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TURBULENCE MODEL FOR THE BOUNDARY LAYER TRANSITION USING LRN MODELS

A.Rahbari
Department of Mechanical Engineering;
Iran University of Science and Technology; Tehran; Iran

ABSTRACT

The predicted location of boundary layer transition is very sensitive to the initial profiles of turbulence quantities and starting location of calculation. To eliminate these effects, a solution approach is proposed in order to solve the boundary layer equations over a flat plate with the starting location of calculation very close to the leading edge of the plate. Computations show that this approach leads to identical results. In this research, well-known low-Reynolds-number (LRN) turbulence models are evaluated with respect to the transition on a flat plate. The obtained results from this formulation are compared with the experimental results and the accurate models are chosen. Also these models illustrate the better compatibility in comparison with other LRN models published in the literature.

KEYWORDS: Turbulence Model, Boundary Layer Transition, LRN Models, Flat Plate.

1 INTRODUCTION

Predicting the onset of turbulent flow is a critical component of many engineering and environmental flows. The characteristics of laminar and turbulent boundary layers are so different that the precise location of this relatively abrupt transition can have a profound influence on the overall drag, heat transfer, and performance properties of devices that operate in the transitional regime. The prediction of boundary layer transition is complicated by the fact that it doesn't correspond very directly to the onset of instability. Stability analysis for boundary layers is well developed and very predictive of the behaviour of small disturbances. However, the instabilities go through a series of complex nonlinear and three-dimensional processes before turbulence itself develops. So transition of flow from the laminar to turbulent regime causes a significant increase in skin friction and heat transfer coefficients. Transition affects the boundary layer separation. The turbulent boundary layer has a greater tendency to remain attached to the surface compared with the laminar boundary layer. Even if a good prediction of the laminar and fully turbulent boundary layer can be made, the prediction is not very useful without a reliable method for predicting transition. Thus, an accurate prediction of transition is invaluable in the calculation of fluid flow and heat transfer and in the resulting design of gas turbines. Transition is influenced by many factors such as free-stream turbulence intensity (FSTI), pressure gradient, surface roughness, streamline curvature, compressibility, and heat transfer.

Mayle [1] has shown that, among these factors, the FSTI and the pressure gradient are the most important ones. Accurate prediction of the following three aspects of transition is important for use in practical applications: (1) the location of the onset of transition, (2) the location of the end of transition, and (3) the manner in which the properties of interest in engineering applications. An alternative approach is to use the turbulence model itself to predict the transition location. This is a very natural approach in bypass transition and prediction the free-stream turbulence. A number of studies of boundary layer bypass transition prediction using low Reynolds number k–ε models have been performed, and [1] gave good reviews of how various flavours of the k–ε models perform. It is noticed that two characteristics of LRN model make it a good candidate for prediction of transition. First, such a model is applicable to laminar, transitional, and turbulent flow regimes without additional modifications. Second, the influence of free-stream turbulence intensity (FSTI) is naturally accounted for through the boundary conditions for k and ε. Moreover, the predicted location of transition is quite sensitive to the initial profiles of k and ε and starting location of calculation. This sensitivity appears to decrease when the starting location of calculation shifts toward the leading edge of the plate. Among the methods for predicting transition, solving the boundary layer equations with the LRN k-ε turbulent model is the most common approach. The first version of the LRN k-ε model was introduced by Jones and Launder [2]. They added the so-called LRN functions to the standard k-ε model such that the near wall damping effects were simulated. Jones and Launder [3] used their LRN model to successfully predict the Reynolds number where pipe flows became turbulent.

Early attempts to simulate the boundary layer transition include the work of Pridden [4], Wilcox [5], Dutoya andMichard [6], Arad and colleagues [7], Hylton and colleagues [8], Wang and colleagues [9], and Rodi and Scheuerer [10]. However, in all these cases, a satisfactory prediction of transition is achieved by using very specific initial profiles of turbulence quantities at a specific starting location of calculation.

Fujisawa [11] performed calculations with several LRN models. He showed that the Launder and Sharma [12] model performs better than the others for flow over a flat plate without a pressure gradient but fails in situations with a pressure gradient. However, he did not report the sensitivity of results to the initial profiles and starting location. Schmidt and Patankar [13] were the first to extensively study the sensitivity of the predictions to both the initial profiles of *k* and e and the starting location of calculation. They showed that the predicted location of transition is quite sensitive to these factors. This sensitivity appears to decrease when the starting location of calculation shifts toward the leading edge of the plate. However, with these early starting positions, the LRN models predicted transition at unrealistically early locations and significantly shorter than those found experimentally.

