

IMPROVED SCHEME FOR PREDICTING FAN TONE NOISE

Masanobu Namba

Department of Aerospace Systems Engineering
Sojo University
Kumamoto, 860-0082, Japan
Email: namba@arsp.sojo-u.ac.jp

Ryohei Nishino

Shino Ohgi
Department of Aerospace Systems Engineering
Sojo University
Kumamoto, 860-0082, Japan

ABSTRACT

This paper proposes an improved scheme for prediction of fan tone noise due to interaction of stator vanes with oncoming rotor wakes. The original scheme called the hybrid method computes the unsteady pressures on the stator vanes interacting with the rotor wakes by using a CFD technique, and then calculates the sound field generated from the stator vanes on the basis of the unsteady linearized lifting surface theory. This original scheme, however, neglects the aeroacoustic interaction between the rotor and the stator, which brings about additional unsteady pressures on the stator vanes and also on the rotor blades. The present paper modifies the formulation of the lifting surface theory. The aeroacoustic components of the unsteady surface pressures on the stator vanes and the rotor blades are expressed in such forms that they are induced by the unsteady stator surface pressures due to interaction with wakes, which can separately be predicted, e.g., by a CFD technique. Numerical examples indicate the significant effect of the aeroacoustic rotor-stator interaction on the generated sound powers.

INTRODUCTION

Interaction of stator vanes with oncoming rotor wakes is the primary source of the fan tone noise of turbofan engines. In earlier models the upstream rotor is treated only as a wake generator and therefore interaction of the rotor blades with sound waves generated from the stator vanes was neglected. Later Topol [1] and Hanson [2] studied the problem including the effect of mutual aeroacoustic interaction between

the rotor and the stator, and it was shown that the scattering of acoustic modes and frequencies due to the interaction is of essential importance. But their models compute the scattering effects on the basis of 2-dimensional cascade theory. On the other hand a noise prediction scheme on the basis of genuine 3-dimensional lifting surface theory for a contra-rotating annular cascades was developed by the top author [3].

Recently Tsuchiya, et al. [4] presented a hybrid method of prediction of fan tone noise. Their scheme uses CFD NS solver to compute the unsteady loading on stator vanes due to interaction with oncoming viscous wakes from the rotor blades, and calculates the acoustic field generated from the stator vanes on the basis of unsteady lifting surface theory. The concept is clever, because CAA (computational aeroacoustics) requires computational schemes of higher sophistication and higher cost than CFD (computational fluid dynamics), and the interaction of stator vanes with vortical disturbances is essentially of hydrodynamic phenomenon, which can be evaluated accurately in the scope of CFD. Unfortunately, however, their scheme neglects the components of the unsteady loadings on the stator vanes and the rotor blades resulting from aeroacoustic interaction between the rotor and the stator.

This paper proposes an improved hybrid method. The original complete analytical method [3] is modified as follows. The unsteady loading on the stator vanes is resolved into two components: the primary component due to interaction with vortical disturbances of wakes from rotor blades and the secondary components due to acoustic interaction between the

rotor and the stator. On the other hand the unsteady loading on the rotor blades is composed of the secondary component alone. The calculation process consists of three steps.

1. Calculation of the primary component of the unsteady loading on the stator vanes.
2. Calculation of the secondary components of the unsteady loadings on the stator vanes and the rotor blades.
3. Calculation of the acoustic field generated from the stator vanes and the rotor blades by treating the unsteady loadings on the stator vanes and the rotor blades as the sound sources.

The second and third steps are conducted on the basis of the lifting surface theory. On the other hand the first step can be conducted either on the basis of a lifting surface theory or on the basis of more accurate CFD. The latter case is called a hybrid method. The original hybrid method skips the second step and neglects the secondary components of the unsteady loadings. Inclusion of the second step is the essence of the improvement proposed here.

