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Numerical simulations of forced convective heat transfer of Nanofluids in circular tubes with uniform heat flux

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ABSTRACT

Forced convective of a nanofluid consists of water and Al_2O_3 in horizontal circular tubes has been studied numerically. Computed results are validated with existing well established correlation. Two phase eulerian model has been implemented for the first time to study such a flow field. A single phase model and two phase mixture model formulations are also used for comparison. The comparison of calculated results with experimental values shows that the mixture model is more precise than the other two models. It is illustrated that single phase model and two phase eulerian model underestimates the Nusselt number more than the mixture model. Effects of nanoparticles concentration on the thermal parameters are also presented and discussed.

KEYWORDS:

Nanofluids, Forced convective heat transfer, Laminar flow, Turbulent flow, Numerical Simulation

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INTRODUCTION

Fluids are essential for heat transfer in many engineering equipments. Although various techniques are applied to enhance the heat transfer, the low heat transfer performance of these conventional fluids obstructs the performance enhancement and the compactness of heat exchangers. The use of solid particles as an additive suspended into the base fluid is a technique for the heat transfer enhancement. The enhancement of thermal conductivity of conventional fluids by the suspension of solid particles, such as millimeter- or micrometer-sized particles, has been well known for more than 100 years. However, they have not been of interest for practical applications due to problems such as sedimentation, erosion, fouling and increased pressure drop of the flow channel. The recent advance in materials technology has made it possible to produce nanometer-sizes particles that can overcome these problems. Innovative heat transfer fluids suspended by nanometer-sized solid particles are called 'nanofluids'. These suspended nanoparticles can change the transport and thermal properties of the base fluid.

Xuan and Li (2003) studied the single-phase flow and heat transfer performance of nanofluids under turbulent flow in tubes. Their experimental results showed that the convective heat transfer coefficient and the Nusselt number of nanofluids increase with the Reynolds number and the volume fraction of nanoparticles under turbulent flow. Compared with water, the Nusselt number of the nanofluids with a 2.0 vol% of Cu nanoparticles is more increased than 39%. Wen and Ding (2004) focused on the entry region under laminar flow condition using nanofluids containing γ-Al_2O_3 nanoparticles of various concentrations. It is shown that the enhancement increases with the Reynolds number as well as the volume concentration of nanoparticle. Heris et al. (2006 and 2007) investigated the convective heat transfer coefficient of Al_2O_3–water and CuO–water nanofluids for laminar flow in annular

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tube under a constant wall temperature boundary condition. The thermal and physical properties of the nanofluids were calculated using the following formulas: the Einstein equation for viscosity, the Xuan and Roetzel (2000) equation for specific heat, and Yu and Choi (2003) correlation for thermal conductivity and Hamilton and Crosser (1962) model. The results showed that the heat transfer coefficient increased with an increasing Peclet number and increasing volume fraction while Al_2O_3–water nanofluid showed larger enhancement than CuO–water nanofluids.

In addition to the experimental works some numerical studies have been done to investigate the effect of nanofluids in thermal enhancement of fluids. (Roy et al., 2004, Behzadmehr et al, 2007Namburu et al., 2008, Oztop et al., 2008).

In the present study forced convective of a nanofluid consists of water and Al_2O_3 in a horizontal circular tube has been studied numerically. Three different approaches for simulating nanofluids, which are a single phase model and two phase mixture model have been implemented. Two phase eulerian model has been implemented for the first time to study such a flow field. The comparison of calculated results with experimental values shows that the mixture model is more precise than the other two models.

