

FLUTTER CHARACTERISTICS OF FUNCTIONALLY GRADED MATERIAL PANELS SUBJECT TO MECHANICAL AND THERMAL LOADS

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ABSTRACT

A nonlinear finite element model is provided for the flutter limit-cycle oscillations of functionally graded material panels subject to combined aerodynamic, thermal and random acoustic loads. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The governing equations are derived using the classical plate theory with von Karman geometric nonlinearity and the principle of virtual work. The first-order piston theory is adopted to model aerodynamic pressures induced by supersonic airflows. The thermal load is assumed to be steady state constant temperature distribution, and the acoustic excitation is considered to be a stationary white-Gaussian random pressure with zero mean and uniform magnitude over the plate surface. The governing equations are transformed to modal coordinates to reduce the computational efforts. Newton-Raphson iteration method is employed to obtain the dynamic response at each time step of the Newmark scheme for numerical integration. Finally, numerical results are provided to study the effects of volume fraction exponent, aerodynamic pressure, temperature rise, and random acoustic load on the panel response.

KEYWORDS:

Functionally graded materials, Limit-cycle oscillations, Nonlinear FEM.

INTRODUCTION

Functionally graded materials (FGMs) are non-homogeneous composites characterized by a smooth and continuous change of material properties from one surface

to the other. This is achieved by gradually varying the volume fraction of the constituent materials. Functionally graded materials are usually composed of two or more materials whose volume fractions are changing smoothly and continuously along desired direction(s). This continuous change in the compositions leads to a smooth change in the mechanical properties, which has many advantages over the laminated composites, where the delamination and cracks are most likely to initiate at the interfaces due to the abrupt variation in the mechanical properties between laminas. One of the advantages of using these materials is that they can survive environments with high temperature gradients, while maintaining structural integrity. Accordingly, one of the most important applications of functionally graded materials is in the skin panels of supersonic and hypersonic flight vehicles, which have to survive the harsh thermal and mechanical loadings.

Panel flutter is a phenomenon that is usually accompanied by temperature elevation on the outer skin of high-speed air vehicles. Panel flutter is a self-excited oscillation of a plate or shell in supersonic flow. Because of aerodynamic pressure forces on the panel, two eigen modes of the structure merge and lead to this dynamic instability. Panel flutter differs from wing flutter only in that the aerodynamic force resulting from the air flow acts only on one side of the panel. Most flutter analyses can be placed in one of four categories based on the structural and aerodynamic theories employed, 1) linear structural theory; quasi-steady aerodynamic theory, 2) linear structural theory;

full linearized (inviscid, potential) aerodynamic theory, 3) nonlinear structural theory; quasi-steady aerodynamic theory, 4) nonlinear structural theory; linearized (inviscid, potential) aerodynamic theory. Analyses of the first type have two major weaknesses: *a*) it does not account for structural nonlinearities, hence it can only determine the flutter boundary and can give no information about the flutter amplitudes, *b*) the use of quasi-steady aerodynamics neglects the three dimensionality and unsteadiness of the flow, hence it cannot be used in the transonic region where the flutter is most likely to occur. Analyses of the second type are intended to remedy weakness *b* but this type still has weakness *a*. The third type remedies weakness *a*, but still possesses weakness *b*. The fourth type remedies both *a* and *b* [1].

Thin plates are a commonly used form of structural components especially in aerospace vehicles, such as high-speed aircraft, rockets, and spacecrafts, which are subjected to aerodynamic loads, thermal loads due to aerodynamic and / or solar radiation heating, and random acoustic loads due to engine and / or aerodynamic transonic noise. This results in a temperature and pressure distributions over the panel surface. The presence of these thermal and pressure fields results in a flutter motion at a lower aerodynamic pressure, or a larger flutter limit-cycle amplitude at the same aerodynamic pressure. In addition, a high temperature rise may cause large thermal deflections (thermal buckling) of the skin panels, which could affect flutter response. Accordingly, it is important to consider the interactive effect of aerodynamic, thermal and random acoustic loads.

Extensive research work has been carried out on the FGM since its concept was proposed in the late 1980s. Reddy [2] developed both theoretical and finite element formulations for thick FGM plates according to the Higher-order Shear Deformation Plate Theory, and studied the nonlinear dynamic response of FGM plates subjected to a suddenly applied uniform pressure. El-Abbassi and Meguid [3] provided a new thick shallow shell element to study the thermo-elastic behavior of functionally graded structures made of shells and plates. The element accounts for the varying elastic and thermal properties across its thickness. Zenkour [4] presented a new generalized shear deformation theory to study the static response for a simply supported functionally graded rectangular plate subjected to a transverse uniform load. Dai et al. [5] developed a mesh free model for the active shape control and the dynamic response suppression of a functionally graded material plate containing piezoelectric sensors and actuators. He et al. [6] have presented the vibration control of FGM thin plate with integrated piezoelectric sensors and actuators. Yang et al. [7] investigated the geometrically non-linear bending behavior of FGM plates with integrated piezoelectric layers and subjected to transverse loads and a temperature gradient through the plate thickness. Batra and Jin [8] adopted the first-order shear deformation theory coupled with finite

