POISEUILLE FLOW OF FLUID WHOSE VISCOSITY IS TEMPERATURE DEPENDENT

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ABSTRACT

We investigate fluid flow between two fixed parallel horizontal plates. The fluid is assumed to depend exponentially on temperature.

We also assume that the fluid is reacting according to the Arrhenius law. We non-dimensionalize both the momentum and energy equations and the steady state equations were solved analytically and numerically in the limit of large activation energy.

We prove that the steady state problem has a solution using the shooting method technique. It is shown that the constant parameters have appreciable impact on the solutions. A major result of the paper is the existence of two solutions for the velocity equation. Graphs feature prominently in the thesis.

1.0 INTRODUCTION

The hydrodynamics equations may be obtained as a first approximation to the Boltzman equation [1]. Poiseuille flow constitutes a very simple class of parallel flows in fluid mechanics with many applications in mathematical modeling of several biological and engineering systems. This type of flow normally occurs between two parallel planes due to an imposed constant and pressure gradient or uniform motion of both planes. (Batchelor [2].)

The improvement in the thermal system as well as energy utilization during the convection in any fluid is one of the fundamental problems of the engineering processes, since improved thermal systems will provide better material processing, energy conservative and environmental effects. [Makinde, (3)].

Zheng, Garcia and Alder [4] investigated kinetic theory and hydrodynamics for pressure-driven Poiseuille flow.

They examined the failure of the acceleration driven Poiseuille flow to reproduce the central minimum in the temperature profile and a non-constant pressure profile, which are both predicated by kinetic theory and observed in numerical simulations down to Knudsen numbers of 10^{-2}. They did not observe such failure in the pressure driven poiseuille flow.

Also, extensive work has been carried out on the subject for various shapes of the cross-section of the thermal explosions, Vityazer (2004), Gainutdinov (2001), Merzhanor and Abramor (1981). Hence the study of hydrodynamic and thermal explosion within a channel is very important from a practical point of view.

In this paper, we investigate the properties of the velocity profile of a fluid whose viscosity depends strongly on temperature.

1.0 MATHEMATICAL MODEL

We consider the flow of an incompressible viscous fluid between two parallel plates

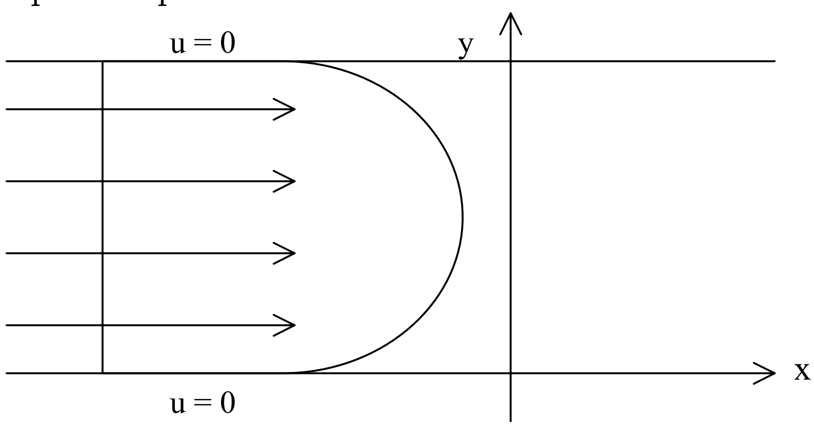

Fig (1) Poiseuille Flow

The equations governing the motion of fluid are:

Momentum Equation

$$\rho\left(\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x} \qquad \ldots(1)$$

Energy Equation

$$\rho c_p\left(\frac{\partial T}{\partial t} + V_0 \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(K \frac{\partial T}{\partial y}\right) + Qe^{\frac{-E}{RT}}, \qquad \ldots(2)$$

where ρ is density, μ is viscosity, C_p is heat capacity, u and V_o are velocity components along x and y axis respectively, T is the temperature, P is pressure, K is thermal conductivity, x is the Co-ordinate in the direction of flow, E is the Activation energy, R is the universal gas constant and Q is heat released per unit mass during reactions.