2. Mathematical modeling

2.1. Physical properties of the nanofuids

By assuming the nanoparticles are well dispersed within the base-fluid, the effective physical properties of the mixtures studied can be evaluated using some classical formulas as well known for two-pahse fluids.

$$\rho_{eff} = (1-\phi)\rho_f + \phi\rho_p \quad (1)$$

$$(C_p)_{eff} = (1-\phi)(C_p)_f + \phi(C_p)_p \quad (2)$$

$$\frac{\lambda_{eff}}{\lambda_f} = 4.97\phi^2 + 2.72\phi + 1 \quad (3)$$

$$\frac{\mu_{eff}}{\mu_f} = 123\phi^2 + 7.3\phi + 1 \quad (4)$$

Eqs. (1) and (2) are general relationships used to compute the density and specific heat for a classical two-phase mixture. Eq. (3) for calculating the thermal conductivity of the nanofluid has been obtained by Hamilton and Crosser (1962) by the use of spherical particles assumption. Although this model was first developed based on millimeter and micrometer size particles, it is believed its use with nanofluids be acceptable. So despite its probable underestimation with nanofluids, because of its simplicity, this model has been adapted for this study. It should be noted that for the case of 1% volume concentration of Al_2O_3 in water, the experimental result of Wen and Ding (2004) is used for calculating the thermal conductivity of nanofluid to have a unique input for comparison. The dynamic viscosity of nanofluids has been calculated through Eq. (4), which is obtained, by Maiga et al. (2005) performing a least-square curve fitting of some experimental data available for the mixtures considered.

As it will be shown in next sections, in this paper it is concluded that the mixture approach in comparison with single phase and eulerian approaches, shows more reliable results for numerical simulation of nanofluids. So in the next section the mathematical formulation of the mixture model is discussed.

2.2. Mixture model

The mixture model, based on a single fluid two phase approach, is employed in the simulation by assuming that the coupling between phases is strong, and particles closely follow the flow. The two phases are assumed to be interpenetrating, meaning that each phase has its own velocity vector field, and within any control volume there is a volume fraction of primary phase and also a volume fraction of the secondary phase. Instead of utilizing the governing equations of each phase separately, the continuity, momentum and energy equations for the mixture are employed. A nanofluid composed of water and Al_2O_3 nanoparticles flowing in a long tube with uniform heating at the wall boundary is considered. Therefore, the dimensional equations for steady state mean conditions are

Continuity

$$\nabla.(\rho_m V_m) = 0 \quad (5)$$

Momentum

$$\nabla.(\rho_m V_m V_m) = -\nabla p_m + \nabla.[\tau - \tau_t] + \rho_m g + \nabla.\left(\sum_{k=1}^{n} \phi_k \rho_k V_{dr,k} V_{dr,k}\right) \quad (6)$$

Energy

$$\nabla.(\phi_k V_k(\rho_k h_k + p)) = \nabla.(\lambda_{eff}\nabla T - C_p \rho_m \overline{vt}) \quad (7)$$

while considering Φ is the volume fraction of phase k, we have:

$$\rho_m = \sum_{k=1}^{n} \phi_k \rho_k \quad (8)$$

$$\mu_m = \sum_{k=1}^{n} \phi_k \mu_k \quad (9)$$

In Eq. (6), $V_{dr,k}$ the drift velocity for secondary phase k, i.e. the nanoparticles, is defined as:

$$V_{dr,k} = V_k - V_m \quad (10)$$

and also τ and τ_t are defined as follows:

$$\tau = \mu_m \nabla V_m \quad (11)$$

$$\tau_t = -\sum_{k=1}^{n} \phi_k \rho_k \overline{v_k v_k} \quad (12)$$

In the case of turbulent flow, to model turbulence, the Launder and Spalding $k-\varepsilon$ turbulence model has been used:

$$\nabla.(\rho_m V_m k) = \nabla.(\frac{\mu_{t,m}}{\sigma_k}\nabla k) + G_{k,m} - \rho_m \varepsilon \quad (13)$$

$$\nabla.(\rho_m V_m \varepsilon) = \nabla.(\frac{\mu_{t,m}}{\sigma_\varepsilon}\nabla \varepsilon) + \frac{\varepsilon}{k}(C_1 G_{k,m} - C_2 \rho_m \varepsilon) \quad (14)$$

where

$$\mu_{t,m} = \rho_m C_\mu \frac{\kappa^2}{\varepsilon}, \quad G_{k,m} = \mu_{t,m}(\nabla V_m + (\nabla V_m)^T) \quad (15)$$

$$C_1 = 1.44,\ C_2 = 1.92,\ C_\mu = 0.09,\ \sigma_k = 1,\ \sigma_\varepsilon = 1.3 \quad (16)$$