element method to study the vibration of functionally graded anisotropic rectangular plate with different edge support conditions. The functional grading was achieved through continuously changing the fiber orientation angle through the thickness. Kim [9] developed an analytical technique to investigate the effect of temperature on the vibration characteristics of thick functionally graded rectangular plates, taking into account the temperature-dependence of the material properties. However, the studies pertaining to flutter instability of FGM panels are rather sparsely treated in the literature compared to vibration analysis. The influence of thermal environment on the critical flutter dynamic pressure of a flat FGM panel was studied by Prakash and Ganapathi [10]. Navazi and Haddadpour [11] provided an analytical formulation to determine the aero-thermoelastic stability margins of FGM panels. The effect of compressive in-plane loads and both uniform and through the thickness nonlinear temperature distributions were considered in their formulation. Haddadpour et al. [12] extended the work presented in [11] by studying the effect of the constituents volume fraction exponent on the flutter limit-cycle oscillation amplitudes of semi-infinite FGM plates. Ibrahim et al. [13] investigated the thermal buckling and flutter boundaries of thin FGM panels at elevated temperature. They adopted an incremental finite element technique to capture the effect of the temperature-dependence of material properties on the panel response. Ibrahim et al. [14] developed a frequency-domain solution to predict the flutter limit-cycle oscillation amplitudes of thin FGM panels under combined aerodynamic and thermal loads. To account for the temperature-dependence of material properties, the thermal strain was modeled as an integral quantity of thermal expansion coefficient with respect to temperature. Ibrahim et al. [15] extended the formulation presented in [14] by including the shear deformation effect to make it capable of handling thick FGM panels. Sohn and Kim [16] studied the static and dynamic stabilities of FGM panels under supersonic airflows and elevated temperatures environment, while assuming temperature-independent material properties. However, to the best of authors' knowledge, the combined effect of aerodynamic, thermal and acoustic loads on the flutter limit-cycle oscillation of FGM panels has not been yet accomplished in the literature. As the use of panels made of functionally graded materials is expected to increase, it is worth investigating the nonlinear response of such panels under combined aerodynamic, thermal and random acoustic loads.

In this work, a time-domain solution is presented for the nonlinear flutter response of clamped FGM panels under combined aerodynamic, thermal, and random acoustic loads. A nonlinear finite element model is provided based on the classical plate theory with von Karman geometric nonlinearity and the principle of virtual work. Material properties are assumed to be temperature-

dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The first-order piston theory is adopted to model aerodynamic pressures induced by supersonic airflows. The thermal load is assumed to be steady state constant temperature distribution, and the acoustic excitation is considered to be a stationary white-Gaussian random pressure with zero mean and uniform magnitude over the plate surface. The governing equations are transformed to modal coordinates to reduce the computational efforts. Newton-Raphson iteration method is employed to obtain the dynamic response at each time step of the Newmark numerical integration scheme. Finally, numerical results are provided to study the effects of volume fraction exponent, aerodynamic pressure, temperature rise, and random acoustic load on the panel response.

FINITE ELEMENT FORMULATION

The equation of motion with the consideration of large deflection and temperature dependence of material properties are derived for a functionally graded plate subject to aerodynamic, thermal and random acoustic loads. To account for temperature dependence of material properties, the thermal strain is modeled as an integral quantity of the thermal expansion coefficient. The element used in this study is the rectangular four-node-Bogner-Fox-Schmidt (BFC) C^1 conforming element (for the bending DOFs) [17].

Nonlinear Strain-Displacement Relations

The nodal degrees of freedom vector $\{\theta\}$ of the rectangular plate element having 6 degrees of freedom at each node can be written as:

$$\{\theta\} = \left\{ \left\{ w, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial^2 w}{\partial x \partial y} \right\}, \{u, v\} \right\}^T = \left\{ \begin{matrix} \{w_b\} \\ \{w_m\} \end{matrix} \right\} \quad (1)$$

where $\{w_b\}$ is the transverse displacement vector and $\{w_m\}$ is the membrane displacement vector. Inplane strains and curvatures, based on von Karman moderately large deflection and classical thin plate theory, are given by:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \\ \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \\ \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} = \{\varepsilon_m\} + \{\varepsilon_\theta\} + z\{\kappa\} \quad (2)$$

Parameters u , v and w are displacements in x , y and z directions, respectively. ε_m , ε_θ , and $z\kappa$ are the membrane

linear strain vector, the membrane nonlinear strain vector, and the bending strain vector, respectively.

Functionally Graded Materials

Typically, the FGMs are made of a mixture of two materials; a ceramic, which is capable of withstanding high temperature environments due to its low thermal conductivity and a metal which act as a structural element to support loading and prevent fractures. Without losing generality, it is usually assumed that the top surface of an FGM plate is ceramic rich and the bottom is metal rich. The region between the two surfaces consists of a blend of the two materials which is assumed in the form of a simple power law distribution as [5]:

$$P_e(z) = P_c V_c + P_m (1 - V_c) \quad (3)$$

$$V_c = \left(0.5 + \frac{z}{h} \right)^n, \quad (-h/2 \leq z \leq h/2, \quad 0 \leq n \leq \infty) \quad (4)$$

where z is coordinate in the thickness direction of a plate; (P_e , P_c , P_m) are effective material properties of the FGM, the properties of the ceramic and the properties of the metal respectively. V_c is the ceramic volume fraction, h is the plate thickness, and power n is the volume fraction exponent.

