radiuses in the rotating flow passage of turbomachinery.

## 7. INTERRELATION BETWEEN PUMP HEAD AND MASS WEIGHT FLOW RATE

From above discussion, it is clear that magnitude of centrifugal force $F_{OB}$ corresponds to pump head H. Imaginary circular line AB, which produces the imaginary centrifugal force $F_{AB}$, is formed at rotational radius $r_{AB}$ and its rotational center locates on the circular line A. This indicates that magnitude of imaginary centrifugal force $F_{AB}$ is the largest at outside circular line B of the radial line because its circular line touches internally with the outside circular line B and the minimum (zero) at the inside circular line A because its rotational center of circular line AB locates on the surface of inside circular line A. In other words, substantial pump head H is the largest at the surface of outside circular line B at the trailing edge of impeller outlet and the minimum (zero) at the inside circular line A at the leading edge of impeller inlet.

Transferred hydraulic energy $E_{HT}$ is constructed from two physical categories: mass weight flow rate $\rho gQ$ and pump head H [1]. Substantial pump head H is the maximum at the outside rotational radius $r_{OB}$ and zero at the inside rotational radius $r_{OA}$. This means that flow rate Q is the maximum at the inside rotational radius $r_{OA}$. Then rectangular velocity triangle is formed at the leading edge of impeller inlet. This indicates that radius $r_{AB}$ between the outside and inside radiuses, $r_{OB}$ and $r_{OA}$, of the radial line corresponds to pump head H and radius $r_{OA}$ measured from the rotational center O corresponds to flow rate Q, and their summation $r_{OB}$ corresponds to transferred total hydraulic energy $E_{HT}$. This indicates that energy transfer is explained along the radial line.

This indicates that if inlet radius $r_{OA}$ is selected at a large value in the practical design of centrifugal pump, it indicates that leading edge of impeller inlet is designed to locate far away from the center O. Then flow rate Q becomes large but pump head H becomes small. If inlet radius $r_{OA}$ is selected at a small value, it indicates that leading edge of impeller inlet is designed to locate near the center O. Then flow rate Q becomes small, but pump head H becomes large.

From these viewpoints, fundamental interrelation between pump head H and flow rate Q could be explained as follow: If the inlet radius $r_{OA}$ at the leading edge of impeller inlet locates at the rotational center O, inlet radius $r_{OA}$ becomes zero. Then flow rate Q might become zero and the pump head H obtained at the discharge radius $r_{OB}$ at the trailing edge of impeller outlet might become the largest. If the inlet radius $r_{OA}$ at the leading edge of impeller inlet moves away from the rotational center O, inlet radius $r_{OA}$ becomes large. Then flow rate Q increases but pump head H obtained at the discharge radius $r_{OB}$ decreases. Therefore, if the leading edge of impeller inlet reaches to the location to the discharge radius at the trailing edge of impeller outlet, flow rate Q due to theoretical consideration might become the largest and the pump head H obtained at the discharge radius $r_{OB}$ at the trailing edge of impeller outlet might become the minimum (zero). This might be clear if we consider the forced vortex motion.

 

In this case, if the inlet radius point $r_{OA}$ at the leading edge of impeller inlet moves away from the rotational center O and the inlet radius $r_{OA}$ at the leading edge of impeller inlet becomes large, substantial rotational radius $r_{AB}$ becomes small. This indicates that effective blade length between the impeller inlet and outlet becomes small. In other words, substantial fluid particles rotational motion caused by the impeller blades becomes small. Then substantial fluid particles rotational motion due to infinitely small but infinitely large number fluid particles portions own rotational vortex motions around themselves at infinitely small rotational radiuses, caused by the impeller blades rotational motion at rotational radius $r_{AB}$ along the circular line AB, becomes small. Then resultant substantial pump head H produced by impeller blades in the rotating flow passage may become small.

If the impeller blade is filled with the impeller blades between the outside circular line B and rotational center O, impeller blades operating condition changes from that at the outside circular line B to that at rotational center O. It changes from that at the minimum (zero) pump head at the maximum flow rate to that at the maximum pump head at the minimum (zero) flow rate with a decrease in flow rate. However, in the practically designed impeller blades, as the impeller blade is luck between the inside circular line A and rotational center O, the operating condition starts at the location of inside circular line A.

Pump head is the minimum (zero) at the location of inside circular line A. This indicates that flow rate Q produced is the maximum because supplied or transferred hydraulic energy is constructed from two different kinds hydraulic energies: mass weight flow rate $\rho gQ$ and pump head H. While pump head produced by the impeller blade at this operating condition does not become zero. If we look at the rotating flow passage between the impeller inlet and outlet, pressure head increases from leading edge of impeller inlet toward radial outward and reaches to the maximum value at the trailing edge of impeller outlet. This indicates that although the inlet pump head at the leading edge of impeller inlet becomes zero, the pump head does not become zero at the largest flow rate. Pump head becomes large if we choose radial blade length long while its fundamental maximum flow rate becomes small. Usually we consider that if the flow rate becomes the maximum, pump head becomes the minimum (zero). However, in the practical operation of centrifugal pump, the pump head does not become zero even at the maximum flow rate. This result of discussion is very meaningful and important.

If the impeller blade is filled with the impeller blades between the outside circular line B and rotational center O, operating condition moves from outside circular line B to rotational center O through inside circular line A along the radial line with a decrease in flow rate. Therefore, we can image the energy transfer mechanism as follow: At the maximum flow rate its operating condition locates point B in Fig.2. With a decrease in flow rate, operating condition moves from point B toward rotational center O through the points A and E along the radial line. That is energy transfer caused at the minimum (zero) flow rate is caused in the rotating flow passage of area BB'O in Fig.2. Which expresses the maximum pump head caused in the rotating flow passage of impeller blades.