Some numerical examples are presented in this paper. Comparison in the acoustic powers between the original hybrid method and the improved one indicates that the secondary components of the unsteady loadings significantly modify the fan noise level.

MATHEMATICAL FORMULATIONS

Model Description

Consider a stage of blade rows composed of a rotor and a stator in an annular duct of infinite axial extent and of an outer radius r_T^* and an inner radius hr_T^* as shown in Fig. 1. The oncoming undisturbed flow is of a uniform axial velocity W_a^* , a uniform density ρ_0^* and a uniform axial Mach number M_a^* . Hereafter unstarred symbols denote dimensionless values. Lengths, velocities, pressures, densities, and times are normalized by r_T^* , W_a^* , $\rho_0^*W_a^{*2}$, ρ_0^* , and r_T^*/W_a^* respectively. Let the numbers of rotor blades and the stator vanes be N_R and N_S respectively, and the rotational angular velocity of the rotor be $\Omega (= \Omega^* r_T^*/W_a^*)$.

Let the cylindrical coordinate system fixed to the rotor and that fixed to the stator (i.e., the duct) be denoted by (r, θ_R, z_R) and (r, θ, z) , respectively. Then it holds that

$$\theta = \theta_R - \Omega t, \quad (1)$$

$$z = z_R - G, \quad (2)$$

where G denotes the distance between the rotor and stator centers. Further we define a helical coordinate η_R by

$$\eta_R = \theta_R - \Omega z_R. \quad (3)$$

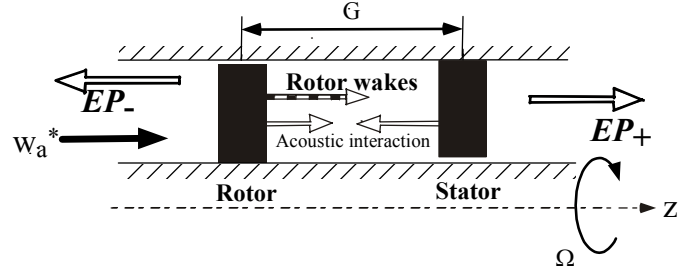


Figure 1. Model for predicting the coupled fan noise.

Here $\eta_R = \text{constant}$ stands for an undisturbed streamline in the frame of reference fixed to the rotor. All disturbances are assumed to be small so that the governing equations can be linearized.

Formulation of Lifting Surface Theory

The vortical disturbances generated from the rotor blades as viscous wakes are steady with respect to (r, θ_R, z_R) . The disturbance velocity can be expressed in a Fourier series form:

$$q_w = \sum_{\substack{\mu=-\infty \\ \neq 0}}^{\infty} W^{(\mu)}(r, z_R) e^{i\mu N_R \eta_R}. \quad (4)$$

Rewriting Eq. (4) in terms of the duct-fixed coordinates (r, θ, z) , one obtains

$$q_w = \sum_{\substack{\mu=-\infty \\ \neq 0}}^{\infty} W^{(\mu)}(r, z+G) e^{i\omega_{S,\mu} t + i\sigma_{S,\mu}(\theta - \Omega z - \Omega G)}, \quad (5)$$

where

$$\omega_{S,\mu} = \mu N_R \Omega, \quad \sigma_{S,\mu} = \mu N_R. \quad (6)$$

This implies that the vortical disturbances are sensed by the stator vanes as multiple sinusoidal gust of reduced frequencies $\mu N_R \Omega$ and circumferential wave numbers μN_R . Interaction of the vortical disturbances with the stator vanes produces unsteady loadings on the stator vanes, which generate acoustic waves. Then there occur acoustic interactions between the rotor and the stator. The unsteady loadings on the m -th stator vane and the m -th rotor blade resulting from the interactions can be expressed by [5]

$$\sum_{\substack{\mu=-\infty \\ \neq 0}}^{\infty} \Delta p_{S,\mu}(r, z) e^{i\omega_{S,\mu} t + i2\pi\sigma_{S,\mu} m / N_S} : m = 0, 1, \dots, N_S - 1, \quad (7)$$