Stress-Strain Relationship of an FGM panel

The relationship of inplane forces $\{N\}$ and bending moments $\{M\}$ in terms of the strain vectors can be written as:

$$\{N\} = [A] \{\varepsilon_m + \varepsilon_\theta\} + [B] \{\kappa\} - \{N_T\} \quad (5)$$

$$\{M\} = [B] \{\varepsilon_m + \varepsilon_\theta\} + [D] \{\kappa\} - \{M_T\} \quad (6)$$

where

$$([A], [B], [D]) = \int_{-h/2}^{h/2} (1, z, z^2) [Q(z, T, n)] dz$$

$$[Q(z, T, n)] = \begin{bmatrix} \frac{E(z, T, n)}{1 - \nu^2(z, n)} & \frac{\nu E(z, T, n)}{1 - \nu^2(z, n)} & 0 \\ \frac{\nu E(z, T, n)}{1 - \nu^2(z, n)} & \frac{E(z, T, n)}{1 - \nu^2(z, n)} & 0 \\ 0 & 0 & \frac{E(z, T, n)}{2(1 + \nu(z, n))} \end{bmatrix}$$

$$\begin{Bmatrix} N_T \\ M_T \end{Bmatrix} = \int_{-h/2}^{h/2} \left[\int_{T_{ref}}^T [Q(z, \tau, n)] \{\alpha(z, \tau, n)\} d\tau \right] \begin{Bmatrix} 1 \\ z \end{Bmatrix} dz$$

where $[A]$, $[B]$ and $[D]$ are the extensional stiffness matrix, extensional-bending coupling stiffness matrix and flexural stiffness matrix, respectively. T denotes the temperature, while a constant temperature distribution in the x , y and z directions are assumed. In addition, α denotes the thermal expansion coefficient.

Aerodynamic Loading

The aerodynamic pressure caused by a supersonic airflow will be approximated by the first-order quasi-steady piston theory [14], which is valid for $\sqrt{2} < M_\infty$ and shows good accuracy in the range of $\sqrt{2} < M_\infty < 5$. The aerodynamic pressure using the first-order piston theory is expressed as:

$$P_a = -\left(\frac{g_a}{\omega_o} \frac{D_{11}}{a^4} \frac{\partial w}{\partial t} + \lambda \frac{D_{11}}{a^3} \frac{\partial w}{\partial x}\right) \quad (7)$$

With

$$g_a = \sqrt{\lambda C_a}, \quad C_a = \frac{(M_\infty^2 - 2)^2}{(M_\infty^2 - 1)^2} \frac{\rho_a a}{\rho h \beta}, \quad \lambda = \frac{2qa^3}{\beta D_{11}},$$

$$q = \frac{\rho_a V^2}{2}, \quad \beta = \sqrt{M_\infty^2 - 1}, \quad \text{and} \quad \omega_o = \left(\frac{D_{11}}{\rho h a^4}\right)^{\frac{1}{2}}$$

where P_a is the aerodynamic pressure loading, V is the airflow velocity on one side of the panel, M_∞ is the Mach number, q is the dynamic pressure, ρ and ρ_a is the panel and air mass densities, g_a is the non-dimensional aerodynamic damping, C_a is the aerodynamic damping coefficient, λ is the non-dimensional dynamic pressure, D_{11} is the first entry in the flexural stiffness matrix D (1, 1), and a is the stream wise panel length.

Governing Equations

By using the principle of virtual work and Eqs. 2, 5, and 6, the governing equation can be derived as follows:

$$\delta W = \delta W_{int} - \delta W_{ext} = 0 \quad (8)$$

The internal virtual work δW_{int} is given as:

$$\delta W_{int} = \int_A \left(\{\delta \varepsilon_m\}^T \{N\} + \{\delta \kappa\}^T \{M\} \right) dA$$

$$= \{\delta \theta\}^T \left([k] - [k_T] + \frac{1}{2}[n1] + \frac{1}{3}[n2] \right) \{\theta\} - \{\delta \theta\}^T \{p_T\} \quad (9)$$

where $\{\theta\} = [w \ w_x \ w_y \ w_{xy} \ u \ v]$ is the nodal displacement vector; $[k]$ and $[k_T]$ are the linear stiffness matrix and the thermal geometric stiffness matrix; $[n1]$ and $[n2]$ are the first- and second-order nonlinear stiffness matrices, respectively [20]. In addition, $\{p_T\}$ is the thermal load vector. On the other hand, the virtual work done by external forces on a plate element, considering inertia and random pressure excitations, is:

$$\delta W_{ext} = \int_A (\delta w(-\rho h \ddot{w} + P_a + p(t)) + \delta u(-\rho h \ddot{u}) + \delta v(-\rho h \ddot{v})) dA \quad (10)$$

where $p(t)$ represents the acoustic load which is modeled as a white-Gaussian random pressure. The virtual work of aerodynamic loading, P_a , using the quasi-steady first-order piston theory can be written as [20]:

$$\int_A \delta w P_a dA = - \int_A \delta w \left(\frac{g_a}{\omega_o} \frac{D_{11}}{a^4} \frac{\partial w}{\partial t} + \lambda \frac{D_{11}}{a^3} \frac{\partial w}{\partial x} \right) dA \quad (11)$$

$$= - \{\delta w_b\}^T [g_a] \{\dot{w}_b\} - \{\delta w_b\}^T \lambda [a_a] \{w_b\}$$

The finite element form of the virtual work done by inertia and aerodynamic forces on a plate element takes the form:

$$\delta W_{ext} = - \left\{ \begin{matrix} \{\delta w_b\} \\ \{\delta w_m\} \end{matrix} \right\}^T \left[\begin{matrix} [m_b] & 0 \\ 0 & [m_m] \end{matrix} \right] \left\{ \begin{matrix} \{\dot{w}_b\} \\ \{\dot{w}_m\} \end{matrix} \right\} - \left\{ \begin{matrix} \{\delta w_b\} \\ \{\delta w_m\} \end{matrix} \right\}^T \left[\begin{matrix} [g_a] & 0 \\ 0 & 0 \end{matrix} \right] \left\{ \begin{matrix} \{w_b\} \\ \{w_m\} \end{matrix} \right\}$$

$$- \lambda \left\{ \begin{matrix} \{\delta w_b\} \\ \{\delta w_m\} \end{matrix} \right\}^T \left[\begin{matrix} [a_a] & 0 \\ 0 & 0 \end{matrix} \right] \left\{ \begin{matrix} \{w_b\} \\ \{w_m\} \end{matrix} \right\} + \left\{ \begin{matrix} \{\delta w_b\} \\ \{\delta w_m\} \end{matrix} \right\}^T \left\{ \begin{matrix} \{p_b(t)\} \\ 0 \end{matrix} \right\}$$

