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A General Correlation for Mean Velocity Distribution With Drag Reducing Polymers

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ABSTRACT

In the present study, a general correlation for dilute polymer solutions, depending on Prandtl's mixing length model, of the form $U_P^+ = \alpha' A_N \ln y^+ + B_N + \Delta B$ is developed to correlate the velocity data obtained previously by many investigators. Comparing the above equation with that for Newtonian fluids, it is shown that the parameters α' and ΔB present the additional slope with an upward shifting of the logarithmic velocity profile for dilute polymer solutions. The present study predicts that each of these two parameters is function of the amount of drag reduction DR only. For the special case, when the amount of drag reduction DR = 0, the obtained correlation is valid for Newtonian (solvent) fluids. The usefulness of the proposed correlation for turbulent velocity of dilute polymer solutions is that the turbulent velocity profile for any polymer solution can be predicted from the pressure drop/flow rate measurements. Good agreement was obtained between independent sets of experimental data and predictions of the velocity profile using the proposed correlation.

KEYWORDS:

Drag Reduction, Polymer, Log Law, Velocity Correlation.

INTRODUCTION

Energy saving is one of the top concern on the agenda of the decision makers all over the world in resent years. Consequently, due to their effective contribution on energy saving in many applications, interest in studying the flow of dilute polymer solutions has increased. These applications include long distance transport of crude oil (Ram et al. [1]) and mineral suspensions [Herod and Tiederman [16], Poreh et al. [29] and Gust [9]], sewers and open channels [Sellin [38], Sellin and Barnard [39] and Overfield et al. [20]], hydraulic machinery [Kato et al. [13], Pallabazzer [43] and [44]], fire fighting systems (Rubin [14]), marine applications (Canham et al. [11]) and many other.

The introduction of minute quantities of high molecular weight polymers in the flow systems results in a phenomenon which has been universally termed "drag reduction phenomenon". The phenomenon of drag reduction by polymer additives was discovered by Toms [3]. The polymer solution offered less resistance to flow in turbulent conditions, under constant pressure, than the solvent itself.

Hoyt [23] reviewed and discussed the drag reduction phenomena, While, Lumley ([18] and [19]) dealt with the drag reduction from a fluid mechanics viewpoint and proposed various mechanisms. Moreover, Virk [34] presented comprehensive experimental study and established important features relating to the physical mechanism of drag reduction. In addition, Berman [31] discussed the dynamics of drag reduction. Furthermore, Sellin et al ([40] and [41]) reemphasized some of the basic aspects of drag reduction and discussed the existing industrial applications. Meanwhile, den Toonder [21] presented the existing theories concerning drag reduction phenomenon.

Most of the experimental drag reduction studies conducted in smooth pipes yielding three types of data (Tam et al. [24]), namely:

1- Pressure drop versus flow rate.
2- Axial velocity profiles.
3- Dynamic data in the form of spectra.

The first type is the most common and is reported in almost all drag reduction studies. It is easy to measure but lacks the details needed to describe the complex behavior in turbulent flows. Velocity profiles provide information about the nature of the boundary layer near the wall of the pipe. Many researchers have correlated a change in the boundary layer thickness with the reduction in drag at the wall of the pipe [Bandyopahyay [32] and Gad-el-Hak [26]]. Detailed study of drag reduction is performed by examining the dynamic response of the velocity fluctuations at various radial positions within the duct [Walker and Tiederman [6] and Wei and Willmarth [48]].

A thorough review of the previous studies on the drag reduction phenomena and related aspects is given by Naoum [7]. However, the following paragraphs will contain a brief review of some closely related investigations to the present study.

Kato et al. [12] analyzed the turbulent pressure loss in the pipe flow of a dilute polymer solution. They confirmed theoretically that velocity profile for dilute polymer solutions is not in-parallel to that for Newtonian fluids due to the effect of drag reduction. Hassid and Poreh [45] used a simple turbulent energy model, with a variable damping parameter, $A_{\square}$ ($A_{\square}$ is a function of drag reduction amount) to describe the effect of high molecular weight polymers on the velocity profile and turbulent energy distribution in channel and pipe flows. Tiederman and Reischman [52] developed a procedure for predicting mean velocity profiles in drag reducing, as well as, Newtonian channel flows. The procedure based upon the eddy viscosity model of Cess [37], and it requires only pressure drop, flow rate and geometry information. McComb and Rabie [51] studied the effect of homogeneous polymer solution on the pipe velocity profiles. Their experimental results confirmed the existence of three regions in drag reducing turbulent flow in agreement with the results of Reischman and Tiederman [28] for drag reducing channel flows. These regions are: the viscous sublayer, the buffer layer, and the turbulent core region.