$$\sum_{\substack{\mu=-\infty \\ \neq 0}}^{\infty} \Delta p_{R,\mu}(r, z) e^{i\omega_{R,\mu} t + i2\pi\sigma_{R,\mu} m / N_R} : m = 0, 1, \dots, N_R - 1, \quad (8)$$

where

$$\omega_{R,\mu} = -\mu N_R \Omega, \quad \sigma_{R,\mu} = \mu N_S. \quad (9)$$

The fundamental concept of the lifting surface theory is to regard the blades and the vanes as pressure dipole surfaces with strengths equal to the unsteady loadings and to express the disturbance flow field as induced by those pressure dipoles. In the present case, the disturbance velocity \mathbf{q} is expressed in the form

$$\begin{aligned} \mathbf{q} = & \sum_{\substack{\nu=-\infty \\ \neq 0}}^{\infty} e^{i\omega_{R,\nu} t} \int_0^1 \int_{-C_{aR}/2}^{C_{aR}/2} \Delta p_{R,\nu}(\rho, \zeta) \\ & \times \mathbf{K}_q(r, \eta_R, z_R - \zeta | \rho; N_R, \Omega, \omega_{R,\nu}, \sigma_{R,\nu}) d\zeta d\rho \\ & + \sum_{\substack{\mu=-\infty \\ \neq 0}}^{\infty} e^{i\omega_{S,\mu} t} \int_0^1 \int_{-C_{aS}/2}^{C_{aS}/2} \Delta p_{S,\mu}(\rho, \zeta) \\ & \times \mathbf{K}_q(r, \theta, z - \zeta | \rho; N_S, 0, \omega_{S,\mu}, \sigma_{S,\mu}) d\zeta d\rho, \end{aligned} \quad (10)$$

where C_{aR} and C_{aS} denote the axial chord lengths of the rotor blades and the stator vanes, respectively, and they are assumed to be constant along the span. It is further assumed that the rotor blades and the stator vanes are of no lean and no sweep.

The kernel function $\mathbf{K}_q(r, \eta, z - \zeta | \rho; N, \Omega, \omega, \sigma)$ denotes the disturbance velocity induced by an annular row of pressure dipoles, which are placed at $r = \rho$, $z = \zeta$, $\eta = \theta - \Omega z = 2\pi m / N$; $m = 0, 1, \dots, N-1$ and rotating at a dimensionless angular speed Ω with respect to the duct-fixed coordinated system. The axes of the dipoles are normal to the helical surfaces of $\eta = 2\pi m / N$; $m = 0, 1, \dots, N-1$. The strength of the dipoles is fluctuating with unit amplitude at a reduced frequency ω and a phase difference $2\pi\sigma / N$ between one dipole and the next. The mathematical expression of the kernel function is exactly same as that given for a single cascade model [6] [7]. This is further decomposed into circumferential modes in the form

$$\begin{aligned} \mathbf{K}_q(r, \eta, z - \zeta | \rho; N, \Omega, \omega, \sigma) \\ = \sum_{\nu=-\infty}^{\infty} e^{i(\nu N + \sigma)\eta} \mathbf{K}_q^{(\nu)}(r, z - \zeta | \rho; N, \Omega, \omega, \sigma). \end{aligned} \quad (11)$$

Equation (10) describes the disturbance velocity field in terms of the unsteady loadings as the disturbance sources. We should note, however, that in the present problem the blade loadings are not prescribed but should be determined so that the disturbance velocity given by Eq. (10) satisfies the flow tangency condition on the rotor blade surfaces and the stator vane surfaces.