where $[m_b]$ and $[m_m]$ are bending and inplane inertia matrices, respectively. $[g_a]$ is the aerodynamic damping matrix, and $[a_a]$ is the aerodynamic stiffness matrix. Combining Eqs.9 and 12, the element equation of motion is expressed as:

$$[m] \{\ddot{\theta}\} + [g] \{\dot{\theta}\} + \left(\lambda [\bar{a}_a] + [k] - [k_T] \right) \{\theta\} = \{p(t)\} + \{p_T\} \quad (13)$$

$$+ \frac{1}{2}[n1] + \frac{1}{3}[n2]$$

Assembling the element equations of motion to the system level by summing up the contributions from all elements and applying the boundary conditions, the system equations of motion become:

$$[M] \{\ddot{W}\} + [G] \{\dot{W}\} + \left(\lambda [A] + [K] - [K_T] \right) \{W\} = \{P(t)\} + \{P_T\} \quad (14)$$

$$+ \frac{1}{2}[N1] + \frac{1}{3}[N2]$$

SOLUTION PROCEDURE

In this section, a time-domain solution is presented for the nonlinear flutter response of FGM panels under combined aerodynamic, thermal and random acoustic loads. Equation 14 can be stated as:

$$\left[\begin{matrix} [M_b] & 0 \\ 0 & [M_m] \end{matrix} \right] \left\{ \begin{matrix} \{\dot{W}_b\} \\ \{\dot{W}_m\} \end{matrix} \right\} + \left[\begin{matrix} [G_a] & 0 \\ 0 & 0 \end{matrix} \right] \left\{ \begin{matrix} \{\dot{W}_b\} \\ \{\dot{W}_m\} \end{matrix} \right\}$$

$$+ \left(\lambda \left[\begin{matrix} [A_a] & 0 \\ 0 & 0 \end{matrix} \right] + \left[\begin{matrix} [K_b] & [K_{bm}] \\ [K_{mb}] & [K_m] \end{matrix} \right] - \left[\begin{matrix} [K_{Tb}] & 0 \\ 0 & 0 \end{matrix} \right] \right) \left\{ \begin{matrix} \{W_b\} \\ \{W_m\} \end{matrix} \right\} = \left\{ \begin{matrix} \{P_b(t)\} \\ 0 \end{matrix} \right\} + \left\{ \begin{matrix} \{P_{Tb}\} \\ \{P_{Tm}\} \end{matrix} \right\}$$

$$+ \frac{1}{2} \left[\begin{matrix} [N1_{Nm}(\{W_m\}) + N1_{Nb}(\{W_b\})] & [N1_{bm}] \\ [N1_{mb}] & 0 \end{matrix} \right] \left\{ \begin{matrix} \{W_b\} \\ \{W_m\} \end{matrix} \right\} + \frac{1}{3} \left[\begin{matrix} [N2_b] & 0 \\ 0 & 0 \end{matrix} \right] \left\{ \begin{matrix} \{W_b\} \\ \{W_m\} \end{matrix} \right\}$$

where the subscripts m and b denote membrane and bending, respectively. Separating the membrane and transverse displacement equations in equation (15):

$$[M_b]\{\ddot{W}_b\} + [G_a]\{\dot{W}_b\} + \left(\lambda[A_a] + [K_b] - [K_{Tb}] + \frac{1}{2}[N_{I_{Nm}}(\{W_m\})] \right) \{W_b\} + \left(\frac{1}{2}[N_{I_{Nb}}(\{W_b\})] + \frac{1}{3}[N_{2b}] \right) \{W_b\} + \left([K_{bm}] + \frac{1}{2}[N_{I_{bm}}] \right) \{W_m\} = \{P_b(t)\} + \{P_{Tb}\} \quad (16)$$

$$[M_m]\{\ddot{W}_m\} + \left([K_{mb}] + \frac{1}{2}[N_{1_{mb}}] \right) \{W_b\} + [K_m]\{W_m\} = \{P_{Tm}\} \quad (17)$$

Note that, neglecting the in-plane inertia term will not bring significant error, because in-plane natural frequencies are 2 to 3 order of magnitude higher than bending ones [18]. Therefore, the in-plane displacement can be expressed in terms of bending displacement as:

$$\begin{aligned} \{W_m\} &= [K_m]^{-1} \left(\{P_{Tm}\} - \left([K_{mb}] + \frac{1}{2}[N_{1_{mb}}] \right) \{W_b\} \right) \\ &= [K_m]^{-1} \{P_{Tm}\} - [K_m]^{-1} [K_{mb}] \{W_b\} - \frac{1}{2} [K_m]^{-1} [N_{1_{mb}}] \{W_b\} \\ &= \{W_m\}_o - \{W_m\}_1 - \{W_m\}_2 \end{aligned} \quad (18)$$

It can be shown that [20]:

$$\frac{1}{2}[N_{1_{bm}}] \{W_m\}_o = \frac{1}{2}[N_{1_{Nm}}(\{W_m\}_o)] \{W_b\} \quad (19)$$