2.3. Numerical method

This set of nonlinear differential equations was solved by control volume approach. Control volume technique converts the governing equations to a set of algebraic equations that can be solved numerically. For the convective and diffusive terms, a second order upwind method was used. Pressure and velocity were coupled using Semi Implicit Method for Pressure Linked Equations [SIMPLE].

In the turbulent case study, the selected grid for the present calculations considered of 200 and 150 nodes for r-direction and x-direction, respectively. While in the laminar case, they were selected as 200 and 100 nodes for r-direction and x-direction, respectively. Other combinations of nodes were also tested in the present work, but all of them gave similar values of velocity and temperature as the outlet. Therefore the mentioned nodes were accepted as the optimal ones.

3. Results and discussions

3.1. Validation of the present discussion

The case study presents the hydrodynamic and thermal behaviors of forced convective flow of a conventional fluid inside a circular tube with constant heat flux. The tube has a diameter of 0.01 m and a length of 0.8m. The fluid enters the tube with a constant inlet temperature of 293 K and with uniform axial velocity. The Reynolds number was varied from $10^4 - 10^5$. A uniform heat flux of 50 W/cm^2 was subjected to the walls.

In order to demonstrate the validity and also precision of the model and the numerical procedure, comparisons with the previously published traditional expressions and experimental results have been done.

A traditional expression for calculation of heat transfer in fully developed turbulent flow in smooth tubes is that recommended by Dittus and Boelter :

$$Nu_d = 0.023 \mathrm{Re}_d^{0.8} \mathrm{Pr}^n \quad (17)$$

The properties of this equation are evaluated at the average fluid bulk temperature, and the exponent n has the following values:

n=0.4, for heating of the fluid

n=0.3, for cooling of the fluid

The equation is valid for fully developed turbulent flow in smooth tubes for fluids with Prandtl numbers ranging from about 0.6 to 100 and with moderate temperature differences between wall and fluid conditions. More recent information by Gnielinski suggests that better results for turbulent flow in smooth tubes may be obtained from the following(Holman, 1997):

$$Nu_d = 0.0214(\mathrm{Re}_d^{0.8} - 100)\mathrm{Pr}^{0.4}$$

for $0.5<\mathrm{Pr}<1.5;\ 10^4 < \mathrm{Re} < 5*10^6$ (18)

or

$$Nu_d = 0.012(\mathrm{Re}_d^{0.87} - 280)\mathrm{Pr}^{0.4}$$

for $1.5<\mathrm{Pr}<500;\ 3000 < \mathrm{Re} < 10^6$ (19)

Fig. 1 displays the comparison of Nusselt Number from Dittus – Boelter and Gnielinski formulas and computed values from the present simulation for water with Prandtl number of 6.99.

Next, the Nusselt number for a mixture of ethylene glycol (EG) and water (60:40 by mass) is calculated. The Prandtl number for this case study is 47.54. In this case our simulations were almost all within this range. Fig. 3 displays comparison of Nusselt numbers from the present numerical analysis for forced convection flow with the equations given by Dittus – Boelter and Gnielinski formulas. These data agree with the results published by Namburu et al. (2008).

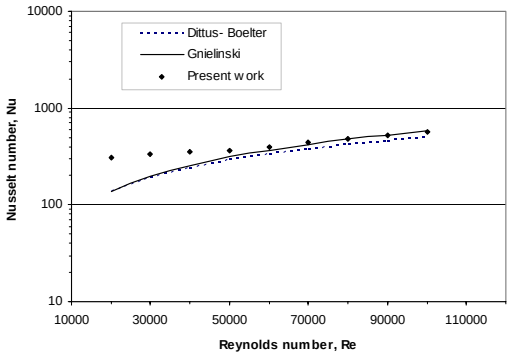

Fig.1. Comparison of Nu No. from D – B and Gnielinski formulas and computed values for water.