Schmidt and Patankar [14] introduced empirical constants and functions into the LRN models to adjust the beginning and end of transition. These modifications were very specific to the problem of a two-dimensional boundary layer and were adjusted to force the results to conform to the empirical correlations for flat plates developed by Abu-Ghannam and Shaw [15]. Using these modifications, Schmidt and Patankar predicted some of the flat plate data very well but experienced difficulties with more complex situations, such as accelerated flow. Abid [16] also showed that the results of his new LRN *k-ε* turbulence model depended on the choice of initial profiles and starting locations. Only when he made these choices in a specific manner was he able to get good agreement with experimental data. Savill [17] reports results achieved by the European Research Community on Flow Turbulence and Combustion (ERCOFTAC) to evaluate the performance of turbulence models in predicting transition over a flat plate under the influence of free-stream turbulence. Although the sensitivity of results to the starting location and initial profiles were shown in this report, no effort was made to obtain a result independent of these factors.

Wilcox [18] used the *k-ε* model to predict the transitional flow. He set the initial profile of *k* to zero throughout the boundary layer and introduced a new concept known as the numerical roughness strip. By changing the width of the numerical roughness strip he could trigger the transition at the desired location.

Biswas and Fukuyama [19] developed a new LRN model and evaluated it along with the other models against experimental data for flat plates and turbine blades. Except for one of the turbine blade cases, they obtained quite satisfactory results. However, they did not address some of the details in their calculations. They adopted Rodi and Scheuerer’s [10] suggestion for initial profile of ε. and *k*, they stated that they used the experimental profile. They did not define the shape of this profile.

In this study, two well-known LRN turbulence models are evaluated with respect to the transition on a flat plate. The obtained results from this formulation are compared with the experimental results and the accurate model is chosen. Also these models illustrate the better compatibility in comparison with other LRN models published in the literature.

2 LRN MODELS

2.1 Yang-Shih k-ε

Yang and Shih (1993) postulated a new time scale concept. By using this time scale to reformulate the dissipation equation, the singularity in the standard dissipation equation is removed and the equation can be integrated to the wall. This overcomes the deficiency of using near-wall pseudo-dissipation rate since the definition of the pseudo-dissipation rate is quite arbitrary. This model uses R_y instead of y^+ as its independent variable in the damping function. This model is governed by the following equations:

$$\upsilon = C_\mu f_\mu k T_t \qquad (1)$$

Where the time scale expression is:

$$T_t = \frac{k}{\varepsilon} + \sqrt{\frac{\upsilon}{\varepsilon}} \qquad (2)$$

The dissipation function is:

$$f_\mu = \left[1 - \exp\left(-1.5\times10^{-4} Ry - 5\times10^{-7} R^3 y - 1\times10^{-10} R^5 y\right)\right]^{0.5} \qquad (3)$$

The turbulent kinetic energy and the dissipation rate equations can be written as:

$$\frac{Dk}{Dt} = P_k - \varepsilon + \frac{\partial}{\partial x_j}\left[\left(\upsilon + \frac{\upsilon_t}{\sigma_k}\right)\frac{\partial k}{\sigma x_j}\right] \qquad (4)$$

$$\frac{D\varepsilon}{Dt} = \left(C_{\in 1} P_k - C_{\in 2}\,\varepsilon\right)/T_t + 2\upsilon\upsilon_t\left(\frac{\partial^2 \upsilon_i}{\partial x^2{}_j}\right) + \left[\left(\upsilon + \frac{\upsilon_t}{\sigma_\in}\right)\frac{\partial_\in}{\partial x_j}\right] \qquad (5)$$

The boundary condition for ε is:

$$\varepsilon_w = \upsilon\frac{\partial^2 k}{\partial x^2{}_j} \qquad (6)$$

The coefficients are shown in Table 1.