Accordingly, by substituting equation (18) into (16), equation (16) can be written as:

$$[M_b]\{\ddot{W}_b\} + [G_a]\{\dot{W}_b\} + \left(\begin{aligned} &\lambda[A_a] + [K_b] - [K_{Tb}] - [K_{bm}][K_m]^{-1}[K_{mb}] \\ &+ [N_{I_{Nm}}(\{W_m\}_o)] + \frac{1}{2}[N_{I_{Nb}}] - \frac{1}{2}[N_{I_{Nm}}(\{W_m\}_1)] \\ &- \frac{1}{2}[N_{I_{bm}}][K_m]^{-1}[K_{mb}] - \frac{1}{2}[K_{bm}][K_m]^{-1}[N_{I_{mb}}] \\ &+ \frac{1}{3}[N_{2b}] - \frac{1}{4}[N_{I_{bm}}][K_m]^{-1}[N_{I_{mb}}] \\ &- \frac{1}{2}[N_{I_{Nm}}(\{W_m\}_2)] \end{aligned} \right) \{W_b\} = \{P_b(t)\} + \{P_{Tb}\} - [K_{bm}][K_m]^{-1} \{P_{Tm}\} = \{P_B\} \quad (20)$$

An effective solution procedure is to transform equation (20) into modal coordinates using reduced system normal modes by expressing the system bending displacement $\{W_b\}$ as a linear combination of some mode shapes as:

$$\{W_b\} \approx \sum_{r=1}^n Q_r \{\phi_r\} = [\Phi] \{Q\} \quad (21)$$

where the r^{th} normal mode $\{\phi_r\}$ and the corresponding natural frequency ω_r are obtained from the linear vibration of the system as:

$$\omega_r^2 [M_b] \{\phi_r\} = \left([K_b] - [K_{bm}][K_m]^{-1}[K_{mb}] \right) \{\phi_r\} \quad (22)$$

Based on the normal modes evaluated in equation (22), all the matrices in equation (20) are transformed into modal coordinates. Accordingly, equation (20) can be written in modal coordinates as:

$$[\bar{M}_b] \{\ddot{Q}\} + 2[\zeta_r \omega_r] [\bar{M}_b] + [\bar{G}_a] \{\dot{Q}\} + \left([\bar{K}] + [\bar{K}_q] + [\bar{K}_{qq}] \right) \{Q\} = \{\bar{P}_B\} \quad (23)$$

where $[\bar{K}]$ is the linear modal stiffness matrix, $[\bar{K}_q]$ and $[\bar{K}_{qq}]$ are the first- and second-order nonlinear modal stiffness matrices, respectively. A modal structural damping matrix $2[\zeta_r \omega_r] [M_b]$ has been added to Eq.23 to account for the structural damping effect on the system. The coefficient ζ_r is the modal damping ratio of the r^{th} mode, while ω_r is the r^{th} natural frequency in Hz.

NUMERICAL RESULTS AND DISCUSSIONS

The nonlinear vibration behavior of an FGM panel is investigated with four parameters in the study; volume fraction exponent n , temperature rise ΔT , sound pressure level SPL , and non-dimensional dynamic pressure λ . The panel is modeled using a 10x10 mesh. All panel edges are assumed clamped. A damping ratio that follows the relation $\zeta_r f_r = \zeta_s f_s$ is used with a fundamental modal damping coefficient ζ_1 equal to 0.02. The aerodynamic damping coefficient C_a is set to 0.1 [19]. Newmark implicit numerical integration scheme is utilized to solve the system differential equations, while Newton-Raphson iteration scheme is adopted to solve the nonlinear algebraic system of equations at each time step [21]. Modal transformation using nine normal modes were found giving converged solution and thus used, noting that linear normal modes can efficiently express the panel nonlinear response in panels with all edges clamped [22]. The dimensions of the FGM panel are chosen to be 0.38 x 0.305 x 0.002m. A uniform temperature rise is applied to the plate, and the reference temperature is assumed to be 21°C. The FGM panel adopted in this study is a mixture of nickel and silicon nitride (Si_3N_4). The properties of the constituent materials are assumed to be temperature-dependent according to the following relation:

$$P = P_o \left(I + P_1 T + P_2 T^2 + P_3 T^3 \right) \quad (24)$$

where, the coefficients P_o , P_1 , P_2 and P_3 for young's modulus E , the Poisson ratio ν and the thermal expansion coefficient α of nickel and silicon nitride are given in Table 1 [23]. It is worth noting that, for all simulation

presented in this study, D_{11} is evaluated at T_{ref} and $n = \infty$ (i.e., D_{11} is evaluated using the nickel properties at room temperature).

Table 1 Temperature-dependence coefficients for nickel and silicon nitride

Properties	Material	P_0	P_1	P_2	P_3
E (MPa)	Si ₃ N ₄	348.43e9	-3.07e-4	2.2e-7	-8.9e-11
	Nickel	223.95e9	-2.79e-4	3.9e-9	0
ν	Si ₃ N ₄	0.24	0	0	0
	Nickel	0.31	0	0	0
α	Si ₃ N ₄	5.8723e-6	9.09e-4	0	0
	Nickel	9.9209e-6	8.71e-4	0	0
ρ (Kg/m ³)	Si ₃ N ₄	2370			
	Nickel	8900			

Validation of the Formulation

To validate the present formulation, flutter limit-cycle oscillation results were compared to those of Dowell [24]. The flutter limit-cycle oscillations for an aluminum square panel, at two different values of the non-dimensional dynamic pressure λ are presented in Fig.1. The panel dimensions are 0.305 x 0.305 x 0.00127m with all edges simply supported. The aerodynamic damping coefficient C_a is set to 0.1. Nine normal modes were utilized with a time step for numerical integration equal to 1/10000 sec. The flutter limit-cycle amplitudes at $\lambda = 700$ and 800 were compared to those of Dowell [24], and good agreement is found between them.