The flow tangency condition at the surfaces of the rotor blades and the stator vanes can be expressed by

$$[q_{\perp R}]_{\eta_R=0} = 0 : -\frac{C_{aR}}{2} \leq z_R \leq \frac{C_{aR}}{2}, \quad (12)$$

$$[q_{\perp S}]_{\theta=0} = -q_{w\perp S} : -\frac{C_{aS}}{2} \leq z_R \leq \frac{C_{aS}}{2}, \quad (13)$$

where $q_{\perp R}$ and $q_{\perp S}$ denote the components of the disturbance velocity \mathbf{q} normal to the rotor blade surfaces and the stator vane surfaces, respectively. Further

$$\begin{aligned} q_{w\perp S} &= \sum_{\substack{\mu=-\infty \\ \neq 0}}^{\infty} \tilde{W}_{\perp S}^{(\mu)}(r, z + G) e^{i\omega_{S,\mu} t - i\sigma_{S,\mu} \Omega(z+G)} \\ &= \sum_{\substack{\mu=-\infty \\ \neq 0}}^{\infty} \tilde{W}_{\perp S}^{(\mu)}(r, z) e^{i\omega_{S,\mu} t}, \end{aligned} \quad (14)$$

denotes the component of the vortical disturbance velocity \mathbf{q}_w normal to the stator vane surfaces.

Substitution of the integral representations of the velocities into Eqs. (12) and (13) gives a set of simultaneous integral equations for the unsteady loading functions. The expression of the integral equations is rather lengthy and is omitted here to save space. Equivalent expressions are found in Ref. [5].

Instead, we write the system integral equations in a compact matrix form as follows:

$$\begin{bmatrix} \mathbf{K}_{RR} & \mathbf{K}_{SR} \\ \mathbf{K}_{RS} & \mathbf{K}_{SS} \end{bmatrix} * \begin{bmatrix} \mathbf{P}_R \\ \mathbf{P}_S \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\tilde{\mathbf{W}} \end{bmatrix}, \quad (15)$$

where

$$\begin{aligned} \mathbf{P}_R &= [\dots, \Delta P_{R,-2}, \Delta P_{R,-1}, \Delta P_{R,+1}, \Delta P_{R,+2}, \dots]^T, \\ \mathbf{P}_S &= [\dots, \Delta P_{S,-2}, \Delta P_{S,-1}, \Delta P_{S,+1}, \Delta P_{S,+2}, \dots]^T, \\ \tilde{\mathbf{W}} &= [\dots, \tilde{W}_{\perp S}^{(-2)}, \tilde{W}_{\perp S}^{(-1)}, \tilde{W}_{\perp S}^{(+1)}, \tilde{W}_{\perp S}^{(+2)}, \dots]^T. \end{aligned} \quad (16)$$

Further \mathbf{K}_{RR} , \mathbf{K}_{SR} , \mathbf{K}_{RS} , and \mathbf{K}_{SS} denote submatrices involving kernel functions, and the symbol $*$ denotes the convolution product of the kernel function and the loading function.

This is the complete formulation of the lifting surface theory. Various methods are available to solve the integral equations numerically.

Formulation of Hybrid Method

To obtain the formulation of the hybrid method, let \mathbf{P}_S be resolved into two parts:

$$\mathbf{P}_S = \mathbf{P}_S^{(w)} + \mathbf{P}_S^{(a)}, \quad (17)$$

where $\mathbf{P}_S^{(w)}$ denotes the loading due to direct interaction with wakes only, and $\mathbf{P}_S^{(a)}$ denotes the additional loading due to aeroacoustic interaction between the rotor and the stator. Then Eq. (15) can be decomposed into two parts;

$$\mathbf{K}_{SS} * \mathbf{P}_S^{(w)} = -\tilde{\mathbf{W}}, \quad (18)$$

$$\begin{bmatrix} \mathbf{K}_{RR} & \mathbf{K}_{SR} \\ \mathbf{K}_{RS} & \mathbf{K}_{SS} \end{bmatrix} * \begin{bmatrix} \mathbf{P}_R \\ \mathbf{P}_S^{(a)} \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_{SR} * \mathbf{P}_S^{(w)} \\ \mathbf{0} \end{bmatrix}. \quad (19)$$

Here let $\mathbf{P}_S^{(w)}$ be called the primary component, whereas let $\mathbf{P}_S^{(a)}$ and \mathbf{P}_R be called the secondary components.