On the other hand, Tam et al. [24] developed a correlation to predict mean velocity distribution in flows of dilute polymer solutions. They correlate the velocity data of both water and dilute solutions of many polymer kinds, with concentrations from 10 to 200 PPMW (part per million by weight). Tam et al. correlation has the form:

$$U_p^+ = \frac{A_N}{\alpha} \ln y^+ + \left(\frac{B_N - \beta}{\alpha} + \Gamma \right) \qquad 1$$

Where, α, β, and Γ = unique functions of ε, $\varepsilon = u^* / \overline{U}$ or $\sqrt{f/2}$, $\overline{U}$ = bulk (average) velocity, and f = Fanning friction loss coefficient.

The above parameters α, β, and Γ have the forms

$$\alpha = -2.864 + 105.5\ \varepsilon - 654.4\ \varepsilon^2 \qquad 2$$

$$\beta = 24.98 - 679.9\ \varepsilon + 4140\ \varepsilon^2 \qquad 3$$

$$\Gamma = -396.1 + 1.629 \times 10^4\ \varepsilon - 1.665 \times 10^5\ \varepsilon^2, \quad \text{for } \varepsilon \le 0.05 \qquad 4\text{-a}$$

$$\Gamma = 2.2, \text{ for } \varepsilon \ge 0.05 \qquad 4\text{-b}$$

Equation 1 showed good agreement when compared with their measurements. However, this correlation is based on measurements in the range of Reynolds number from 4,000 to 15,000. This range is a small with respect to practical applications.

The non-parallelness of the velocity profile in the turbulent core is observed by den Toonder et al. [22] during the measurements of a turbulent pipe flow of a dilute polymer solution by laser Doppler anemometer (LDA). Moreover, Harder and Tiederman [25], in their measurements of the velocity profiles of the channel, found a slightly increased slope for the logarithmic law, for a polymer solution of Polyacrylamide (PAM) in water.

The objective of the present study is to develop and validate a general correlation for turbulent velocity profiles of dilute polymer solution taking into consideration the velocity profile to be in parallel (upward shifting) with the logarithmic law profile for the Newtonian fluids, as well as an additional slope due to modifying of the Prandtl's mixing length theory.

THEORETICAL ANALYSIS

Depending on the Prandtl's mixing length theory, the turbulent shear stress for axial flow of Newtonian fluid in a pipe can be expressed as

$$\overline{\tau}^{(t)}_{rz} = \rho\ L_N^2\ (du_z/dy)^2 \qquad 5$$

And $\quad L_N = K_N\ y \qquad 6$

Where, L_N = Prandtl's mixing length, K_N = universal constant for Newtonian fluids (usually in the range 0.38 to 0.43), and u_z = time-average axial velocity.

For turbulent flows of polymer solutions, the experimental evidence of many investigations [Squire et al. [53], Virk [33], Luchik and Tiederman [47], and den Toonder et al. [22]] leads to the deduction that there is an apparent thickening of the viscous sublayer. Hence, for such flows it is appropriate to rewrite Eq. 6 to hold the polymer additives effect (Tam et al. [24]) as

$$L_P = K_P\ y \qquad 7\text{-a}$$

or $\quad L_P = \alpha\ K_N\ y \qquad 7\text{-b}$

Where, L_P and K_P = modified Prandtl's mixing length and universal constant for polymer fluids, respectively, and $\alpha = K_P / K_N$ = the ratio of Prandtl's mixing length for polymer fluids to that for Newtonian ones.

Hence, Eq. 5 may then be rewritten, for polymeric fluids, as

$$\overline{\tau}^{(t)}_{rz} = \rho\ L_p^2\ (du_z/dy)^2 \qquad 8$$

Substituting Eq. 7-b in Eq. 8, then

$$\overline{\tau}^{(t)}_{rz} = \rho\ \alpha^2\ K_N^2\ y^2\ (du_z/dy)^2 \qquad 9$$

An alternative expression for $\overline{\tau}^{(t)}_{rz}$ in cylindrical coordinates is

$$\overline{\tau}^{(t)}_{rz} = \tau_w\ (1 - y/R) \qquad 10$$

Following the approach analogous to that adopted by Prandtl for Newtonian fluids, the right hand sides of Eqs. 9 and 10 were equated and the resulting expression integrated to give

$$U_P^+ = A_P \ln y^+ + B_P \qquad 11$$

Where $\quad A_P = 1/K_P = 1/(\alpha\ K_N) = (1/\alpha)\ A_N = \alpha'\ A_N \qquad 12$

And $\quad B_P = B_N + \Delta B \qquad 13$

Where, A_P and B_P = constants in the logarithmic law of the wall for the polymeric fluids.