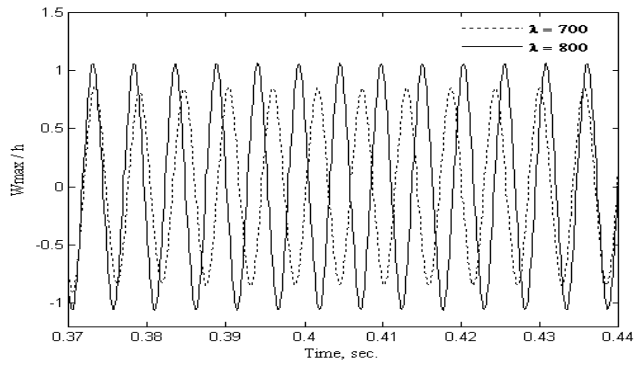


Fig.1 Flutter limit-cycle oscillation for a simply supported aluminum square panel at different non-dimensional dynamic pressures, λ

Nonlinear Vibration Behavior

Time history responses for a panel at room temperature, $\lambda = 0$ and 700, SPL = 110dB, and $n = 0$ (Si₃N₄), 1, and ∞ are presented in Figs. 2 and 3. For $\lambda = 0$, it is seen in Fig.2 that the FGM panel (i.e., $n = 1$) is not favorable regarding the random acoustic response, since the nickel panel (i.e., $n = \infty$) has the lowest deflection r. m. s. value compared to the other panel. It is also noticed that the FGM panel response is not intermediate to those of the

other two panels as was the case in the thermal buckling temperatures and the linear flutter boundaries presented in [15]. Figure 3 illustrates the effect of the presence of an airflow with $\lambda = 700$, which is less than the flutter critical dynamic pressures for all the panels. It is seen that the airflow adds stiffness to the panels resulting in a lower deflection r. m. s. values compared to those of Fig.2.

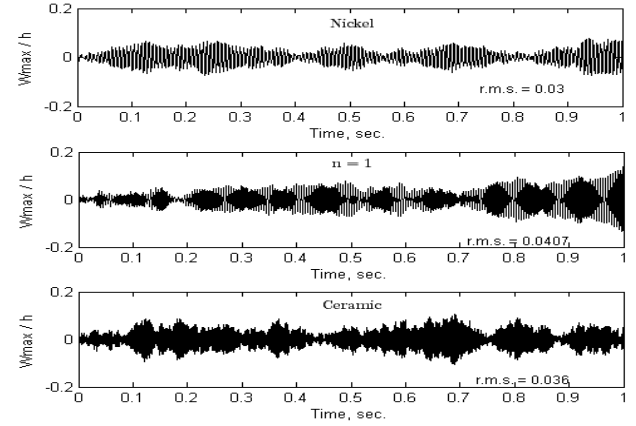


Fig.2 Time histories for an FGM panel at SPL = 110dB, $\lambda = 0$, $\Delta T = 0$ °C, and different volume fraction exponent

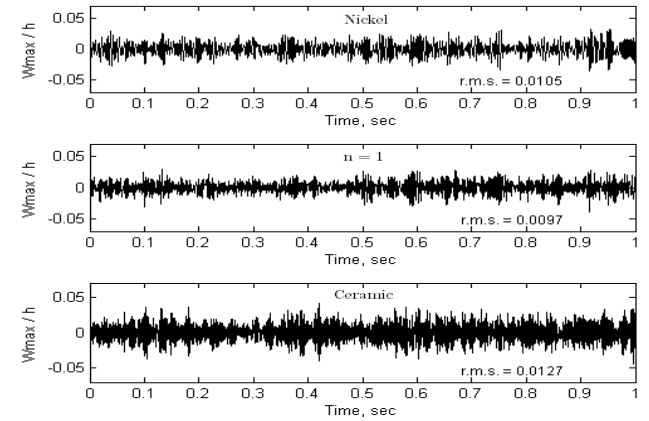


Fig.3 Time histories for an FGM panel at SPL = 110dB, $\lambda = 700$, $\Delta T = 0$ °C, and different volume fraction exponent

Figures 4 and 5 present the time history responses with the same SPL and λ values presented in Figs. 2 and 3, but with a temperature rise value $\Delta T = 8$ °C. before the occurrence of thermal buckling, the nickel panel loses stiffness faster than the FGM and ceramic panels. Accordingly, the FGM and ceramic panels turned to have a better performance than the nickel panel compared to trend shown in Fig.2. As the critical non-dimensional dynamic pressure for the nickel panel is exceeded at $\lambda = 700$ and $\Delta T = 8$ °C, it is seen in Fig.5 that the nickel panel turned to flutter with harmonic limit-cycle oscillations with a vibration amplitude much larger than those of the FGM and ceramic panels.

Accordingly, the FGM panels is found to have favorable performance compared to the nickel panels at $\Delta T = 8^\circ\text{C}$. Moreover, it is also found that the presence of airflow is forcing the FGM panel response to be intermediate to those of nickel and ceramic.

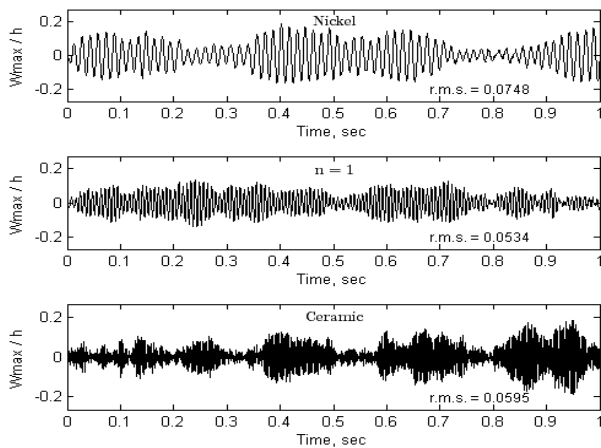


Fig.4 Time histories for an FGM panel at SPL = 110dB, $\lambda = 0$, $\Delta T = 8^\circ\text{C}$, and different volume fraction exponent