The interaction of the stator vanes with the oncoming rotor wakes is essentially a hydrodynamic phenomenon, and to determine the primary component $\mathbf{P}_S^{(w)}$ by solving Eq. (18) can be replaced by more accurate methods based on CFD. The hybrid method developed by Tsuchiya et al. [4] uses a CFD NS solver to compute the primary component $\mathbf{P}_S^{(w)}$, and calculate the acoustic field generated by the primary component alone by using the lifting surface theory.

This paper proposes an improved hybrid method, which determines the primary component $\mathbf{P}_S^{(w)}$ by using CFD and then solves Eq. (19) to determine the secondary components $\mathbf{P}_S^{(a)}$ and \mathbf{P}_R . It should be noted that if one uses CFD or CAA also to determine the secondary components, a quite sophisticated numerical scheme and quite a long computation time will be required. On the other hand to obtain a numerical solution of Eq. (19) on a modern PC will require only a few seconds.

Acoustic Field and Acoustic Power

Once the loading functions are determined, one can calculate the acoustic field quantities by using the loadings on blades and vanes as noise sources. In particular, the acoustic pressure field at upstream (subscript -) and downstream (subscript +) stations can be expressed by

$$p_{\pm} = \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} e^{i\nu N_R \Omega t} \sum_{\ell=0}^{\infty} \exp \left\{ i\nu N_R \Omega \frac{M_a^2}{1-M_a^2} z \mp \Lambda_{\ell}^{(n)} z \right\} \times R_{\ell}^{(n)}(r) FP_{\pm}(n, \ell), \quad (20)$$

where

$$n = \nu N_R + \mu N_S, \quad (21)$$

denotes the circumferential wave number, $R_{\ell}^{(n)}(r)$ denotes the radial eigenfunction of the circumferential wave number n and the radial order ℓ , and

$$\Lambda_{\ell}^{(n)} = \begin{cases} \sqrt{B}: & B > 0: \text{cut-off mode} \\ i \operatorname{sgn}(\nu N_R \Omega) \sqrt{-B}: & B < 0: \text{cut-on mode} \end{cases}, \quad (22)$$

$$B = \frac{1}{1-M_a^2} \left\{ \left(k_{\ell}^{(n)} \right)^2 - (\nu N_R \Omega)^2 \frac{M_a^2}{1-M_a^2} \right\}. \quad (23)$$

Here $k_{\ell}^{(n)}$ denotes the radial eigenvalue. Further $FP_{\pm}(n, \ell)$ denotes the modal pressure amplitude of circumferential wave number n and the radial order ℓ , which is composed of two parts:

$$FP_{\pm}(n, \ell) = FP_{R\pm}(n, \ell) + \exp \left\{ - \left(i\nu N_R \Omega \frac{M_a^2}{1-M_a^2} \mp \Lambda_{\ell}^{(n)} \right) G \right\} FP_{S\pm}(n, \ell). \quad (24)$$

Here $FP_{R\pm}(n, \ell)$ and $FP_{S\pm}(n, \ell)$ involve integration of unsteady loading functions $\Delta p_{R,\mu}(\rho, \zeta)$ and $\Delta p_{S,\nu}(\rho, \zeta) = \Delta p_{S,\nu}^{(w)}(\rho, \zeta) + \Delta p_{S,\nu}^{(a)}(\rho, \zeta)$ respectively, but the details are omitted to save space.