Finally, Eq. 11 may be rewritten as

$$U_P^+ = \alpha'\ A_N \ln y^+ + B_N + \Delta B \qquad 14$$

In Eq. 14, parameter α' related to the slope variations, while parameter ΔB related to the upward shifting of the log law. The difference between the obtained correlation, Eq. 14, and the equations used by other investigators, is the existence of parameter α'.

At the limiting condition of Newtonian fluid (zero drag reduction), α' becomes unity and ΔB becomes zero, and Eq. 14 reduces to that for Newtonian fluids.

$$U_N^+ = A_N \ln y^+ + B_N \qquad 15$$

Figure 1 shows the schematic diagram representing the equations and parameters of the modified Prandtl's mixing length model used in the present study.

The practical application of the developed correlation, Eq. 14, requires the development of the relationship for the parameters α' and ΔB. There are many parameters affecting the polymeric velocity distribution. These parameters include, polymer kind, polymer concentration, degradation of the polymer, geometrical effects, and Reynolds number. Based on the dynamics of the drag reduction phenomenon in turbulent flow, it is reasonable to postulate that all of these parameters are related to the drag reduction amount DR. Hence, the representative parameter for all of these parameters is the amount of drag reduction DR. Details about the relation between these parameters and the percentage of drag reduction are given by Naoum [7]. A relationship exists between each of the two parameters α' and ΔB, and the drag reduction amount DR, i.e.

$$\alpha' = \alpha'(DR), \qquad \Delta B = \Delta B(DR) \qquad 16$$

Consequently, Eq. 14 may be rewritten as

$$U_p^+ = \alpha'(DR)\, A_N \ln y^+ + B_N + \Delta B(DR) \qquad 17$$

Now, it is needed to determine Eq. 16.

DATA USED IN THE PRESENT STUDY

In order to obtain values for the two parameters of polymeric velocity profile, α' and ΔB, used in the present model, a through review of the previous experimental studies of drag reducing polymer solutions was performed. These studies may be classified according to the flow conditions and polymer characteristics as follows, Table 1.

1. Polymer kind: the present study covered many polymer kinds. These are polyethylene oxide (PEO), polyacrylamide (PAM), guar gum (GG), xanthan gum (XG), sodium carboxymethylcellulose (CMC), polyisobutylene (PIB). Each polymer kind has different blends, and hence, different molecular weights.
2. Polymer concentration: several concentration ranges are covered in this study.
3. Geometry: since the law of the wall is universal (Head and Rechenberg [30]), then, the present study covered several geometries. These are, smooth pipes, rough pipes, and channels.
4. Reynolds number: for pipe flow Reynolds number ranged from 6,000 to 474,000. Meanwhile, for channel flow Reynolds number ranged from 2,857 to 22,000.

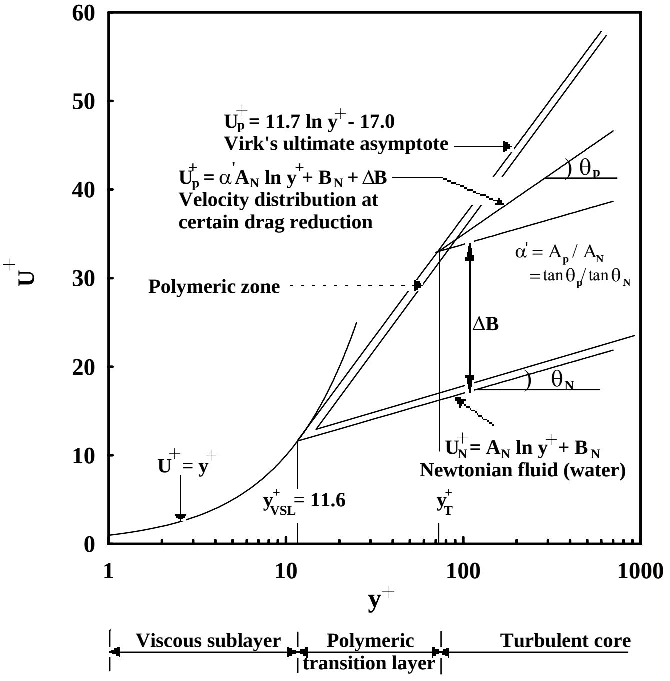

Fig. 1 Schematic diagram of the modified Prandtl's mixing length model used in the present study.