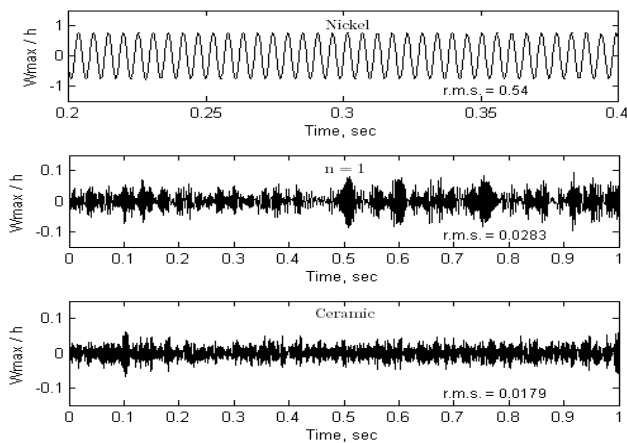


Fig.5 Time histories for an FGM panel at SPL = 110dB, $\lambda = 700$, $\Delta T = 8^\circ\text{C}$, and different volume fraction exponent

At a temperature rise value $\Delta T = 16^\circ\text{C}$, the critical buckling temperatures for both nickel and FGM panels are exceeded [15]. Accordingly, due to the nonlinear stiffness added to the panels through having thermal post-buckling deflections, it is found in Fig.6 that the nickel and FGM panels show small amplitude random vibrations about one of the two equilibrium buckling positions with the occurrence of a snap-through motion in the FGM panel at the beginning of the response. Therefore, it turns to be a compromise between having higher thermal post-buckling deflections with smaller vibration amplitudes and hence more deterioration in the aerodynamic performance, as is

the case of the nickel panel response. Or having lower thermal post-buckling deflections with higher vibration amplitudes and hence a lower fatigue performance, as is the case of the FGM panel. Adding the effect of an air flow with $\lambda = 700$ along with a temperature rise of $\Delta T = 16^\circ\text{C}$, it is found that all the flutter critical dynamic pressures are exceeded for all the panels [15]. Accordingly, it is seen in Fig.7 that both nickel and FGM panels are fluttering with nearly harmonic limit-cycle oscillations, while the random acoustic response still dominating the flutter motion in the ceramic panel. It is also noticed that the FGM panel shows a lower flutter limit-cycle oscillation amplitude with a higher frequency compared to the nickel panel. Therefore, in spite of the lower amplitudes of the FGM panels, the nickel panels may survive longer time under these loads.

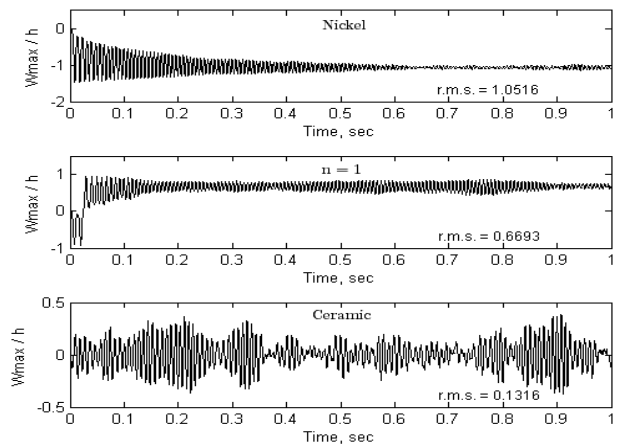


Fig.6 Time histories for an FGM panel at SPL = 110dB, $\lambda = 0$, $\Delta T = 16^\circ\text{C}$, and different volume fraction exponent

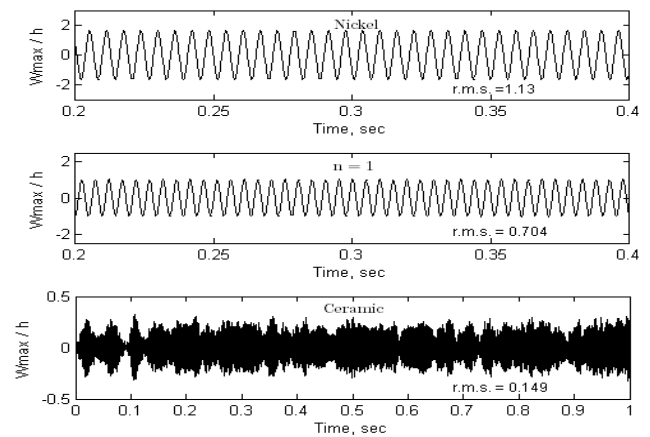


Fig.7 Time histories for an FGM panel at SPL = 110dB, $\lambda = 700$, $\Delta T = 16^\circ\text{C}$, and different volume fraction exponent

Figure 8 and 9 present the time history responses for $\Delta T = 24^\circ\text{C}$ and $\lambda = 0$ and 700. It is seen in Fig.8 the ceramic plate has the smallest thermal post-buckling deflection and

the largest vibration amplitudes. Moreover, it can be concluded from Fig.9 that, at high temperatures and dynamic pressures, decreasing the volume fraction exponent will always result in decreasing the limit-cycle oscillation amplitudes along with increasing the flutter frequency. Figure 10 illustrates the effect of increasing the SPL from 110dB up to 130dB. It is noticed that increasing the SPL has almost no quantitative effect on the flutter amplitude while having a noticeable qualitative effect on the flutter behavior through increasing the number of cycles flutter vibration and changing the type vibration to be non-harmonic or chaotic limit-cycle oscillations as shown in the phase plots of Fig.10. So far, it is concluded that the existence of high sound pressure level values can noticeably affect the panel fatigue life performance.