Finally, the total axial acoustic power and the modal acoustic power normalized by $\rho_0^* W_a^{*3} r_T^{*2}$ is given by

$$EP_{\pm} = \sum_{\nu} \sum_{\mu} \sum_{\ell} E_{\pm}(n, \ell), \quad (25)$$

$$E_{\pm}(n, \ell) = \frac{\pi(1-M_a^2) |\nu N_R \Omega| |\Lambda_{\ell}^{(n)}|}{\left\{ |\nu N_R \Omega| / (1-M_a^2) \mp |\Lambda_{\ell}^{(n)}| \right\}^2} |FP_{\pm}(n, \ell)|^2. \quad (26)$$

Note that the mode parameter ν denotes the component of the frequency $\nu N_R \Omega$ in the frame of reference fixed to the duct. One can define the frequency component of the acoustic power by

$$EP_{\pm}(\nu) = \sum_{\mu} \sum_{\ell} E_{\pm}(\nu N_R + \mu N_S, \ell). \quad (27)$$

NUMERICAL EXAMPLES

In this paper the primary component $\Delta p_{S,\nu}^{(w)}(\rho, \zeta)$ is determined by solving Eq. (18) instead of solving NS equations by CFD. A wake model is defined by applying the diffusion formula by Schlichting [8] to the present rotor model. The mathematical expression is omitted to save space.

The numerical results shown in this paper are obtained by specifying the parameters as follows: $h = 0.5$, $N_R = 24$, $N_S = 32$, $C_{aR} = 0.1409$, $C_{aS} = 0.2618$, and $C_D = 0.02$. Here C_D is the profile drag coefficient of the rotor blades. The viscous wake model requires to specify a numerical value of C_D .

Figures 2 and 3 show frequency components of the upstream axial acoustic power dependent on the rotor-stator distance G for the axial Mach numbers of 0.5 and 0.4

respectively. Here $C_a = (C_{aR} + C_{aS})/2$. Note that the sound frequency is given by $\nu N_R \Omega$. Therefore $\nu = 1$ corresponds to the component of the blade passing frequency $N_R \Omega$, and $\nu = 2$ and $\nu = 3$ denote the second and the third harmonics respectively. Open symbols with dotted lines indicate the calculation without the secondary components $P_S^{(a)}$ and P_R , i.e., the calculation by acoustically decoupling the rotor and the stator.

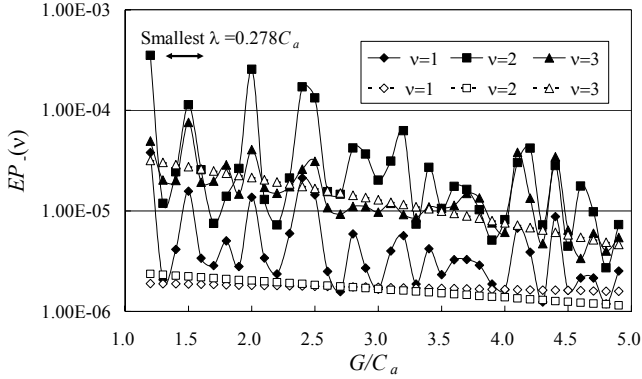


Figure 2. Upstream acoustic powers $EP_-(\nu)$ of the blade passing frequency BPF ($\nu = 1$) and higher harmonics ($\nu = 2, \nu = 3$). G denotes the distance between the blade rows. Dotted lines with open symbols indicate computation neglecting the acoustic coupling between the rotor and the stator. $M_a = 0.5$.

In the decoupled case the acoustic powers monotonically decrease with increasing distance G between the blade rows. This is just because of decrease of the wake velocity depression with streamwise distance from the rotor blades. On the other hand, the acoustic coupling between the rotor and the stator gives rise to wavy variation of the acoustic power with the distance G . The acoustic coupling occurs via the acoustic waves of cut-on modes except the case of very small distance G , where the cut-off modes are not completely diminished. Then this wavy variation is considered to be related to the axial wave lengths of the cut-on modes, which are given by

$$\lambda^\pm = 2\pi / \left(\nu N_R \Omega \frac{M_a^2}{1 - M_a^2} \mp \text{sgn}(\nu \Omega) |\Lambda_\ell^{(n)}| \right). \quad (28)$$

In Figs. 2 and 3, the shortest axial wave length, which is given by the mode of ($\nu = 3, n = 8, \ell = 0$), is indicated. One can observe the periodical distance of the wavy variation approximately corresponds to the axial wave length.