The boundary and initial condition of the flow are.

$$\left.\begin{aligned} &u(h,t) = u(-h,t) = 0 \\ &u(y,0) = 0 \\ &T(h,t) = T_o \\ &T(-h,t) = T_o \\ &T(y,0) = T_o \end{aligned}\right\} \qquad \ldots(3)$$

We assume a temperature dependent viscosity

$$\mu = \mu_0 e^{\alpha (T - T_o)} \qquad \ldots(4)$$

2. NON – DIMENSIONALIZATION.

Let $\theta = (T - T_o)\dfrac{E}{RT_o^2}$

$\phi = \dfrac{u}{V_o}$, $\bar{y} = \dfrac{y}{h}$, $\bar{t} = \dfrac{t}{t_o}$

Where t_o is a reference time

Then, equations (1) and (2) become (dropping "—")

$$\frac{\partial \phi}{\partial t} + a\frac{\partial \phi}{\partial y} = b\frac{\partial}{\partial y}\left(e^{\lambda\theta}\frac{\partial \phi}{\partial y}\right) + M \qquad \ldots(5)$$

$$\frac{\partial\theta}{\partial t} + a\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} + fe^{\frac{\theta}{1+\epsilon\theta}} \qquad \ldots(6)$$

where $a = \frac{v_o t_o}{h}$, $b = \frac{t_o \mu_o}{h}$, $M = \frac{-t_o}{\rho V_o}\frac{\partial p}{\partial x}$, $d = \frac{K t_o}{\rho}$

$\lambda = \alpha\left(\frac{RT_o^2}{E}\right)$ and $f = \frac{t_o E\, Q e^{\frac{-E}{RT_o}}}{\rho C_p RT_o^2}$

Initial conditions are

$$\left.\begin{aligned}\theta(y,0) &= 0\\ \phi(y,0) &= 0\end{aligned}\right\} \qquad \ldots(7)$$

Boundary conditions are

$$\left.\begin{aligned}\theta(-1,t) &= 0\\ \phi(-1,t) &= 0\\ \theta(1,t) &= 0\\ \phi(1,0) &= 0\end{aligned}\right\} \qquad \ldots(8)$$

3. STEADY CASE

We assume that the fluid properties and the variables of this flow are independent of time i.e $\frac{d}{dt} = 0$

Then, we have

$$a\frac{d\phi}{dy} = b\frac{d}{dy}\left(e^{\lambda\theta}\frac{d\phi}{dy}\right) + M \qquad \ldots(9)$$

and

$$a\frac{d\theta}{dy} = d\frac{d^2\theta}{dy^2} + fe^{\frac{\theta}{1+\epsilon\theta}} \qquad \ldots(10)$$

As $\epsilon \to 0$, $\frac{\theta}{1+\epsilon\theta} = \theta$

Now, let $a = 0$, $b = 1$, $\epsilon = 0$, $M = 1$, $d = 1$ and $f = \delta$; the two equations become

$$\frac{d}{dy}\left(e^{\lambda\theta}\,\frac{d\phi}{dy}\right) + 1 = 0 \qquad \ldots(11)$$

$$\phi\,(-1) = \phi\,(1) = 0 \qquad \ldots(12)$$

$$\frac{d^2\theta}{dy^2} + \delta e^{\theta} = 0 \qquad \ldots(13)$$

$$\theta(-1) = \theta\,(1) = 0 \qquad \ldots(14)$$

From equation (13). (Buckmaster and Ludford (1982)

$$\theta = 2\ln\left[e^{\frac{\theta m}{2}}\ \text{sech } cy\right] \qquad \ldots(15)$$

$$e^{\theta} = e^{\theta m}\ \text{sech}^2\, cy, \qquad \ldots(16)$$

$$\sqrt{f/2} = e^{\frac{\theta m}{2}} \cosh^{-1}\left(e^{\frac{\theta m}{2}}\right)$$