Table 1 contains detailed information regarding the previous investigations used in the present study. The data recorded in Table 1, are obtained as they were presented by their authors. These data are for turbulent core only. Hence, the data for the case of maximum drag reduction are not used, since the turbulent core at this case vanishes.

RESULTS AND DISCUSSIONS

The results obtained using data of investigators given in Table 1, will present graphically as a relation between the amount of drag reduction DR, and the two parameters α' and ΔB. The following is the procedure followed to convert the original data to a suitable data for the presentation.

1. The measured velocity in the turbulent core which was given by investigators in ($U^+ - y^+$) coordinates, is fitted by a straight line in the form, $U^+ = A_P \ln y^+ + B_P$. The slope of this fitted velocity profile ($A_P = \alpha' A_N$) and its intercept ($B_P = B_N + \Delta B$), then can be obtained. An example of obtaining the constants of polymeric velocity profile is shown in Fig. 2, which presents one of the polymeric velocity profiles obtained by McComb and Rabie [51]. In this figure, the values of the two velocity constants A_P and B_P, are 2.8 and 8.4, respectively, for DR = 26%. It is worth to note that the constants of velocity profile for Newtonian fluid are the values reported by McComb and Rabie [51] A_N = 2.5 and B_N = 5.5. Consequently, the values of α' and ΔB are 1.12 and 2.9, respectively.

2. The amount of percentage drag reduction DR% in the same measured velocity profile is obtained by two ways, as follows:

- In the case of availability of the amount of percentage drag reduction DR% [Reischman and Tiederman [28], McComb and Rabie [51], and Luchik and Tiederman [47]], it was used as given originally by the investigators.
- For the case of missing the amount of percentage drag reduction DR% [Goren and Norbury [54], Ernst [50], and Wells et al. [5]], it was calculated using the investigators data as follows:

$$DR\% = \{(f_S - f_P) / f_S\}\times 100 \,|_{\text{at the same solvent Re}} \tag{18}$$

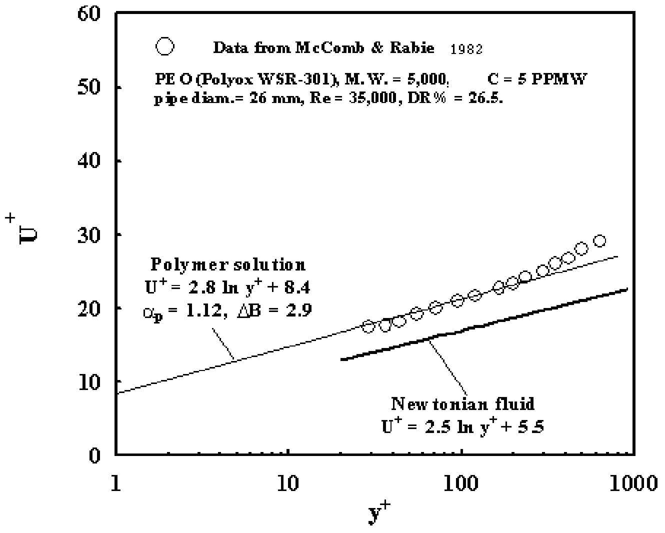

Fig. 2 Mean velocity profile as a function of drag reduction amount.

The friction loss coefficient, f_S, of the flow in pipes or channels if only solvent is flowing, can be obtained from Blasius equation. Meanwhile, the friction loss coefficient, f_P, when polymer solution is flowing, can be obtained either from the presented charts of friction coefficient versus Reynolds number [Goren and Norbury [54] and Ernst [50]], or from the empirical equation developed by the investigators [Wells et al. [5], and Tomita [55]].

Following the procedure of conversion all the investigators data used in the present study, the results are presented graphically in Fig. 3 as a relation between the amount of drag reduction DR, and the two parameters α' and ΔB. It is worth to note that in obtaining α' and ΔB, the values of A_N and B_N are used as they were reported by the original investigators in their papers.

Most of data of these investigators for Newtonian (solvent) flowing, have velocity constants of $A_N = 2.5$ and $B_N = 5.5$, but few of data [Seyer and Metzer [8], and Tam et al [24]] have values slightly lower or greater than the last ones. In the present study, the two parameters, α' and ΔB, are re-obtained based on the typical values of velocity constants (Benedict [42]), i.e., $A_N = 2.5$ and $B_N = 5.5$. This procedure is followed to allow the velocity correlation to be simple and easy to use. Then, the relation between the amount of drag reduction DR, and the two parameters α' and ΔB, is re-plotted in Fig. 4 based on the typical velocity constants.