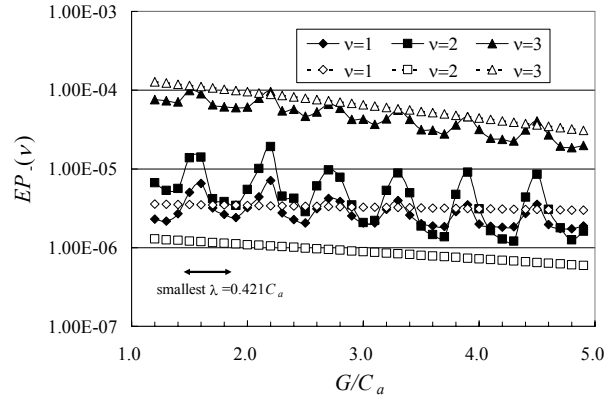


Figure 3. Same as Fig. 2. $M_a = 0.4$.

Furthermore one should note that the effect of the acoustic interaction, which can be measured by difference between the coupled case (solid symbols) and the decoupled case (open symbols), is remarkably large in particular in the case of higher Mach number (Fig. 2), where a lot of duct modes are cut-on. The effect is also dependent on the harmonic number of the frequency. The component of the second harmonic frequency ($\nu = 2$) is most influenced by the acoustic coupling between the rotor and the stator as far as the examples shown here are concerned. The acoustic coupling significantly enhances the acoustic power of $\nu = 2$.

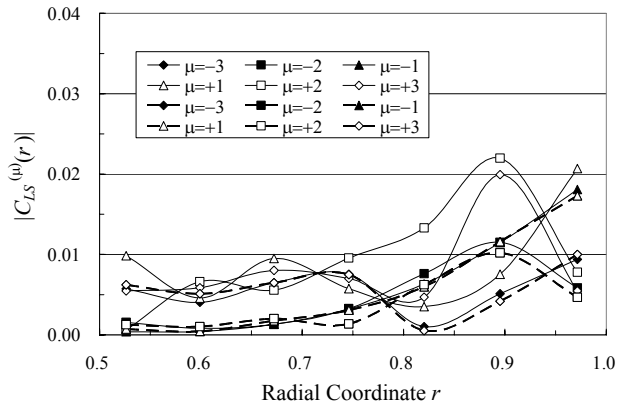


Figure 4. Absolute values of local lift coefficients on stator vanes $|C_{LS}^{(\mu)}(r)|$ for various frequency components $\nu = \pm 1, \pm 2, \pm 3$. Broken lines indicate computation neglecting the acoustic coupling between the rotor and the stator. $M_a = 0.5$, $G = 2.4 C_a$.

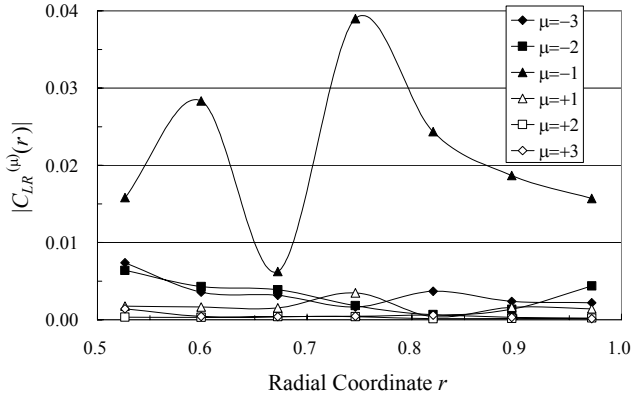


Figure 5. Absolute values of local lift coefficients on rotor blades $|C_{LR}^{(\mu)}(r)|$ for various frequency components $\nu = \pm 1, \pm 2, \pm 3$. $M_a = 0.5$, $G = 2.4C_a$