$$c^2 = \tfrac{1}{2}\delta\, e^{\theta m} \qquad \ldots(17)$$

and $\delta = 0.4$

CASE I, $\lambda = 1$

Equation (12) becomes

$$e^{\theta m}\frac{d}{dy}\left(\text{sech}^2\, cy\,\frac{d\phi}{dy}\right) = -1$$

differentiate;

$$e^{\theta m}\left\{\text{sech}^2\, cy\,\frac{d^2\phi}{dy^2} + \frac{d\phi}{dy}\cdot\frac{d}{dy}\,(\text{sech}^2\, cy)\right\} = -1$$

$$\phi'' - 2\tanh cy\,\phi' = -\cosh^2 cy\, e^{-\theta m} \qquad \ldots(18)$$

$$\phi(-1) = \phi(1) = 0$$

CASE II, $\lambda = ½$

Equation (12) becomes

$$e^{\frac{\theta m}{2}}\,\frac{d}{dy}\left\{\text{sech}^2\, cy\,\frac{d\phi}{dy}\right\} = -1$$

differentiate,

$$e^{\frac{\theta m}{2}}\left\{\text{sech } cy\,\underline{d^2\phi} + \underline{d\phi}\,(-\tanh cy\ \text{sech } cy)\right\} = -1$$

dy² dy

$$\phi'' - \tanh cy\, \phi' = -\cosh cy\, e^{\frac{-\theta m}{2}} \qquad \ldots(19)$$

$$\phi(-1) = \phi(1) = 0$$

4. METHOD OF SOLUTION AND RESULT

We resolve equation (18) and (19) in system of equation and we solve them numerically by using shooting method with their boundary condition.