It can be shown from Figs. 3 and 4, that the discrepancy between the results for each parameter is small. This is due to the fact that only few data have Newtonian velocity constants not equal to the values of typical ones, and the difference between the obtained and typical constants is small.

The scatter of the data presented in Fig. 4 is due to many factors. These include, uncertainty of the measurements, geometric variations as smooth pipes, rough pipes or channels, variations on the measuring techniques as Pitot tubes, hot film anemometer, bubble streak photography, and LDA, as well as error due to data processing to obtain α' and ΔB from given measuring data.

It is clear from Fig. 4 that in the range of drag reduction from 0 to 45%, there is a small variation in the value of α' beside an observed variation in the value of ΔB, compared with the Newtonian values ($\alpha' = 1$ and $\Delta B = 0$). However, when drag reduction exceeds 45%, the parameter α' increases rapidly, while the parameter ΔB decreases rapidly. This is due to the growth of the viscous sublayer thickness which begins to occupy most of the pipe cross section. Finally, when the drag reduction reaches its maximum value (~ 82%), the viscous sublayer filling the pipe cross section and the turbulent core vanishes as it is seen in Fig. 1.

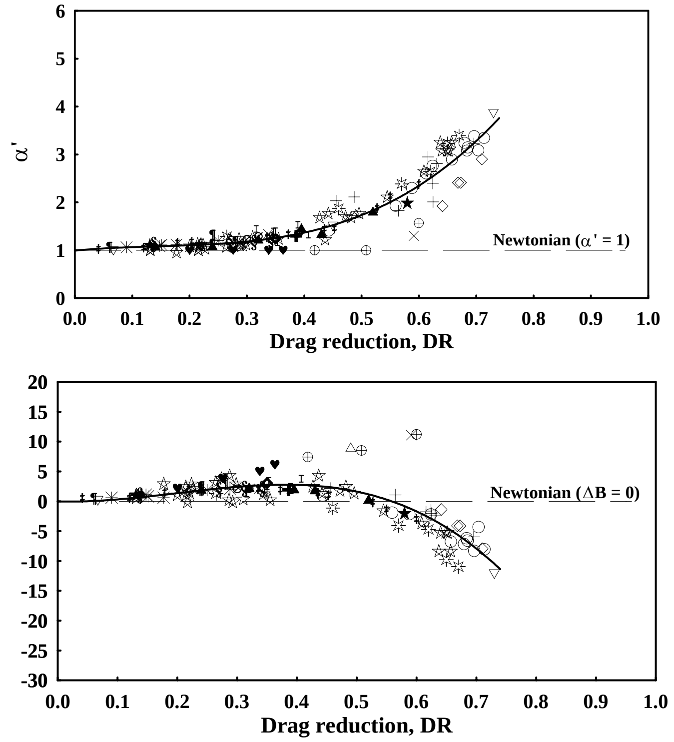

Fig. 3 The parameters α’ and ΔB as functions of drag reduction DR. Constants of Newtonian velocity as obtained by the investigators. Symbols are in Table 1.

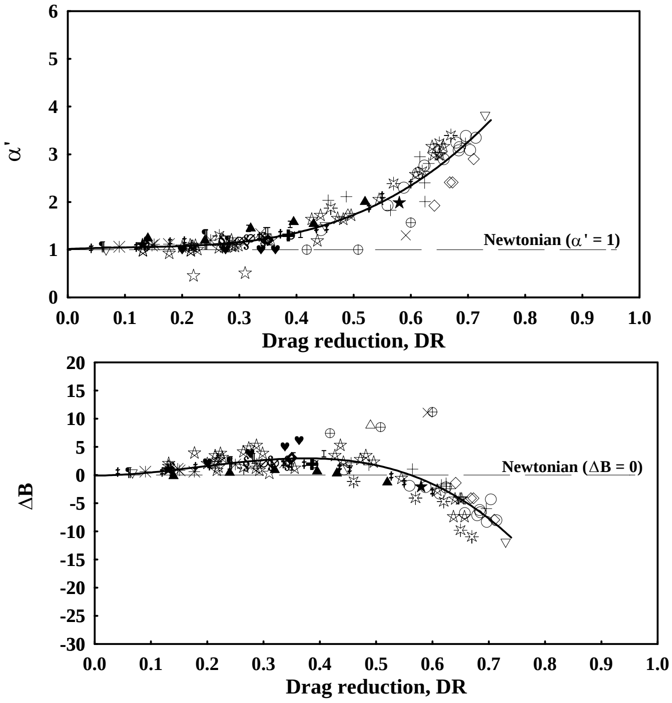

Fig. 4 The parameters α’ and ΔB as functions of drag reduction DR. Constants of Newtonian velocity ($A_N = 2.5$ and $B_N = 5.5$). Symbols are in Table 1.