However, in the practical operation of centrifugal pump, operating condition moves from ABC to EBD for the decreased flow rate, and OBB' at the zero flow rate. From these interrelations it seems that formation of infinitely small but infinitely large number fluid particles portions rotational vortex motions around themselves, that is, production of pump head is related to remaining time of fluid particles in the rotating flow passage with the decrease in flow rate.

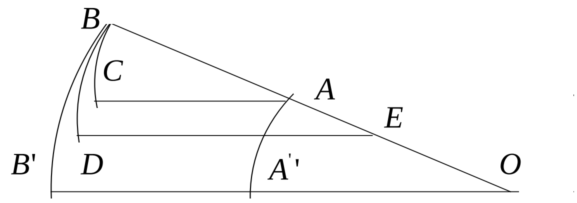


Fig.2 Illustration to show the operating condition.

## 8. APPLICATION TO FLUID FLOW IN CENTRIFUGAL PUMP

In case of fluid particles rotational motion, fluid particle does not always start its rotational motion at center O, but starts at an arbitrary radius points larger than zero. In other words, fluid particle starts its rotational motion at some distances away from the location of rotational center O. This kind rotational motion is seen in the rotating flow passage formed by impeller blades in hydraulic machines, such as pumps, blowers, fans, compressors, water turbines, gas turbines, and so on.

Let us try to consider practical application of conservation low of centrifugal force introduced in this discussion to the fluid flow in the rotating flow passage between impeller blades in centrifugal pumps next. See Fig.2. In this case, rotational radius $r_{OB}$ corresponds to the trailing edge of impeller outlet. Inside rotational radius $r_{OA}$ corresponds to leading edge of impeller inlet.

If the centrifugal pump is set at the depth $H_{WX}$ measured vertical downward from the water surface W-W, the pressure head $F_{WX}$ at the location of impeller discharge is given by

$$F_{WX} = \rho g H_{WX} \tag{18}$$

And this pressure force $F_{WX}$ of rested water at vertical depth $H_{WX}$ due to gravitational acceleration g that directs vertical downward is balanced with the imaginary centrifugal force $F_{AB}$ of the fluids due to impeller blades rotational motion that directs radial outward given by equation (11). While acting directions are opposed. That is,

$$F_{WX} = -F_{AB} \tag{19}$$

Then, theoretical pump head $H_{AB}$ is given by

$$H_{AB} = -H_{WX} = -\frac{1}{g} \cdot \omega_O^2 \cdot r_{AB} \tag{20}$$

Rotational radius $r_{AB}$ of imaginary circular line AB varies between $r_{OA}$ and $r_{OB}$, which corresponds to the distance between the leading edge of impeller inlet and the trailing edge of impeller outlet of the impeller blade. Then, equation (20) indicates that pump head $H_{AB}$, that is, the magnitude of imaginary centrifugal force $F_{AB}$ is the minimum (zero) at radius $r_{OA}$ at the leading edge of impeller inlet and the maximum at outside rotational radius $r_{OB}$ at the trailing edge of impeller outlet.

Fig.1 Illustration to show the example of application of conservation low of centrifugal force caused in the flow field around the center O.

Here attention has to be paid to the facts that the fluid particle that locates on the inside circular line A rotates at peripheral velocity $U_{OA}$ at rotational radius $r_{OA}$ and that locates on the outside circular line B rotates at peripheral velocity $U_{OB}$ at rotational radius $r_{OB}$. However, substantial practical magnitude of fluid particles centrifugal force $F_{OA}$ is not the value given by equation (5) but zero and that of centrifugal force $F_{OB}$ is not the value given by equation (1) but that given by equation (11). This indicates that substantial practical imaginary centrifugal force $F_{AB}$ indicated by imaginary circular line AB is not supplied by the fluid particles rotational motion around the axis of rotation together with impeller blades but supplied by the fluid particles rotational vortex motion around itself. These indicate that there are essentially two different kinds fluid particles rotational motions: appearance and substantial rotational motions. Therefore, these two different kinds rotational motions have to be considered in the rotating flow passage.

For example, magnitude of imaginary centrifugal force $F_{AB}$ obtained as a result of theoretical discussion is explained as the difference between that of centrifugal force $F_{OB}$ and that of centrifugal force $F_{OA}$ by applying the conservation law on centrifugal force.

Resultant imaginary centrifugal force $F_{AB}$ corresponds to theoretical head $H_{Th}$. Magnitude of centrifugal force $F_{OB}$ corresponds to the discharge pump head $H_2$ obtained at the trailing edge of impeller outlet. Magnitude of centrifugal force $F_{OA}$ corresponds to the suction pump head $H_1$ at the leading edge of impeller inlet.

From these it could be said that if conservation law on centrifugal force is applied, theoretical pump head $H_{Th}$ is explained as the difference between discharge and suction heads, $H_2$ and $H_1$.

In other words, substantial pump head $H_{Pr}$ is obtained by $H_2 - H_1$ in the practical operation of turbomachinery. This also indicates that the discharge pump head $H_2$ is not the substantial pressure head but the appearance pressure head at the trailing edge of impeller outlet and the suction pump head $H_1$ is not the substantial pressure head but the appearance pressure head at the leading edge of impeller inlet. These indicate that their difference might be the substantial pump head.

## 9. CONCLUSIONS

Fluid particles rotational motion is constructed from two different kinds: One is the fluid particles appearance rotational motion around the center together with impeller blades and the other is the substantial rotational motion due to infinitely small but infinitely large number fluid particles portions rotational vortex motion around themselves at infinitely small rotational radiuses. It seems that formation of vertex motion, that is, production of pump head is related to remaining time of fluid particles in the rotating flow passage of impeller blades.

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