TABLE 1. Yang-Shih k-ε Coefficients

C_θ	0.09
$C_{\in_1}$	1.44
$C_{\in_2}$	1.92
σ_k	1.0
	1.3

2.2 Abe-Kondoh-Nagano k-ε

This model is governed by the following equations:

$$\upsilon = C_\mu f_\mu \frac{k^2}{\varepsilon} \tag{7}$$

The dissipation function is:

$$f_\mu = \left[1-\exp\left(-\frac{y^*}{14}\right)\right]\left[1+\frac{5}{R_t^{3/4}}\exp\left(-\left(\frac{R_t}{200}\right)^2\right)\right] \tag{8}$$

The turbulent kinetic energy and the dissipation rate equations can be written as:

$$\frac{Dk}{Dt} = P_k - \varepsilon + \frac{\partial}{\partial x_j}\left[\left(\upsilon + \frac{\upsilon_t}{\sigma_k}\right)\frac{\partial k}{\sigma x_j}\right] \tag{9}$$

$$\frac{D\varepsilon}{Dt} = C_{\in 1}\frac{\varepsilon}{k}P_k - C_{\in 2} f_2 \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left[\left(\upsilon + \frac{\upsilon_t}{\sigma_\in}\right)\frac{\partial_\in}{\partial x_j}\right] \tag{10}$$

Where

$$f_2 = \left(1-\exp\left(-\frac{y^*}{3.1}\right)\right)\left[1-0.3\exp\left(-\left(\frac{R_t}{6.5}\right)^2\right)\right] \tag{11}$$

The boundary condition for ε is:

$$\varepsilon_w = \frac{2\upsilon k}{y^2} \tag{12}$$

The coefficients are shown in Table 2.

TABLE 2. Abe-Kondoh-Nagano k-ε Coefficients

C_μ	0.09
$C_{\in_1}$	1.5
$C_{\in_2}$	1.9
σ_k	1.4
	1.4

2.3 Abid k-ε

Myong and Kasagi (1990)'s model postulated the concept of two characteristic length scales for dissipation rate, one very close to the wall and the other remote from the wall. These length scales are then used in the turbulent momentum transport and the damping function for turbulent viscosity it thus obtained as:

$$f_\mu = \left(1+\frac{3.45}{\sqrt{R_T}}\right)\left[1-\exp\left(-\frac{y^+}{70}\right)\right] \tag{13}$$

Speziale, Abid and Anderson (1990)'s model sought to improve the MK model by introducing the hyperbolic tangent function $\tanh\left(y^+/70\right)$ instead of $\left[1-\exp\left(-y^+/70\right)\right]$ in the turbulent viscosity damping function.

2.4 Lan-Bremhorst , Launder-Sharma $k-\tilde{\varepsilon}$

The standard differential equations for continuity, momentum, and energy for a two-dimensional steady boundary layer are used. The turbulent shear stress is expressed using the Boussinesq eddy-viscosity hypothesis with turbulent viscosity defined as:

$$\mu_t = \rho C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}} \tag{14}$$

The LRN form of the turbulent kinetic energy and the dissipation rate equations can be written as:

$$\frac{Dk}{Dt} = \upsilon_t\left(\frac{\partial\upsilon_i}{\partial x_j}+\frac{\partial\upsilon_i}{\partial x_i}\right)\frac{\partial\upsilon_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\left(\upsilon+\frac{\upsilon_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \tilde{\varepsilon} - D \tag{15}$$

$$\frac{D\tilde{\varepsilon}}{Dt} = \frac{\partial}{\partial x_j}\left[\left(\upsilon+\frac{\upsilon_t}{\sigma_k}\right)\frac{\partial\tilde{\varepsilon}}{\partial x_j}\right] - f_2 C_{\varepsilon 2}\frac{\tilde{\varepsilon}^2}{k} \quad f_1 C_{\in_1}\frac{\tilde{\varepsilon}}{k}\upsilon_t\left(\frac{\partial\upsilon_i}{\partial x_j}+\frac{\partial\upsilon_j}{\partial x_i}\right)\frac{\partial\upsilon_i}{\partial x_j} + E \tag{16}$$

Where

$$\tilde{\varepsilon} = \varepsilon - D \tag{17}$$

where *D* is the dissipation of turbulence at the wall and is expressed in some models as a function of *k*. In the fully turbulent regime *D* vanishes. Therefore, $\tilde{\varepsilon}$ and *ε* become the same away from the wall. Use of $\tilde{\varepsilon}$ instead of *ε* is recommended because it allows the convenience of using $\tilde{\varepsilon} = 0$ as the wall boundary condition in the equation. Contained within this set of equations are the five empirical constants C_μ, C_1,C_2, σ_k and σ_ε and the five empirical functions f_μ, f_1, f_2, *D*, and *E*. These functions are expressed in LB and LS models in Table 3.