The two parameters α′ and ΔB can be fitted by polynomials using the method of least square fitting. The empirical equations for each of the parameters α′ and ΔB are suggested to be as follows:

$$\alpha' = 1/\alpha = 1 + 0.71\,(DR) - 4.04\,(DR)^2 + 10.9\,(DR)^3, \quad R^2 = 0.91 \qquad \text{19-a}$$

and $$\Delta B = -1.76\,(DR) + 72.64\,(DR)^2 - 127\,(DR)^3, \quad R^2 = 0.84 \qquad \text{19-b}$$

The extent of data scatter is reflected in the values of coefficient of determination, R^2, which suggests that each of the parameters can be adequately represented by the mathematical expressions as given by Eq. 19. The velocity profile of dilute polymer solution, i.e. Eq. 17, is presented schematically in Fig. 5 at different drag reduction amounts.

The usefulness of the proposed correlation, Eq. 17, for turbulent velocity of dilute polymer solutions rests on the fact that the turbulent velocity profile for any polymer solution can be predicted if only the two parameters Re and f_P are known, namely :

1. The (solvent) Reynolds number Re, that is, Reynolds number that depends on the thermophysical properties of the solvent fluid which is given by

$$Re = \rho_s\,\overline{U}\,D/\mu_s \qquad 20$$

Where, μ_s = solvent dynamic viscosity, ρ_s = solvent density.

Thus, the friction loss coefficient, f_S, of the flow in such geometry (pipe, channel and annulus) if only solvent is flowing, can be obtained.

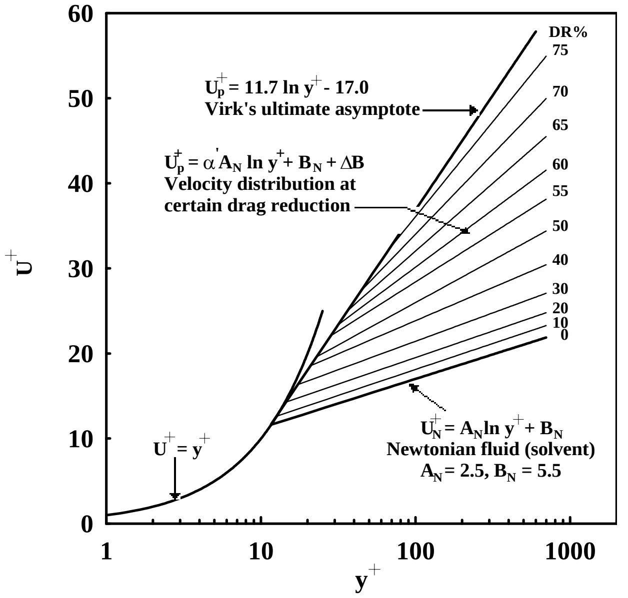

Fig. 5 Mean velocity profile, in case of drag reduction.

2. The friction loss coefficient, f_P, when polymer solution is flowing, at the same (solvent) Reynolds number in the same geometry.

Hence, the amount of drag reduction DR is obtained using Eq. 18.

The majority of the studies of drag reduction phenomenon included measurements of the flow rate and the pressure drop. These measurements allowed to obtain the bulk (average) velocity $\overline{U}$ (consequently Reynolds number Re), and the amount of percentage drag reduction DR%. With the aid of the proposed correlation obtained in the present study, these measurements can be converted into velocity distribution profiles. This add an additional information which can be used in understanding the drag reduction phenomenon, without any need to measure the velocity distribution.

COMPARISON OF EXPERIMENTAL VELOCITY DATA WITH THE PRESENT CORRELATION

Comparisons are made for several independent sets of experimental velocity profiles data obtained by previous investigators [Tomita [55] and Hassid and Poreh [45]], with the present developed correlation, i.e., Eq. 17. All data used for comparison did not use before to obtain the two parameters α′ and ΔB. The present correlation is compared with data of different polymeric solutions flowing in pipes and channels under different conditions.

1. **Pipe flow:** Figure 6 presents a comparison between the present correlation, and the experimental results of Tomita (1970) for 3.5 and 15 PPMW of polyethylene oxide PEO (Alkox-C) solution with drag reduction of 60 and 49%, respectively. The present correlation is in good agreement with the experimental results. Moreover, Fig. 6 shows two velocity asymptotes, the Virk’s asymptote which is the

classical asymptote that commonly used, and the Tomita's asymptote which developed to correlate his experimental results only. The difference between these two asymptotes is only in the values of velocity constants A_p and B_p.