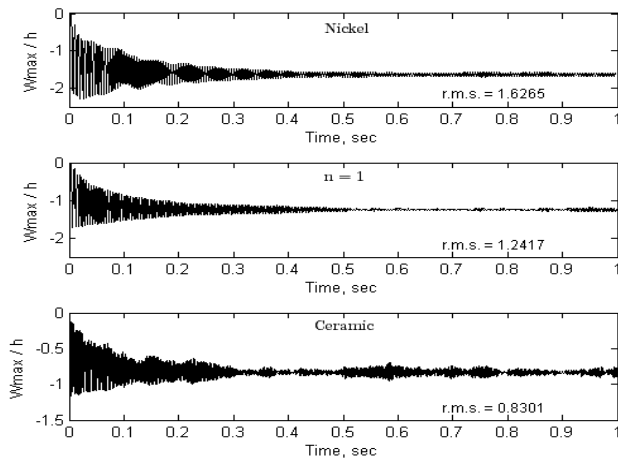


Fig.8 Time histories for an FGM panel at SPL = 110dB, $\lambda = 0$, $\Delta T = 24$ °C, and different volume fraction exponent

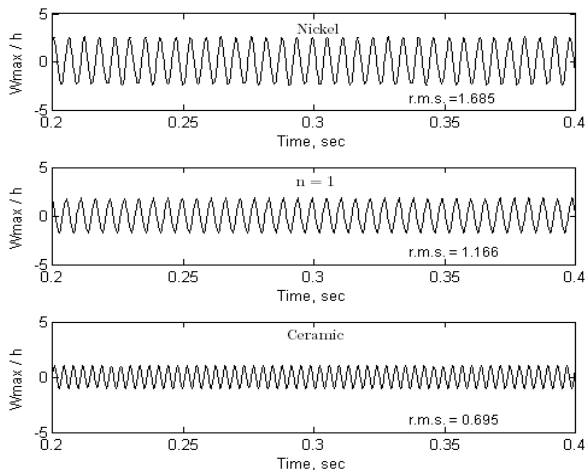


Fig. 9 Time histories for an FGM panel at SPL = 110dB, $\lambda = 700$, $\Delta T = 24$ °C, and different volume fraction exponent

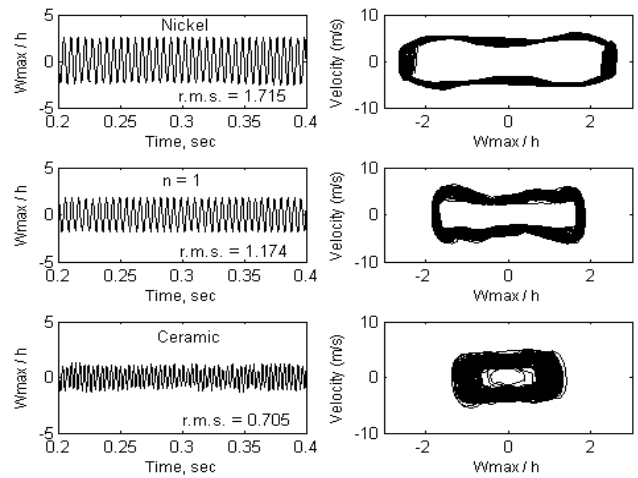


Fig.10 Time histories for an FGM panel at SPL = 130dB, $\lambda = 700$, $\Delta T = 24$ °C, and different volume fraction exponent

CONCLUSIONS

In this work, a time-domain solution is presented for the nonlinear flutter response of clamped FGM panels under combined aerodynamic, thermal, and random acoustic loads. A nonlinear finite element model is provided based on the classical plate theory with von Karman geometric nonlinearity and the principle of virtual work. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The aerodynamic pressure is modeled using the quasi-steady first-order piston theory, while the acoustic load is assumed to be a white-Gaussian random pressure uniformly distributed over the panel surface. The effects of the volume fraction exponent, aerodynamic pressure, temperature rise, and sound pressure level on the nonlinear vibration behavior a clamped FGM panel are studied.

The results showed that, at room temperature, the nickel panel has a better performance regarding the random acoustic response. But, at higher temperatures, it turns to be a matter of compromise between having large thermal post-buckling deflections and hence deterioration in the flight performance, or having higher vibration amplitudes which can affect fatigue life performance. It is also noticed that decreasing the volume fraction results in a reduction in the flutter amplitude and a noticeable increase in the frequency of vibration. Moreover, it is found that the flutter motion frequently dominates the random acoustic response, but high sound pressure levels can affect the flutter behavior through increasing the flutter frequency and changing the type vibration to be non-harmonic or chaotic limit-cycle oscillations. Accordingly, overlooking the existence of acoustic noise can lead to a fatigue failure. Finally, it is concluded that for a panel under aerodynamic, thermal and

random acoustic loads, the FGM panels are not always superior in performance compared to the metal panels.

NOMENCLATURE

[A], [B], [D]	In-plane, coupling, and bending matrices of a laminate
$[A_a]$	Aerodynamic stiffness matrix
[G]	Aerodynamic damping matrix
[K], [M]	System linear stiffness and mass matrices
{N}, {M}	Force and moment resultant vectors
[N1], [N2]	System first- and second-order nonlinear stiffness matrices
$[\bar{Q}]$	Transformed lamina reduced stiffness matrix
T_{ref}	Reference temperature
{W}	System nodal displacement vector
α	Thermal expansion coefficient
λ	Non-dimensional dynamic pressure
h	Thickness
n	Volume fraction exponent
Subscripts	
b	Bending
m	Membrane
x, y, z	Plate Cartesian coordinates

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