TABLE 3. The LRN functions used in the LB and LS models

	LS	LB
f_μ	$\exp\left[\frac{-3.4}{(1+\mathrm{Re}_t/50)^2}\right]$	$\left[1-\exp(-0.016\,\mathrm{Re}_y\right]^2$
f_1	1	$(0.05/f_\mu)^3$
f_2	$1-0.3\exp(-\mathrm{Re}_t{}^2)$	$1-\exp(-\mathrm{Re}_t{}^2)$
D	$2\mu\frac{\partial\sqrt{k}}{\partial x_i}\Big)^2$	0
E	$2\mu\mu_t\frac{\partial^2 u_i}{\partial x_i \partial x_k}\Big)$	0
$B.C.$	0	$\frac{\partial\tilde{\varepsilon}}{\partial y}=0$

3 FLAT PLATE BOUNDARY LAYER IN ZERO PRESSURE GRADIENT

The process of transition is studied by looking at the evolution of the friction coefficient along the streamwise direction. The friction coefficient is a very sensitive indicator of transition that increases dramatically as transition occurs. The model predictions are compared to experimental data with different turbulence intensities. The mean velocity is initially uniform flow for all cases and the initial values of velocity U_0, turbulence Reynolds number $\mathrm{Re}_T = k^2/\upsilon\varepsilon$, turbulence intensity level $T_u = (2/3)^{1/2}/U_\infty$ for for two experimental cases with different turbulence intensities are given in Table 4. The initial values of the turbulent kinetic energy k are determined using $k = 3/2(T_u.U_0)^2$.

The initial turbulent dissipation rate ε is calculated from Re_T using $\varepsilon = k^2/\upsilon\,\mathrm{Re}_T$ where the value of Re_T is assumed.

TABLE 4. Initial flow parameters

	Tu(%)	U(m/s)	Re_T
T3A	3	5.4	200
T3B	6	9.4	200

4 RESULTS

Figures (1) shows the generated mesh for zero pressure gradient (T3A,T3B). As seen in these figures, the obtained cells are more concentrated near the wall in order to better demonstrate the flow circumstances in this zone. The utilized mesh illustrates that the variation of y^+ with Re is always less than unity which is observed in Figure (2).

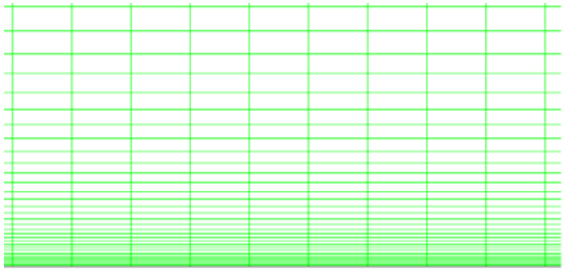

Figure 1. Generated mesh for T3A,T3B

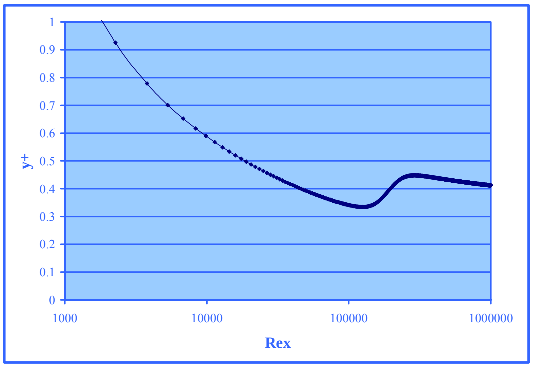

Figure 2. Variation of y^+ with Re_x

4.1 The Obtained Results for Test Case T3A

Figure (3) represents a result for skin friction coefficient as a function of Re_x for T3A. As observed, three well-known models for prediction of transition are compared with the experimental results. The comparison elucidates that among these models, YS and LB models predict the onset of transition in advanced in contrary to the LB model. Also it can be realized that the YS model has the better compatibility with the experimental data. In order to investigate the importance of the three models described above, other existing theoretical results for skin friction coefficient with Re_x are plotted in Figure (4). It is apparent that among all of these models, YS, LB and Abid models show the better adjustment in comparison with other models. As seen in this Figure the onset of transition anticipated with other models aren't acceptable. It is needed to notice that among these models, k-ε can't predict the onset of transition and LS and k-ω forecast the transition too sooner than the experimental result.