2. **Channel flow:** the comparison between the present correlation and the experimental results of Reischman (Given by Hassid and Poreh [45]) for drag reduction of 35.3% obtained by 100 PPMW of polyethylene oxide PEO (Polyox WSR-301) solution flowing in rectangular channels, is presented in Fig. 7. The calculated velocity profile for the Reischman data, resulting from the theoretical model developed by Hassid and Poreh [45], is also plotted in this figure for comparison. It seems that the present correlation, in general, well predicts the experimental data. However, in this figure, it is worth to note that the turbulent core profile of Hassid and Poreh is not in-parallel with the Newtonian one, which confirms the present model.

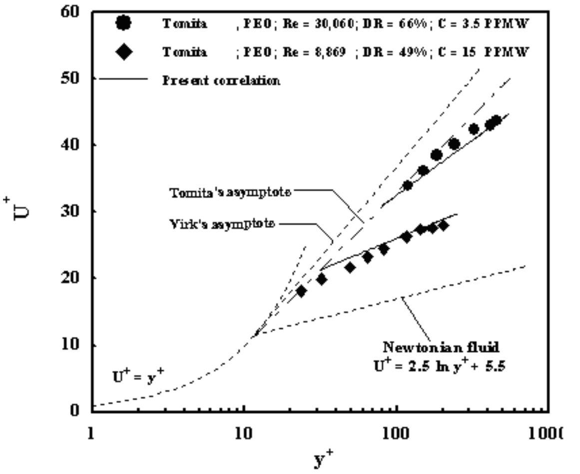

Fig. 6 Comparison with experimental data of Tomita [55].

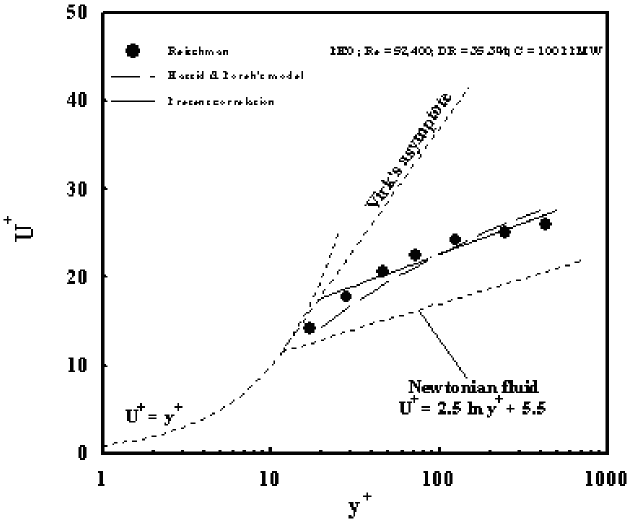

Fig. 7 Comparison with experimental data of Reischman (Given by Hassid and Poreh [45]).

Finally, good agreement is evident from Figs 6 and 7 suggesting that the correlation developed in the present study provides a simple method for predicting the turbulent velocity profile for polymer solutions, and can be applied for Reynolds number higher than the values covered by previous investigators.

CONCLUDING REMARKS

In the present study a general correlation, based on Prandtl's mixing length theory, of the form

$U_P^+ = \alpha' A_N \ln y^+ + B_N + \Delta B$ is developed as a simple means of predicting the velocity data for any dilute polymer solution. Where the two parameters ΔB and α' present the action of drag reducing polymer additives, by upward shifting the logarithmic velocity profile for dilute polymer solutions (ΔB parameter) with an additional slope (α' parameter). The present study suggested that each of these two parameters is function of the amount of drag reduction DR. The following concluding remarks are drawn from the present study:

1. The developed correlation is simple to use, and needs only the knowledge of the amount of drag reduction.
2. The proposed correlation is general, that is, for the special case when DR = 0, the correlation yields the equation for Newtonian fluids (non-drag reducing fluids).
3. Using the proposed correlation the turbulent velocity profile for any polymer solution can be predicted from the pressure drop/flow rate measurements.
4. Good agreement was obtained between independent sets of experimental data and predictions of the velocity profile using the proposed correlation.

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Table 1 Summary of turbulent velocity profile measurements on polymer solutions used in the present study.