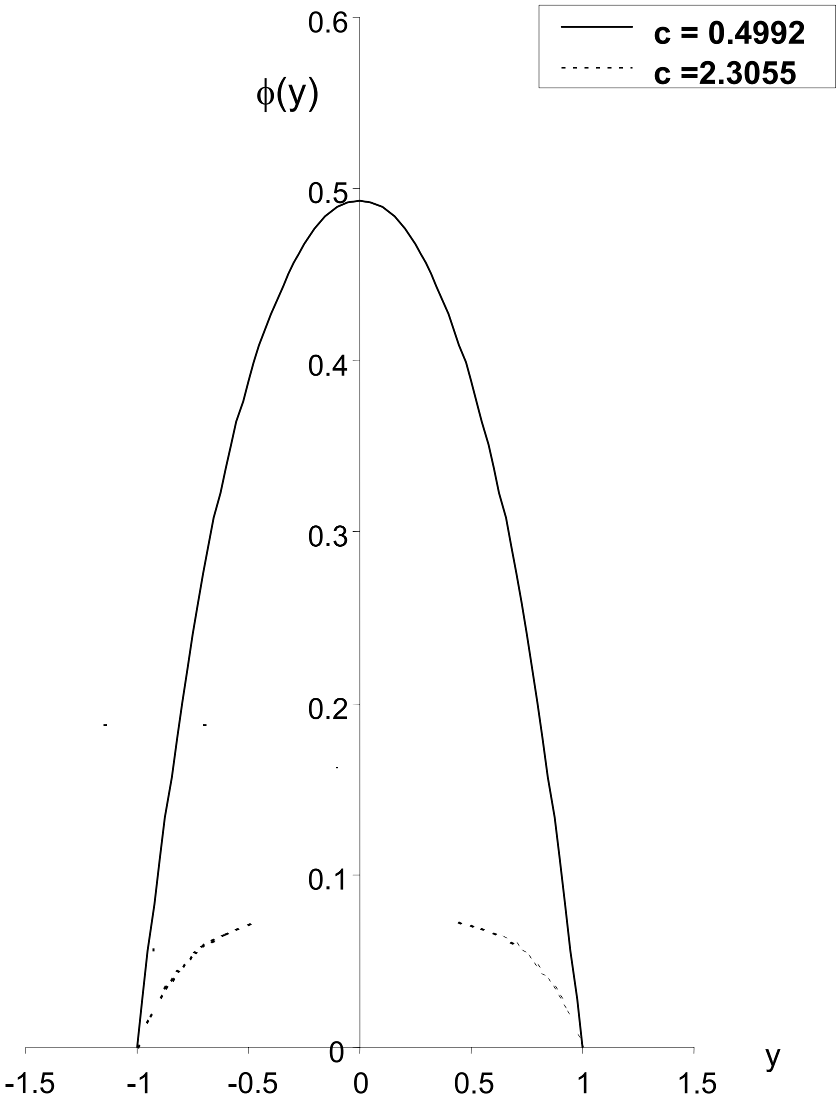

Fig 1: The velocity profile for $\theta_m = 0.22$, c = 0.4992 and for $\theta_m = 3.28$, c = 2.3055 when $\lambda = 1.0$

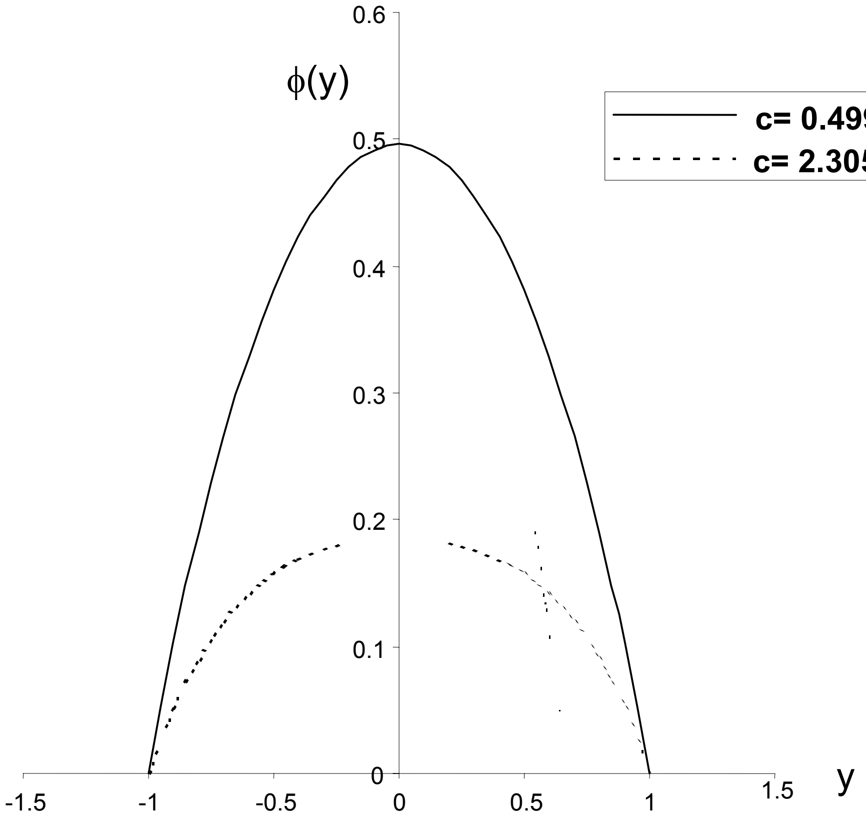

Fig. 2: The velocity profile for $\theta_m = 0.22$, c = 0.4992 and for $\theta_m = 3.28$, c = 2.3055 when $\lambda = 0.5$