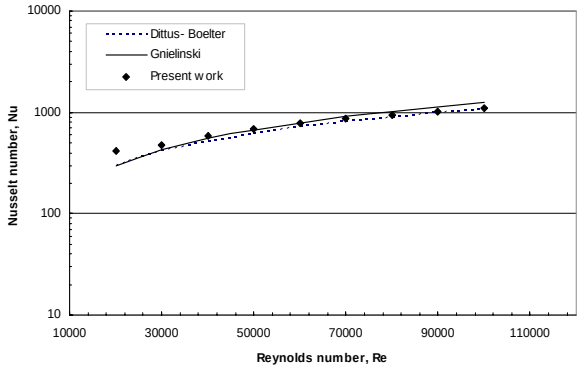

Fig.2. Comparison of Nu No. from D – B and Gnielinski formulas and computed values for EG/water.

3.2. Application of the model

After confirming that the computational model is generating correct results, nanofluids were analyzed at various Reynolds numbers. The case study presents the hydrodynamic and thermal behaviors of forced convective flow of a nanofluid inside a circular tube with constant heat flux. The nanofluid consists of Al2o3 nanoparticles with an average diameter of 42 nm. The tube has a diameter of 45mm and a length of 970mm. The fluid enters the tube with a constant inlet temperature of 295 K and with uniform axial velocity. The Reynolds number was varied from 700 to 1800. It is assumed that the power supply has a maximum power of 300 W. The heat transfer performance of flowing nanofluids was defined in terms of the following convective heat transfer coefficient (h) and the Nusselt number (Nu):

$$h(x) = q/(T_w(x) - T_f(x)) \qquad (20)$$

$$Nu(x) = h(x)D/\lambda \qquad (21)$$

Where q is the heat flux, T_w and T_f are, respectively, the wall and fluid temperatures, D is the tube diameter, λ is the fluid thermal conductivity and x represents axial distance from the entrance of the test section.
The results were compared with the predictions of the well-known Shah equation for laminar flows under the constant heat flux boundary condition:

$$Nu = 1.953(\mathrm{Re}\,\mathrm{Pr}\frac{D}{x})^{1/3} \quad \text{for} \quad (\mathrm{Re}\,\mathrm{Pr}\frac{D}{x}) \geq 33.3 \qquad (22)$$

$$Nu = 4.364 + 0.0722\,\mathrm{Re}\,\mathrm{Pr}\frac{D}{x} \quad \text{for} \quad (\mathrm{Re}\,\mathrm{Pr}\frac{D}{x}) < 33.3 \qquad (23)$$

The numerical simulation is run in three different approaches, which are single phase, mixture and eulerian approaches. The nanofluid consists of water and one percent volume fraction Al_2O_3. The convective heat transfer (h) and Nusselt number is calculated for two different axial distances from the entrance of the test section, which are Position P1 (x=285mm, x/D~63) and also Position P2 (x=524 mm, x/D~116). Figures 3 and 4 display the results for P1 and P2 respectively. Predictions by the classical Shah equation for laminar flows and Dittus-Boelter equation for turbulent flows are also included for comparison. Moreover the results of experimental study of Wen and Ding (2004) are included for comparing the efficiency of three different numerical approaches. As it is obvious from the figures the mixture model predicts the experimental results more accurately than the single phase and also the eulerian models.