Symb.	Investigator (s)	Pipe or channel size, D (mm)	Concentration, C (PPMW)	Reynolds number, Re	Drag reduction DR (%)	Measurement technique	Polymer blend	M.W. $\times 10^{-6}$
+	Wells [4]	16.51	500-1,000	33,200-101,000	56.5-69.6	Pitot tube	GG (J-2P)	1.7
		36.32	1,000	108,000-235,000	45.6-62.4			
×	Elata et al. (1966)*	50.7	400; 800	474,000; 430,000	33.7; 59.1	Pitot tube	GG (--)	1.7
♥	Ernst [49]	16.51	500	69,900; 211,000	27.6; 36.3	Pitot tube	CMC (--)	0.7
		36.25	500	98,000; 459,000	20; 33.8			
✳	Ernst [50]	19.33	500	27,200-111,000	14.8-28.8	Pitot tube	CMC (--)	0.7
		38.16	500	33,500-68,000	9-17.7			
◇	Goren and Norbury [54]	50.8	2.5-20	147,500-151,000	64.1-71.0	Pitot tube	PEO (Polyox WSR-301)	4
◆	Virk et al. [35]	32.1	1,000	180,000	35	Pitot tube	PEO (Polyox WSR-N-3000)	0.69
§	Wells et al. [55]	19.33	500	12,060-108,900	13.7-34.2	Pitot tube	CMC (--)	0.7
¶	Patterson and Florez [10]	25.4	2,000-4,000	27,000-55,000	6-28	Hot film anemometer	PIB (Vistanex L-200)	4
▽	Seyer and Metzner [8]	25.4	100	13,500-144,000	6.7-45	Bubble streak photography	PAM (ET-597)	3.7
			1,000	31,500	73			
★	Spangler [17]	19.33	31	11,400; 157,000	13; 58	Pitot tube	PAM (Polyhall P-295)$^{+}$	5 ÷ 6
‡	Tomita [55]	24.82	0.5-15	7,828-25,790	4.1-60	Pitot tube	PEO (Alkox-C)	6
⊕	Rollin and Seyer [2]	25.4	100	34,400; 80,000	50,8; 60	Bubble streak photography	PAM (Separan AP-30)	3
		69.8	100	97,600	41.8			
△	Rudd [27]	12.7^{S}	100	50,000	49	LDA	PAM (Separan AP-30)	3
I	Reischman and Tiederman [28]	25.4×305^{C}	100	3,459-8,119	24-40.7	LDA	PAM (Magnifloc 837-A)	4
			100	10,983	35.3		PEO (Polyox WSR-301)	4
			100	2,857-5,680	31.6-38.9		PAM (Separan AP-273)	4÷6

Table 1 Continued.

Symb.	Investigator (s)	Pipe or channel size, D (mm)	Concentration, C (PPMW)	Reynolds number, Re	Drag reduction DR (%)	Measurement technique	Polymer blend	M.W. × 10^{-6}
○	Mizushina and Usui [46]	19.9	30-100	51,700-97,800	62.4-71.4	LDA	PEO (--)	4.57
		25.3	20-100	11,500-38,000	44.3-69.7			
✳	McComb and Rabie [51]	26	5	35,000	26.5-67	LDA	PEO (Polyox WSR-301)	5
☆	Luchik and Tiederman [47]	25 × 250^{C}	1.3-2.1	17,800-22,000	22-31	LDA	PAM (Separan AP-273)	4÷6
✫	Tam et al. [24]	9	10-200	6,000-15,000	13.1-65.7	LDA	PEO (Polyox coagulant); PAM (Separan AP-30); PAM (Separan AP-302); XG (Keltrol)	5; 4; 8; 3.7
∅	Wei and Willmarth [48]	25.73 × 304.8^{C}	10	11,776-12,628	30-32	LDA	PEO (Polyox WSR-301)	4
▲	Bewersdorff and Thiel [15]	50 (smooth)	20	30,000; 50,000	39.5; 52	LDA	PAM (Separan AP-45)	4÷5
		50 (rough, d-type)	20	30,000; 50,000	32; 43			
		50.5 (rough, k-type)	20	30,000; 50,000	14; 24			
✚	den Toonder et al. [22]	40.37	20	23,281	38.6**	LDA	PAM (Superfloc A-110)	6÷8

PEO = polyethylene oxide; PAM = polyacrylamide; GG = guar gum; XG = xanthan gum; CMC = sodium carboxymethylcellulose;
PIB = polyisobutylene (non-aqueous solution, dissolved in cyclohexane).
Polyhall^{+} = an anionic copolymer of polyacrylamide and polyacrylic acid.
“ * ” = data from Virk et al. [36]; “ ** ” = the amount of percentage drag reduction DR% is recalculated according to Eq. 18; “M.W.” = molecular weight;
“(--)” = no data found; “ ^{C} ” = channel; “ ^{S} ” = square pipe.