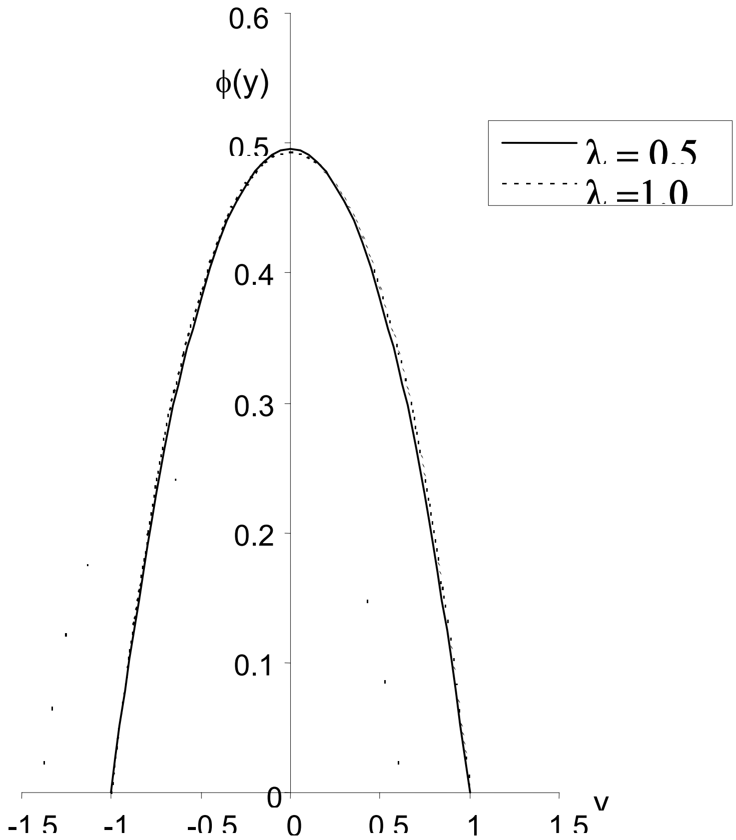

Fig. 3: Velocity profile for various λ when $\theta_m = 0.22$, c= 0.4992

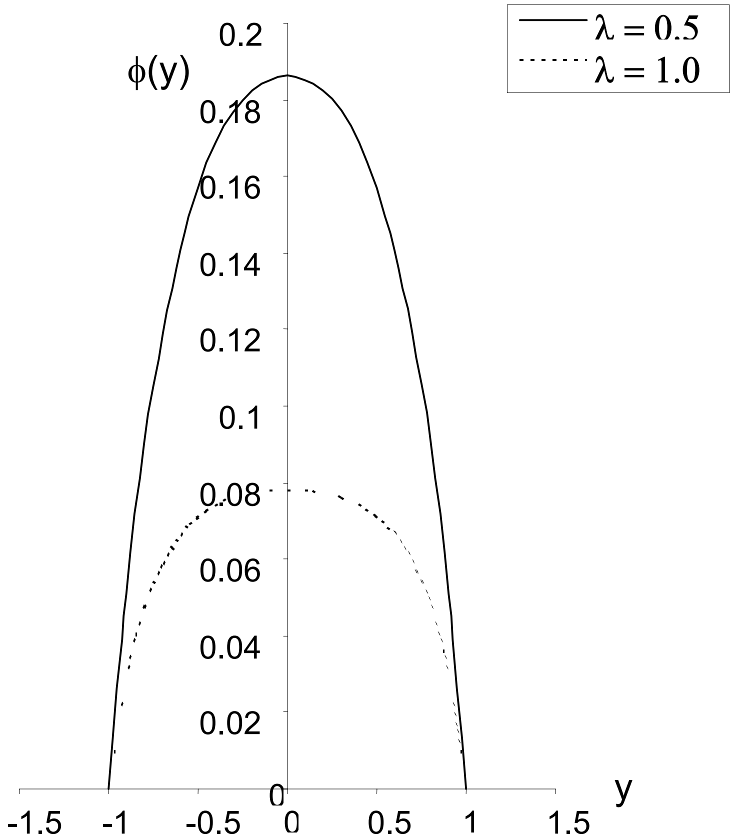

Fig. 4: Velocity profile for various λ when θ_m = 3.28, c= 2.3055

5. CONCLUSION

It has been shown (Buckmaster and Ludford) that the temperature θ has two solutions. We investigated the behaviour of the velocity when viscosity μ depends exponentially on temperature θ. The existence of two velocity solutions is just discovered here. From our results, it shows that the smaller value of maximum temperature [θm] corresponds to the higher value of velocity. The graphs show that both velocity and temperature reaches the maximum at the middle of the channel (y = 0).

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