To investigate how the unsteady loading on the rotor blades and the stator vanes are influenced by the aeroacoustic interaction between the rotor and the stator, the radial (spanwise) distributions of the absolute values of the local lift coefficients defined by

$$C_{LS}^{(\mu)}(r) = \frac{1}{C_{aS}} \int_{-C_{aS}/2}^{C_{aS}/2} \Delta p_{S,\mu}(r, z) dz, \quad (29)$$

$$C_{LR}^{(\mu)}(r) = \frac{1}{C_{aR}} \int_{-C_{aR}/2}^{C_{aR}/2} \Delta p_{R,\mu}(r, z) dz, \quad (30)$$

for various harmonic numbers μ of the frequency are shown in Figs. 4 and 5. In Fig. 4, broken lines indicate the unsteady loading for the decoupled condition, i.e., the unsteady loading due to the interaction with wakes only. We can observe that the components of the unsteady loadings on the stator vanes due to aeroacoustic coupling are of the same order of magnitude as those of the decoupled condition. On the other hand the unsteady loadings on the rotor blades (Fig. 5), which solely result from the acoustic rotor-stator coupling, are generally small except the fundamental frequency component $\mu = -1$, which is remarkably high.

CONCLUSIONS

An improvement of the hybrid scheme of predicting the fan tone noise is proposed and the calculation process is presented.

The effect of the acoustic interaction between the rotor and the stator brings about significant change of the acoustic power. The effect periodically varies with the distance between the rotor and the stator since the interaction is brought about by the acoustic waves of the cut-on modes. The effect also depends on the harmonic number of the blade passing frequency.

REFERENCES

- [1] Topol, D.A., 1997. "Development and Evaluation of a Coupled Fan Noise Design Systems," *AIAA Paper*, AIAA-97-1611.
- [2] Hanson, D.B., 1997. "Acoustic Reflection and Transmission of Rotors and Stators Including Mode and Frequency Scattering," *AIAA Paper*, AIAA-97-1610-CP, pp.199-210.
- [3] Namba, M., 2001. "What is the Role of Analytical Methods in Modern Unsteady Aerodynamics of Turbomachines?," *The 9th International Symposium on Unsteady Aerodynamics, Aeroacoustics and Aeroelasticity of Turbomachines*, Pascal Ferrand and Stephane Aubert, eds., Presses Universitaires de Grenoble, pp.12-23.
- [4] Tsuchiya, N., Nakamura, Y., Yamagata, A., Kodama, H., Nozaki, O., Nishizawa, T. and Yamamoto, K., 2002. "Fan Noise Prediction Using CFD Analysis," *AIAA Paper*, AIAA-2002-2491.
- [5] Namba, M. and Nishino, R., 2006. "Unsteady Aerodynamic Response of Oscillating Contra-Rotating Annular Cascades, Part I: Description of Model and Mathematical Formulations," *Transactions of Japan Society for Aeronautical and Space Sciences*, **Vol. 49**, No. 165, to be published.
- [6] Namba, M. and Ishikawa, A., 1983. "Three-Dimensional Aerodynamic Characteristics of Oscillating Supersonic and Transonic Annular Cascades," *ASME J. of Engineering for Power*, **Vol. 105**, pp. 138-146.
- [7] Namba, M., 1987. "Three-Dimensional Flows," *AGARD Manual on Aeroelasticity in Axial Flow Turbomachines, Vol. 1, Unsteady Turbomachinery Aerodynamics*, **AGARD-AG-298**, M.F. Platzer and F.O. Carta, eds., Neuilly sur Seine, France.
- [8] Schlichting, H., 1966. *Boundary Layer Theory*, McGraw-Hill, Chap. 24.