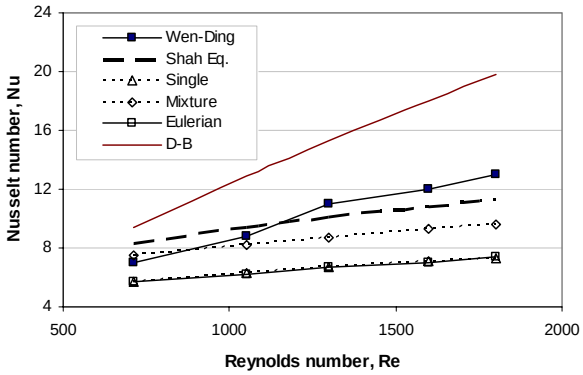

Fig.3. Dependence of Nusselt number on Reynolds number at Position P1 (x/D=63).

It should be noted that in the study of Behzadmehr et al. (2007), they showed the single phase model overestimates the Nusselt number for turbulent forced convective of nanofluids consisting of water and one percent volume fraction Cu with 42

nm mean diameter. While In the present study it is illustrated that both of the single phase and eulerian approaches underestimates the Nusselt number in comparison with experimental results.

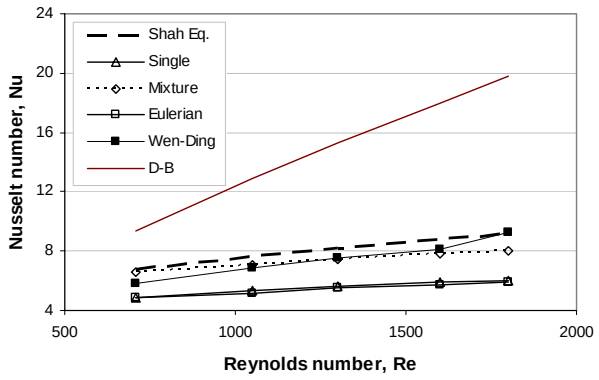

Fig.4. Dependence of Nusselt number on Reynolds number at Position P2 (x/D=116).

To study the effect of concentration of nanoparticles in thermal enhancement of nanofluids, the numerical simulation has been run for six other volume fractions, which are ranged from 2 vol% to 7 vol%. Fig. 5 displays the dependence of tube wall temperature to nanoparticles volume concentration. It is clear from this figure that the rate of thermal enhancement decreases with the increase of nanoparticles volume concentration.

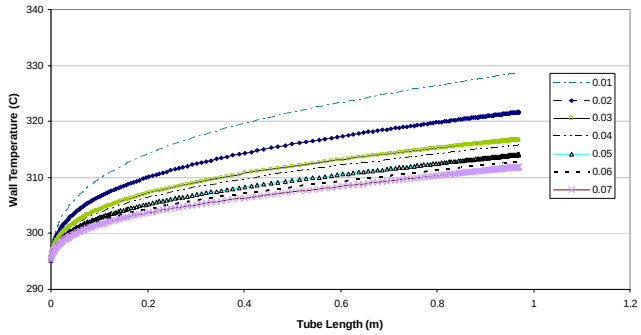

Fig.5. Effect of nanoparticles volume concentration on Wall Temperature of the tube

NOMENCLATURE

C_p fluid specific heat (J kg^{-1} K^{-1})
D tube internal diameter (m)
g acceleration of gravity (m s^{-2})
I turbulent intensity
k turbulent kinetic energy (m^2 s^{-2})
L Length of tube (m)
Nu Nusselt number (=hD/ λ)
Pr Prandtl number (=$\mu C_p/\lambda$)
q_w uniform heat flux at the solid–fluid interface (W m^{-2})
Re Reynolds number (=$V_0 D/\nu$)
x distance from the inlet(m)

Greek letters

ε dissipation of turbulent kinetic energy (m^2 s^{-3})
Φ volume fraction
λ thermal conductivity (W m^{-1} K^{-1})
μ dynamic viscosity (N s m^{-2})
ν kinematic viscosity (m^2 s^{-1})
ρ density (kg m^{-3})

Subscripts

eff effective
f fluid, primary phase
k the kth phase
m mean
p particle, secondary phase
t turbulent
w wall

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