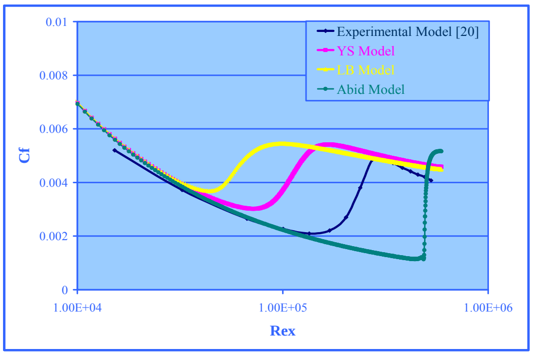

Figure 3. Variation of theoretical and experimental value of C_f with Re_x for test case T3A

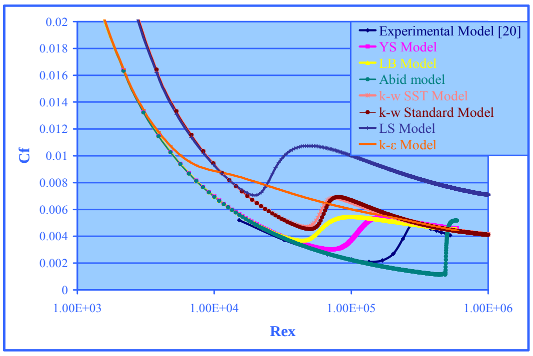

Figure 4. Variation of theoretical and experimental value of C_f with Re_x for test case T3A

4.2 The Obtained Results for Test Case T3B

Figures (5) manifests the variation of skin friction coefficient with Re_x. This Figure shows this variation for three YS, LB , Abid models with the experimental results. As perceived, Abid model has a great compatibility with experimental result. The qualitative characteristics of the variation of C_f during transition seem reasonable for all three models. A continuous and smooth increase in *Cf* from laminar to turbulent region can be observed. The LB prediction of onset of transition is too far upstream compared with the experimental data.

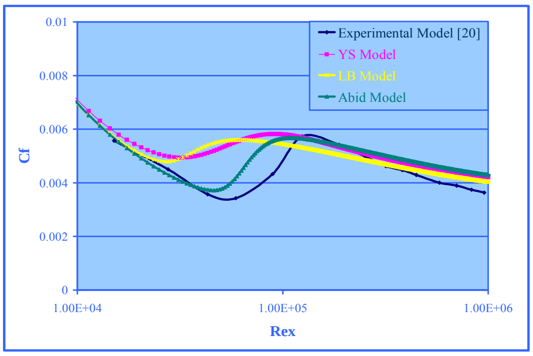

Figure 5. Variation of theoretical and experimental value of C_f with Re_x for test case T3B

Figure (6) claimes that Abid model is the best model in order to determine the skin friction coefficient in comparison with other theoretical results in this test case. Also some models such as k-ε and k-w anticipate the skin friction coefficient as a fully turbulent regime.

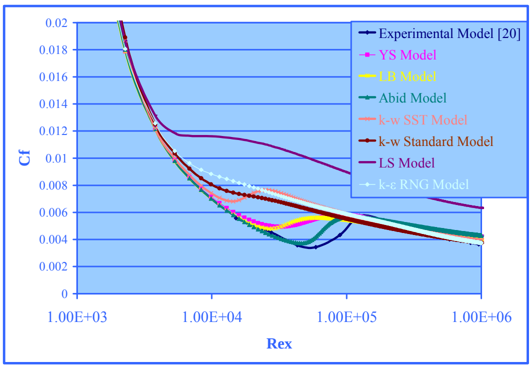

Figure 6. Variation of theoretical and experimental value of C_f with Re_x for test case T3B

6. DISCUSSION

Predicting the onset of turbulent flow is a critical component of many engineering and environmental flows. The characteristics of laminar and turbulent boundary layers are so different that the precise location of this relatively abrupt transition can have a profound influence on the overall drag, heat transfer, and performance properties of devices that operate in the transitional regime.

Accurate prediction of the following three aspects of transition is important for use in practical applications: (1) the location of the onset of transition, (2) the location of the end of transition, and (3) the manner in which the properties of interest in engineering applications.

In this research, well-known low-Reynolds-number (LRN) turbulence models are evaluated with respect to the transition on a flat plate. The obtained results from this formulation are compared with the experimental results and the accurate models are chosen. Also these selected models illustrate the better compatibility in comparison with other LRN models published in the literature. From the above analysis, following conclusions are derived:

1. LRN models are used in order to demonstrate and compare these models for prediction the transition on a flat plate.
2. The variation of C_f with Re_x for theoretical and experimental results is compared for four test cases.
3. The obtained result for T3A represents the great ability of YS model for predicting the skin friction coefficient in comparison with other theoretical models.
4. It is shown that the Abid model is the best model for the test case T3B due to it's harmony with the experimental data.

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